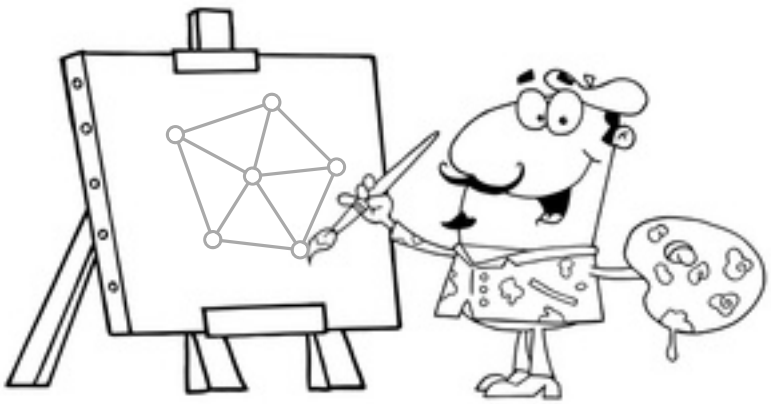
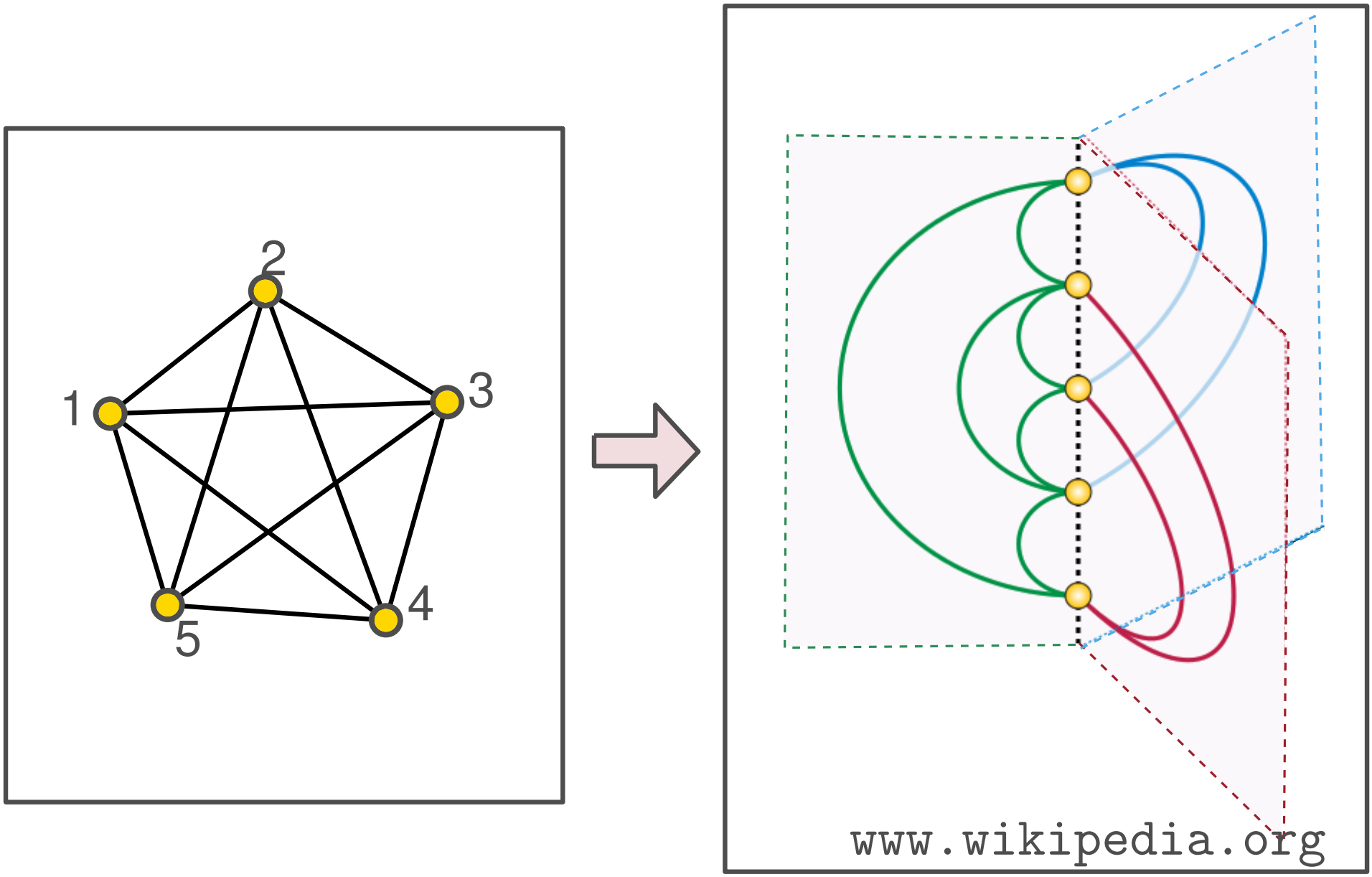
23rd International Symposium on
Graph Drawing & Network Visualization
September 24-26, 2015, Los Angeles, CA, USA

