Spectral Graph Theory and Algorithms, Summer 2017

Course Information

19 June 2017

1 Instructor Information

Instructor:	Prof. David P. Williamson
Office:	3.317
Office hours:	By appointment
Email:	`dpw@cs.cornell.edu`

2 Course content

This course will consider connections between the eigenvalues and eigenvectors of graphs and classical questions in graph theory such as cliques, colorings, cuts, flows, paths, and walks. Both older structural results and recent algorithmic results will be presented. Topics to be covered include the matrix-tree theorem, the Colin de Verdière invariant, Cheeger's inequality, Trevisan's max cut algorithm, Laplacian solvers, electrical flow and its applications to max flow, and (possibly) spectral sparsifiers.

3 Course website

The course website will be at `https://moodle.univie.ac.at/course/view.php?id=64274`. Various materials from the course will be posted there.

4 Prerequisites

There is no formal prerequisite. In practice, I will be assuming some previous exposure to linear algebra, and either to algorithms or combinatorial optimization, and some ability to do mathematical proofs. If you've had some linear algebra and a good undergraduate algorithms class that had proofs about the algorithms, you should be set. Please talk to me if you have questions about whether you have the necessary background.

5 Course materials

There is no required textbook. I taught a semester-long version of this course at Cornell last fall, and will be drawing on materials from it; you can see notes here at `https://people.orie.cornell.edu/dpw/orie6334/`. I will put appropriately modified versions of these notes on our course website as the class progresses.

In addition, there have been a number of excellent courses taught elsewhere on this topic, and I have drawn heavily on the lecture notes posted from these classes. I will point out where I am drawing my material from as the course proceeds. Some of these resources are listed below.

- Lap Chi Lau, University of Waterloo, CS 798, Algorithmic Spectral Graph Theory, Fall 2015, `https://cs.uwaterloo.ca/~lapchi/cs798/`.

- Luca Trevisan, Stanford University, CS 359G, Graph Partitioning and Expanders, Winter 2011, `http://theory.stanford.edu/~trevisan/cs359g/`.
- Luca Trevisan, UC Berkeley, Lecture Notes on Expansion, Sparsest Cut, and Spectral Graph Theory, `https://people.eecs.berkeley.edu/~luca/books/expanders.pdf`.
- Dan Spielman, Yale University, CS 662, Spectral Graph Theory, Fall 2015, `http://www.cs.yale.edu/homes/spielman/561/`.
- Nisheeth Vishnoi, EPFL, $Lx = b$, `http://research.microsoft.com/en-us/um/people/nvishno/site/Lxb-Web.pdf`.
- Chris Godsil and Gordon Royle, *Algebraic Graph Theory*, Springer, 2001. `http://link.springer.com/book/10.1007%2F978-1-4613-0163-9`.
- Dragoš Cvetković, Peter Rowlinson, Slobodan Simić, *An Introduction to the Theory of Graph Spectra*, Cambridge University Press, 2009, `https://www.cambridge.org/core/books/an-introduction-to-the-theory-of-graph-spectra/16366DE4108AE3DEE0E8690ED16`

6 Requirements

There will be one problem homework set released per day starting with the second class. Students are strongly encouraged to attempt the problems as they are the best way to learn the material.

7 Collaboration

Your work on problem sets should be your own. You may discuss approaches to problems with other students. You may write up solutions in groups of at most two. You should acknowledge anyone with whom you discussed the problem by writing their names on your problem set. I suggest that you do not use papers or books or other sources (e.g. material from the web) to help obtain your solution, as this is the best way to learn the material for yourself.

8 Schedule

Here is a rough schedule for the course, which will is subject to change without notice. All lectures will be held in Oskar-Morgenstern Platz 1, Room 3.307. Depending on the background of the class, I may go either faster or slower than this schedule indicates.

Montag 19.06.	10:00 - 12:00	An introduction to spectral graph theory. Some eigenvalue basics.
Dienstag 20.06.	13:30 - 15:30	Raleigh quotients, spectral theorems, the Perron-Frobenius theorem
Mittwoch 21.06.	10:00 - 12:00	Bipartite graphs. Interlacing: clique and chromatic numbers. Laplacians.
Donnerstag 22.06.	10:00 - 12:00	Laplacians, algebraic connectivity, the matrix-tree theorem.
Freitag 23.06.	10:00 - 12:00	Planarity and the Colin de Verdière invariant. Cheeger's inequality.
Montag 26.06.	10:00 - 12:00	Trevisan's MAX CUT algorithm.
Dienstag 27.06.	13:30 - 15:30	Electrical flows.
Mittwoch 28.06.	10:00 - 12:00	A simple Laplacian solver.
Donnerstag 29.06.	10:00 - 12:00	Multiplicative weights and max flows in undirected graphs.
Freitag 30.06.	10:00 - 12:00	Spectral sparsifiers.

9 Your information

Please fill out the information below and return it by the end of the lecture.

Name _____

Preferred email address _____

I have studied linear algebra, Yes or No:

I have already studied some amount of spectral graph theory (Yes/No; if Yes, how much):

I would like to see the following material covered: