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A SAT Approach to Branchwidth

Neha Lodha[1], Sebastian Ordyniak, Stefan Szeider
[neha,ordyniak,sz]@ac.tuwien.ac.at

Problem Description

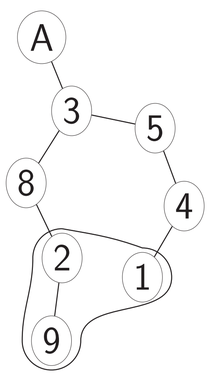
- Finding branch decompositions of small width using SAT
- Extending SAT approach to admit large instances

Branch Decompositions

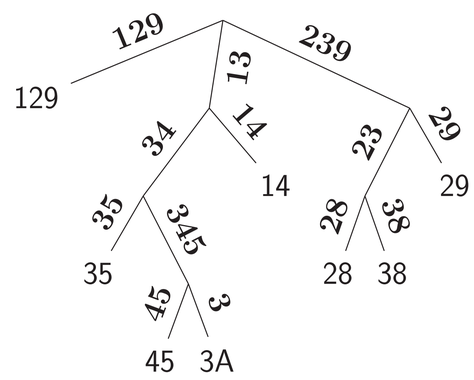
- Introdued by Robertson and Seymour, similar to tree decomposition
- Used for decomposing combinatorial objects
  - Graphs, Matroids, Integer-valued symmetric submodular functions, CNF formulas
- Problems solved efficiently using dynamic programming on branch decomposition of small width
  - Traveling salesman problem, #P-complete problem of propositional model counting, Generation of resolution refutations for unsatisfiable CNF formulas

Definition

Hypergraph G



Branch Decomposition B



Width =3

Branch Decomposition of a hypergraph is a ternary tree with bijection between the edges of the hypergraph and the leaves of the tree

Encoding Partition Based Characterization

Partition Based Characterization:

$\{\{129, 35, 45, 3A, 14, 28, 38, 29\}\}$

$\{\{129\}, \{35, 45, 3A, 14\}, \{28, 38, 29\}\}$

$\{\{129\}, \{35, 45, 3A\}, \{14\}, \{28, 38\}, \{29\}\}$

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- Equivalence classes: To encode refinement as underlying tree is implicitly represented by refinement of partitions
- Cut along the edges: Set a flag for every vertex that is a cut vertex
- Cardinality constraints: To bound the number of cut vertices, sequential counter

Output

For a hypergraph G and an integer k we produce formula $F(G, k)$ which is satisfiable iff G has a branch decomposition of width k.

Results

Single Encoding

Graph	$\|V\|$	$\|E\|$	bw
Watsin	50	75	6
Kittell	23	63	6
Holt	27	54	9
Shrikhande	16	48	8

Local Improvement using SAT

Graph	$\|V\|$	$\|E\|$	hbw	fbw	diff
inithx.i.2-pp	363	8897	55	45	10
graph13	458	1877	141	134	7
bn_31-pp	1148	3317	40	36	4
water-wpp	22	96	11	8	3

bw: branchwidth, hbw: branchwidth of heuristic decomposition, fbw: branchwidth after local improvement using SAT

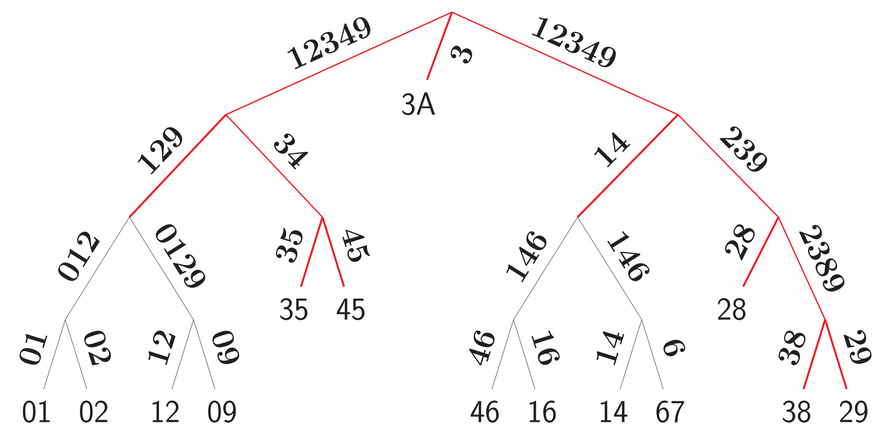
How to extend to large Instances

Use SAT locally!

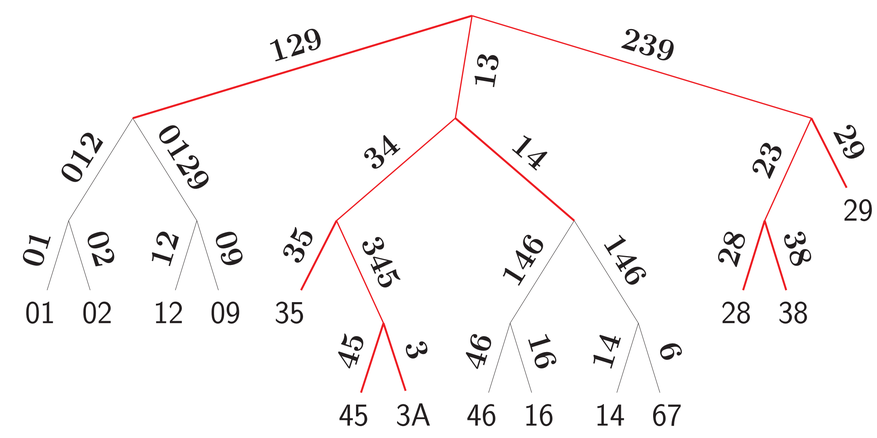
How?

1. Generate heuristic decomposition
2. Pick local branch decomposition around large cut (using specialized DFS procedure)
3. Use SAT to improve local branch decomposition and plug it back in

Repeat till no more improvement possible or timeout



Heuristic Decomposition



Improved Decomposition

Our Contribution

- SAT encoding for branchwidth based on new partition based characterization
- SAT-based local improvements for branch decompositions
  - Provides the means for scaling the SAT-approach to much larger instances
  - New application field of SAT solvers

Future Work

1. Extending the encoding to obtain specialized decomposition to aid local improvement
2. Encoding various other parameters such as boolean width, rank width (similar decomposition scheme)
3. Extending the branch decomposition approach to apply in field of knowledge compilation
4. Extending current approach with incremental and MAXSAT solving

References

- Robertson, Neil and Seymour, P. D., Graph minors X. Obstructions to tree-decomposition, (JCombTB 1991).
- Hicks, I.V., Branchwidth Heuristics, (Congressus Numerantium 2002)

[1]TU Wien. Neha Lodha acknowledges support by the Austrian Science Fund (FWF, projects W1255-N23 and P-27721).