GD challenge 2015: crossings in book embeddings



vs

Book Embeddings



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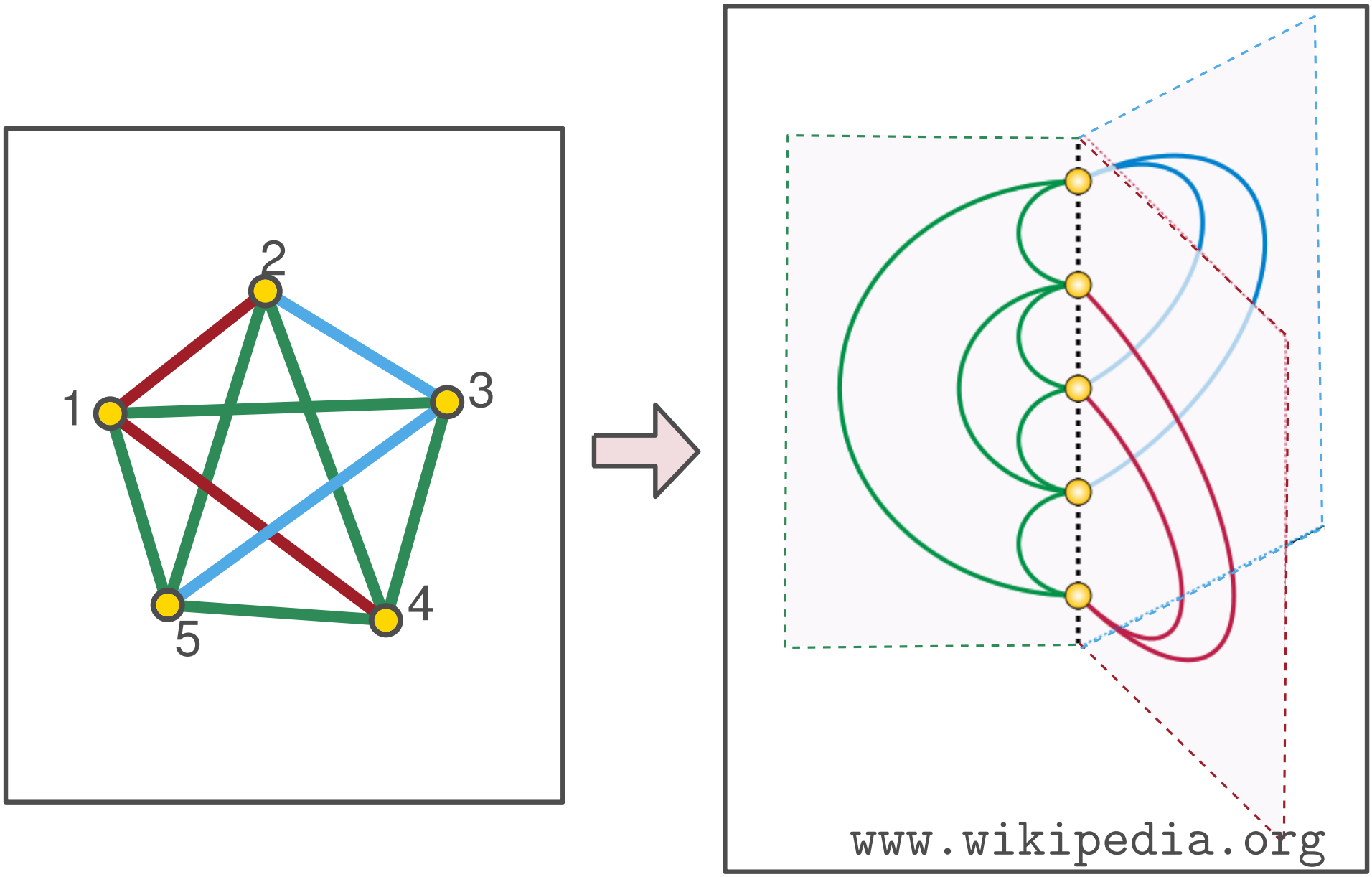

