Spectral Graph Theory and Algorithms, Summer 2017 June 21, 2017

Problem Set 2

Due Date: June 22, 2017

1. Let $\lambda_1(M) \leq \lambda_2(M) \leq \cdots \leq \lambda_n(M)$ be the eigenvalues of any matrix M. The *Courant-Weyl* inequalities state that for symmetric real matrices A and B,

$$\lambda_i(A+B) \leq \lambda_j(A) + \lambda_{i-j+n}(B)$$

for $1 \leq i \leq j \leq n$ and

$$\lambda_i(A+B) \geq \lambda_j(A) + \lambda_{i-j+1}(B)$$

for $1 \leq j \leq i \leq n$.

Prove the inequalities. (Hint: recall the proof of the interlacing theorem. Now we need to think about three different vector spaces, for $A+B$, A, and B).