Subexponential-time complexity and the ETH

Ronald de Haan

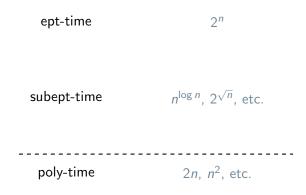


Poly-time, subept-time, ept-time

ept-time 2^n

poly-time $2n, n^2$, etc.

Poly-time, subept-time, ept-time



Poly-time, subept-time, ept-time

ept-time	2 ⁿ
subept-time	$n^{\log n}$, $2^{\sqrt{n}}$, etc.
poly-time	$2n, n^2$, etc.

Subexponential time

Subexponential time: $2^{o(n)}$ time, where:

a function is o(n) if it can be expressed as $\frac{n}{s(n)}$ for some unbounded, nondecreasing computable function s

Example:
$$\sqrt{n} = o(n)$$
, because $\sqrt{n} = n/\sqrt{n}$

FTH

Exponential Time Hypothesis (ETH): 3SAT is not solvable in subexponential time.

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- ▶ 3-colorability needs time $2^{O(n)}$
- k-clique needs time $n^{O(k)}$

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Variant: Strong ETH; use to show optimality for quadratic time algorithms.