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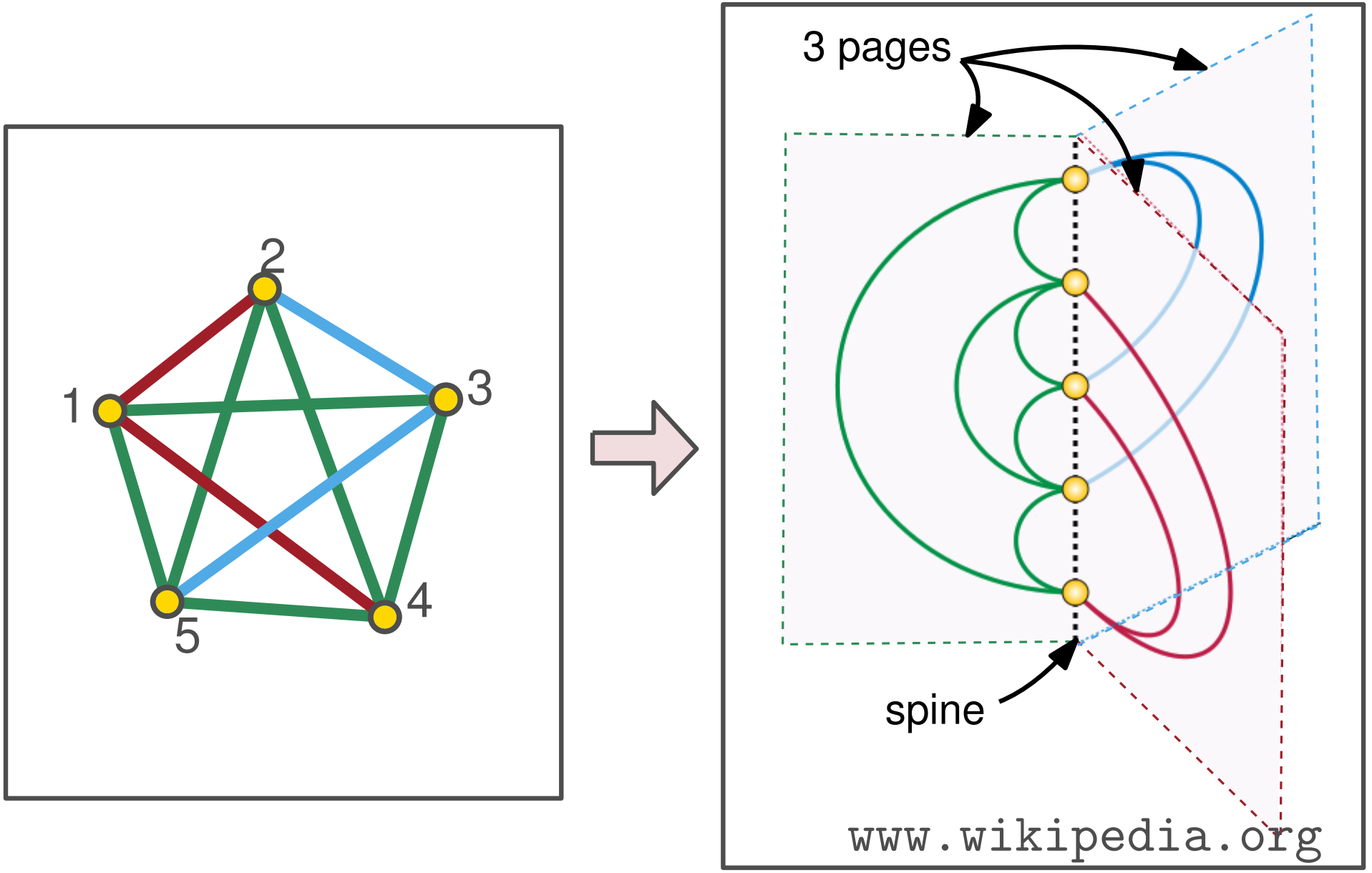
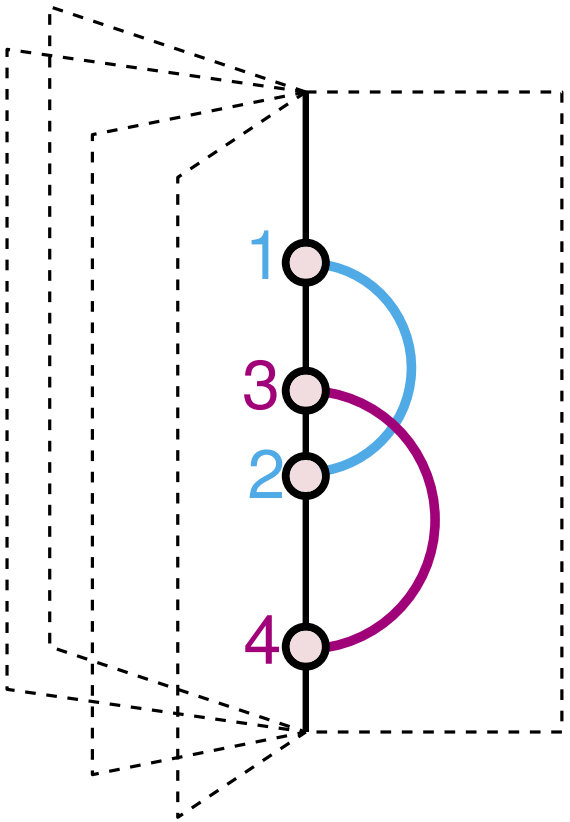


