Knowledge Compilation

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Algorithms and Complexity Group Retreat
Semmering, June 29-May 1, 2015

Area

- Artificial Intelligence

  AAAI, IJCAI, KR, . . .
- Logic in Computer Science

  SAT, . . .
- Theory

  CCC, ICALP, STACS, . . .

Generalities

- FWF Project currently led by Stefan
  (supporting Ronald and Simone)
- International Workshop recently organized at TU
  (`http://www.vcla.at/kc2015`)

Compilation

$$\phi \longmapsto c(\phi)$$

where:

- ϕ represents some propositional knowledge encoded in some *source* language (say CNFs);
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Allow a computationally hard compilation map $c(\cdot)$, as long as $c(\phi)$ is "succinct" and "tractable".

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The high compilation cost is amortized,
by reusing the compiled knowledge over multiple queries.

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is outperformed by

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Theory

Let S and T be representation languages
(a *representation language* is a class of circuits).

S is *polysize compilable* into T (or T *is at least as succinct as S*) if for all $C \in$ S there exists $D \in$ T st:

- D is equivalent to C;
- D is polysize in C.

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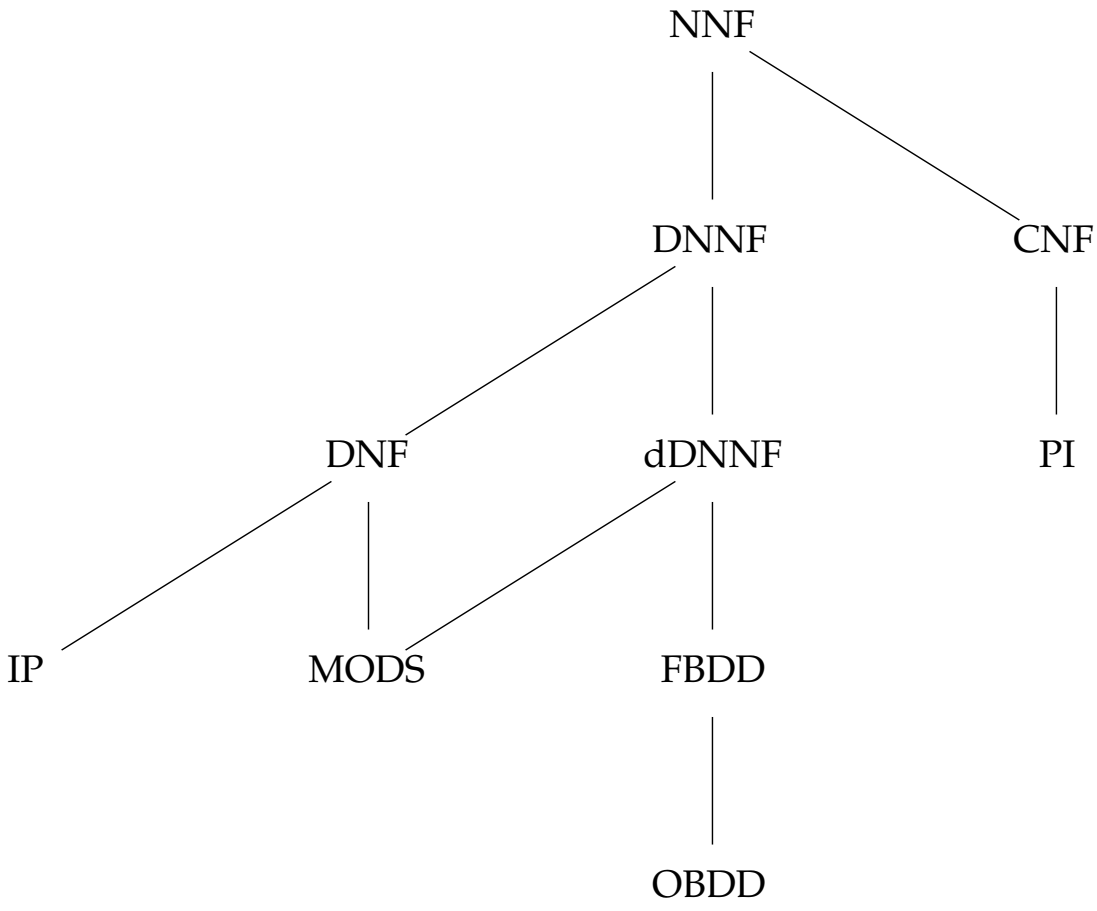
CNFs and OBDDs are incomparable in succinctness.

