Spectral Graph Theory and Algorithms, Summer 2017 28 June 2017

Problem Set 7

Due Date: 29 June 2017

1. Let $\mathcal{E}(f)$ be the energy of s-t electrical flow f in a graph G. Let p be the associated potentials. Recall that $\mathcal{E}(f) = p^T L_G p$, and that $p(s) - p(t) = r_{\text{eff}}(s,t)$. Prove that for all vectors $x \in \Re^n$ for which $x(s) - x(t) = r_{\text{eff}}(s,t)$,

$$p^T L_G p \leq x^T L_G x.$$

   That is, the potentials p minimize the quadratic form $x^T L_G x$ among all vectors such that the potential difference $x(s) - x(t) = r_{\text{eff}}(s,t)$.

2. (Rayleigh's Monotonicity Principle) Given a graph G, let $\mathcal{E}(f,r)$ be the energy of a flow for supply vector b under resistances r. Let f be the electrical flow for supply vector b under resistances r, and let f' be the electrical flow for the same supply vector b under resistances $r' \geq r$. Prove that $\mathcal{E}(f',r') \geq \mathcal{E}(f,r)$.