Model Checking

Simone Bova

Algorithms and Complexity Group Retreat
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TU WIEN
ac ALGORITHMS AND COMPLEXITY GROUP

Area

- Logic in Computer Science

  LICS, CSL, . . .
- Database Theory

  PODS, ICDT, . . .
- Theory

  FOCS, STOC, ICALP, STACS, . . .

Problem

Problem	MODEL-CHECKING (MC)
Instance	A finite structure **A** and a logical sentence ϕ.*
Question	$\mathbf{A} \models \phi$?

*In this talk, ϕ is a first-order sentence.

Exercise (Board)

E is a binary relation symbol.

$\mathbf{A}$ is a digraph on $n+1$ vertices $\{0, 1, \ldots, n\}$ and arcs:

$$E^{\mathbf{A}} = \{(0,1), \ldots, (0,n)\}$$

$\mathbf{A} \models \exists y \forall x E(y,x)$?

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MC is in PSPACE and decidable in time

$$O(\mathrm{size}(\mathbf{A})^{\mathrm{size}(\phi)})$$

Example

$\mathbf{G} = (G, E^{\mathbf{G}})$ is a graph.
$E^{\mathbf{G}} \subseteq G^2$ symmetric irreflexive.

$\phi_k = \exists x_1 \cdots \exists x_k \bigwedge_{1 \leq i < j \leq k} (x_i \neq x_j \wedge \neg E(x_i, x_j))$.

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$\mathbf{G}$ contains k distinct nonadjacent vertices iff
$(\mathbf{G}, k) \in$ INDEPENDENT-SET

Algorithms and Complexity

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Algorithms and Complexity

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   Classify fixed-parameter tractable cases.

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If **A** is a gigantic database and ϕ a small query, then

$$f(\text{size}(\phi))\text{size}(\mathbf{A})^{O(1)}$$

is an interesting runtime.

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1. MC restricted to a fixed sentence. (Why?)
2. MC restricted to a fixed graph class and primitive positive logic is in P iff all graphs in the class are bipartite.

Fixed-parameter tractability results:

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†Closed under subgraphs.

‡Having bounded arity.

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Fixed-Parameter Tractability

Fixed-parameter tractability results:

1. MC restricted to a fixed graph class[†] is in FPT iff "the graph class is nowhere dense".
2. MC restricted to a fixed class of primitive positive sentences[‡] is in FPT iff "the treewidth of the cores of the sentences in the class is bounded".

[†] Closed under subgraphs.
[‡] Having bounded arity.

Thank you for your attention!