Representation Languages



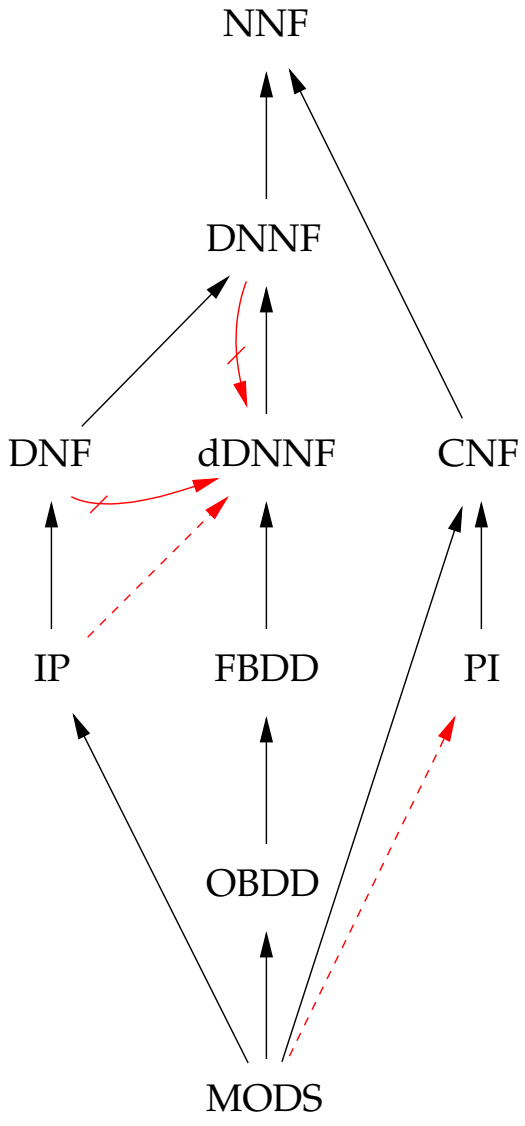
Inclusion relation on representation languages.

Succinctness Relation

The succinctness relation is (partially) established in a series of results:

- Quine (1959),
- Chandra and Markowsky (1978),
- Bryant (1986),
- Wegener (1987),
- Gergov and Meinel (1994),
- Gogic, Kautz, Papdimitriou, and Selman (1995),
- Selman and Kautz (1996),
- Cadoli and Donini (1997),
- Darwiche (1999), and
- B, Capelli, Mengel, and Slivovsky (2015).

Succinctness Relation



S $\dashrightarrow$ T means S $\rightsquigarrow$ T unknown. S $\not\to$ T means S $\not\rightsquigarrow$ T unless PH collapses.

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   - Separate SDDs and OBBDs,
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2. Find new languages challenging existing ones.
   - Find language supporting counting
     but incomparable to deterministic DNNFs.

Parameterizations

S is *fpt size compilable* into T wrt parameterization $\kappa\colon \mathrm{S} \to \mathbb{N}$ if for all $C \in \mathrm{S}$ there exists $D \in \mathrm{T}$ st:

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- Study the succinctness/tractability tradeoff in the parameterized setting.
- Find language supporting fpt size compilation of functions wrt *expression width* (smallest treewidth over equivalent circuits).

Thank you for your attention!