Subexponential-time complexity and the ETH

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Poly-time, subept-time, ept-time

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poly-time	$2n$, n^2, etc.

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Subexponential time

Subexponential time: $2^{o(n)}$ time, where:

a function is $o(n)$ if it can be expressed as $\frac{n}{s(n)}$ for some unbounded, nondecreasing computable function s

Example: $\sqrt{n} = o(n)$, because $\sqrt{n} = n/\sqrt{n}$

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Variant: Strong ETH; use to show optimality for quadratic time algorithms.