Non-Clashing Teaching in Graphs

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Introduction

LEARNING PROBLEM

Input: A fixed set of concepts (concept class) $\mathcal{C}$, an unknown concept $C \in \mathcal{C}$ chosen in advance

Goal: Identify the target concept C using only partial information about it (for example, labeled examples)

A concept is a function $C : X \to \{0, 1\}$ that classifies instances from an instance space X.

BATCH TEACHING: the teacher mapping T assigns a finite set $T(C)$ of correctly labeled examples, called a teaching set, s.t. the learner can reconstruct C from $T(C)$.

[pos version: only positive labels are allowed]

Introduction

Collusion-avoidance property. [Goldman and Mathias'96]

A teacher mapping T for a concept class $\mathcal{C}$ avoids collusion on $\mathcal{C}$ if there exists a persistent learner mapping L in the sense that $L(S) = C$ for all sets S of labeled examples that include $T(C)$ and are consistent with C.

The teaching complexity is the worst-case number of examples needed for teaching a concept in the underlying concept class $\mathcal{C}$.

Question: what is the smallest teaching complexity, called teaching dimension, that can be achieved while respecting the Goldman-Mathias's collusion-avoidance criterion?

[this why it's called non-clashing]

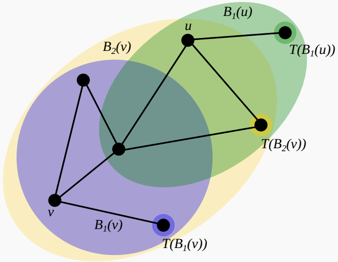
Introduction

The value NCTD (or NCTD^+ for only positive examples) is motivated by:

- it represents the limit of data efficiency in teaching;
- an open question is whether the VC-dimension characterizes teaching complexity, i.e. the problem of determining whether VC-dimension is an upper bound for NCTD.

NCTD on Graphs

Let G be a simple, finite, and undirected graph.



Def. For an integer $r \geq 0$ and a vertex $v \in V(G)$, the ball $B_r(v)$ is the set of all vertices at distance at most r from its center v.

Let $\mathcal{B}$ be a set of balls of G.

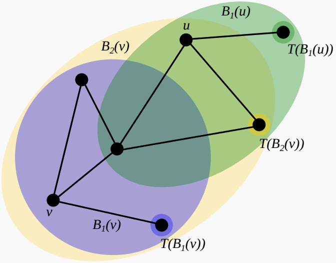
Def. A non-clashing teaching map $T(\mathcal{B})$ is a mapping which assigns to each ball $B \in \mathcal{B}$ a teaching set $T(B) \subseteq V(G)$:
for each pair $B, B' \in \mathcal{B}$, $\exists\, w \in T(B) \cup T(B')$ s.t. $w \notin B \cap B'$ and $w \in B \cup B'$,
we say w distinguishes B and B'. [non-clashing condition]

Def. The dimension of T is $\max_{B \in \mathcal{B}} |T(B)|$.

NCTD$^+$ on Graphs

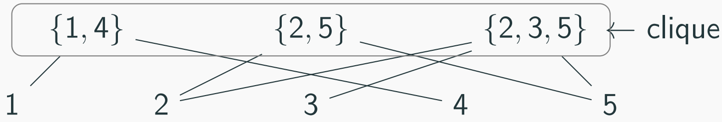
Def. A positive teaching map T for $\mathcal{B}$ is a mapping which assigns to each ball $B \in \mathcal{B}$ a teaching set $T(B) \subseteq B$, i.e., a subset of the vertices of B.

Def. A positive teaching map T is non-clashing for $\mathcal{B}$ if, for each pair of distinct balls $B, B' \in \mathcal{B}$, there exists a vertex $w \in T(B) \cup T(B')$ such that $w \notin B \cap B'$.



Def. NCTD (NCTD$^+$) of $\mathcal{C}$ is the smallest size of a (positive) NCTM for $\mathcal{C}$.

Balls and Closed Neighborhoods (CN)



Graph G for a binary concept class $\mathcal{C} = \{\{1, 4\}, \{2, 5\}, \{2, 3, 5\}\}$ and $\mathcal{B} = \{N[C]\}_{C \in \mathcal{C}}$.

N-NCTD / N-NCTD$^+$
Input: A graph G, a set $\mathcal{B}$ of closed neighborhoods in G, a positive integer k.
Question: Is NCTD($\mathcal{B}$) $\leq k$? / Is NCTD$^+$($\mathcal{B}$) $\leq k$?