Book Thickness and Crossing Minimization



Can I always get a crossing-free drawing?

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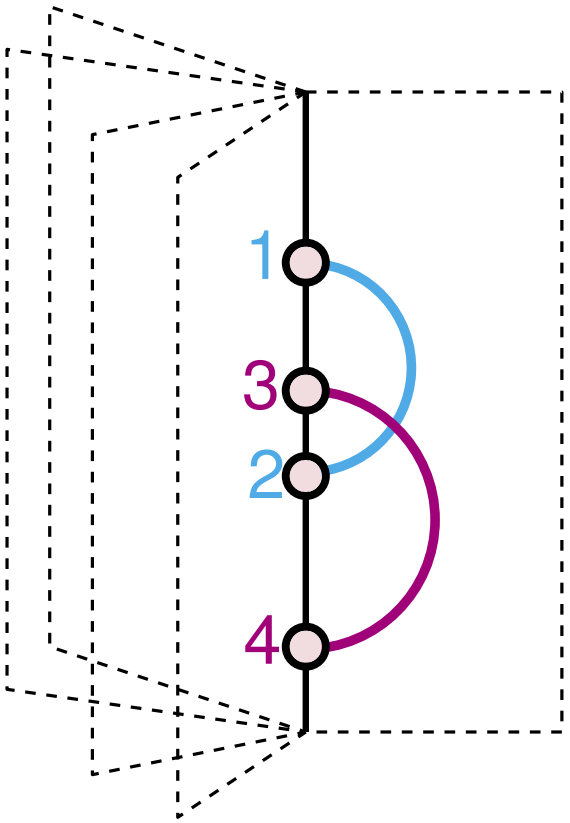

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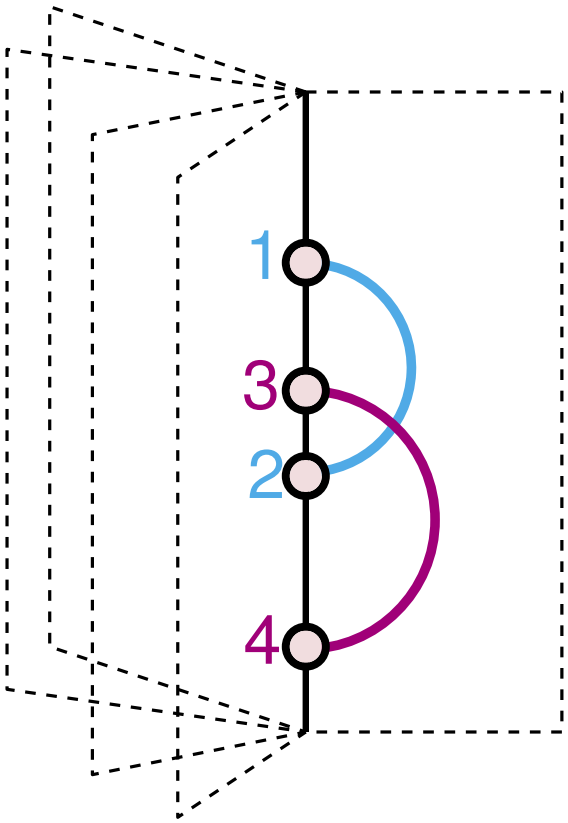

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The smallest k s.t. G has a crossing-free book embedding with k pages is the **book thickness** or **page number** of G

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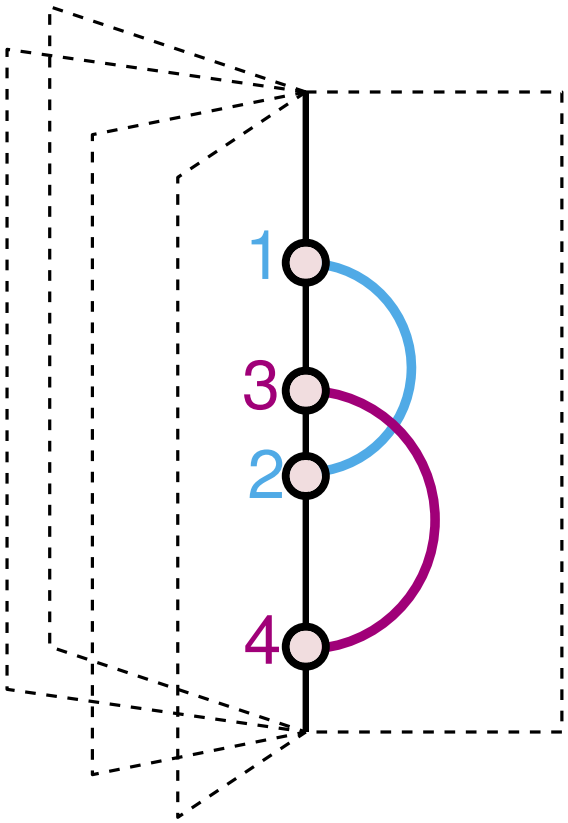

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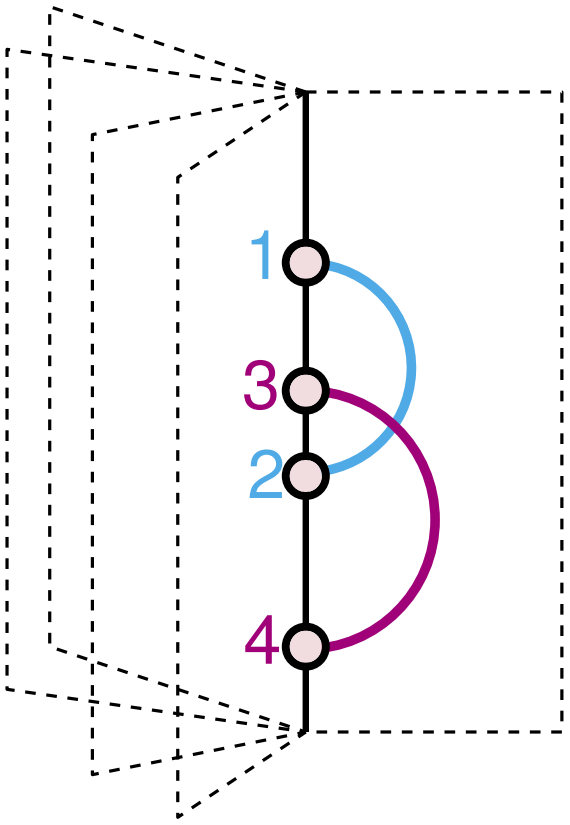

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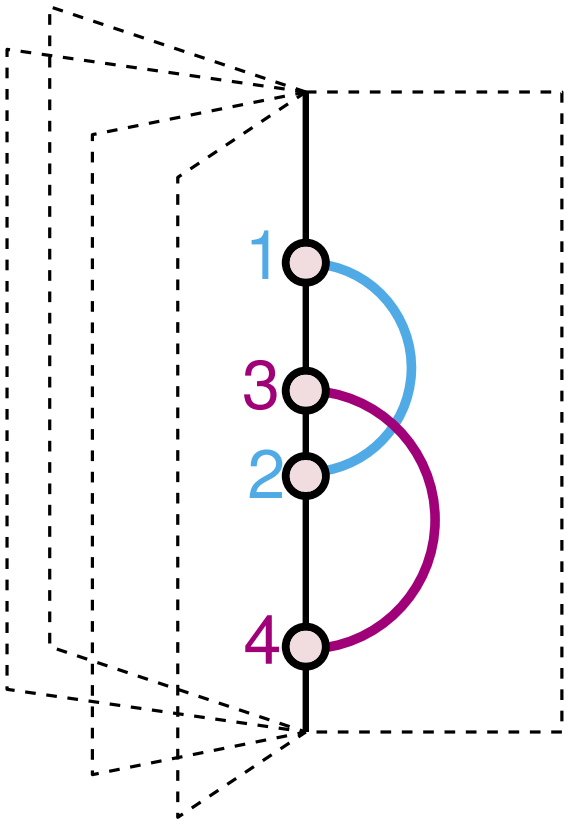

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Crossing Min. in Book Embedding:

Given: Graph G and an integer $k > 0$

Find: A k-page book embedding with minimum number of crossings

- Find a permutation of the nodes on the spine,
- find an assignment of the edges to the pages,
- so that the total number of crossings is minimized

Some Known Results

- complete graphs K_n have book thickness $\lceil n/2 \rceil$
- outerplanar graphs have book thickness 1
- graphs with book thickness 2 are exactly the subhamiltonian graphs [Bernhart, Kainen 79]
- every planar graph has book thickness at most 4 [Yannakakis 89]
- so far no planar graph with book thickness > 3 is known
- graphs with bounded treewidth have bounded book thickness [Dujmovic, Wood 07]
- computing the book thickness is generally NP-hard, even if spine ordering is given [Garey et al. 80], [Unger 88]

Techniques for Crossing Minimization

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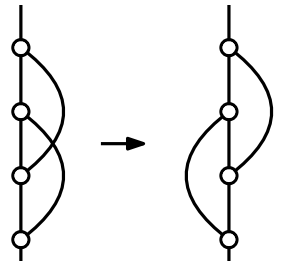
Let $G = (V, E)$, $e \in E$ and $p \in \{1, \ldots, k\}$ be a page index; assume a fixed spine order $<$ is given on V

- variables $x_e^p = \begin{cases} 0 & e \text{ not on page } p \\ 1 & e \text{ on page } p \end{cases}$



- $\forall e \in E : x_e^1 \vee \cdots \vee x_e^k$
- for any p and any two edges $e_1 = (u, v)$, $e_2 = (w, x)$ with $u < w < v < x$ producing a crossing on the same page: $\neg x_{e_1}^p \vee \neg x_{e_2}^p$
- put weights, s.t. only crossing constraints may be violated

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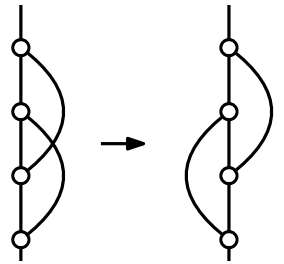

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$\rightarrow$ can be extended to variable spine order

Invitation to Participate

If you want to participate in the challenge with some piece of software, a remote participation on Sept. 24 is possible.

`http://www.csun.edu/gd2015/challenge.htm`