Overview

	NCTD$^+$ Balls	NCTD$^+$ CN	NCTD Balls	NCTD CN
Solution size k	NP-h if $k=1$	NP-h if $k=1$	NP-h if $k=1$	NP-h if $k=1$
Vertex cover	FPT [1]	FPT	?	FPT
Vertex integrity	FPT [1]	FPT	?	?
Treedepth	?	FPT	?	?
Treewidth	$W[1]$-hard [1]	?	?	?
Alg. up. bound	$2^{\mathcal{O}(k \cdot d \cdot \lvert V \rvert \cdot \log \lvert V \rvert)}$ [1]	$2^{\mathcal{O}(\lvert E \rvert)}$	$2^{\mathcal{O}(k \cdot d \cdot \lvert V \rvert \cdot \log \lvert V \rvert)}$ [1]	$2^{\mathcal{O}(k \cdot \lvert V \rvert \cdot \log \lvert V \rvert)}$ [1]
Alg. low. bound	$2^{o(k \cdot d \cdot \lvert V \rvert)}$ [1]	$2^{o(f(k) \cdot (\lvert V \rvert + \lvert E \rvert))}$	$2^{o(f(k) \cdot \lvert V \rvert)}$	$2^{o(f(k) \cdot \lvert V \rvert)}$
Planar	Unbounded by cycle [2]	≤ 7	≤ 615 [2]	≤ 5
Unit square	Unbounded by cycle [2]	≤ 4	≤ 615 [2]	≤ 4

NP-h, CN, [1], and [2] denote NP-hard, closed neighborhoods, Ganian et al., and Chalopin et al., respectively.

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Alg. up. bound	$2^{\mathcal{O}(k \cdot d \cdot \|V\| \cdot \log \|V\|)}$ [1]	$2^{\mathcal{O}(\|E\|)}$	$2^{\mathcal{O}(k \cdot d \cdot \|V\| \cdot \log \|V\|)}$ [1]	$2^{\mathcal{O}(k \cdot \|V\| \cdot \log \|V\|)}$ [1]
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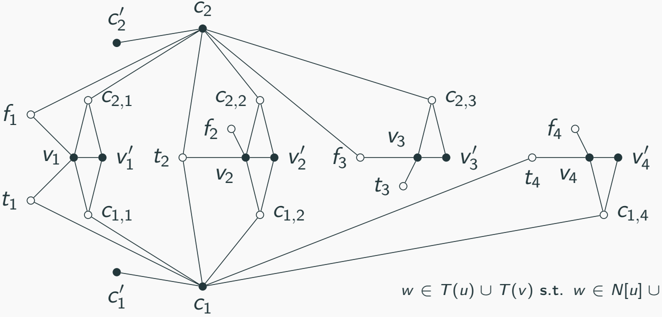
NCTD$^+$ [CN & Balls]: NP-h. for solution size $k = 1$

Theorem. For any computable function f, there is no $2^{o(f(k)\cdot(|V(G)|+|E(G)|))}$ time algorithm for $\mathrm{N\text{-}NCTD}^+$ assuming ETH.

Proof.



$w \in T(u) \cup T(v)$ s.t. $w \in N[u] \cup N[v]$ and $w \notin N[u] \cap N[v]$

Example of the graph G constructed from the 3-SAT instance $\varphi = (\overline{x_1} \vee \overline{x_2} \vee \overline{x_4}) \wedge (x_1 \vee \overline{x_2} \vee x_3)$.
Only the closed neighborhoods of filled vertices are in $\mathcal{B}$.

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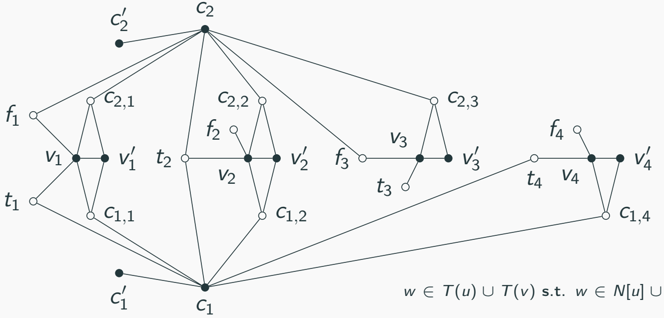

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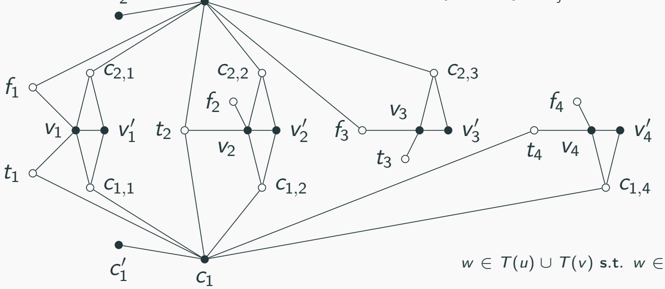
Proof.

Step 1: $\forall i \in [n],\ T(N[v_i]) \subset \{t_i, f_i\}$ to distinguish $N[v_i]$ & $N[v_i']$

Step 2: $\forall j \in [m],\ T(N[c_j]) \subset N(c_j) \setminus \{c_j'\}$ to distinguish $N[c_j]$ & $N[c_j']$



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Only the closed neighborhoods of filled vertices are in $\mathcal{B}$.

False Twins Lemma

Lemma. Let $u_1, \ldots, u_4 \in V(G)$ are pairwise false twins with $N[u_1], \ldots, N[u_4] \in \mathcal{B}$. Then, for any NCTM T of size 1 for $\mathcal{B}$, there exists $i \in [4]$ such that $T(N[u_i]) = \{u_i\}$.

Proof.

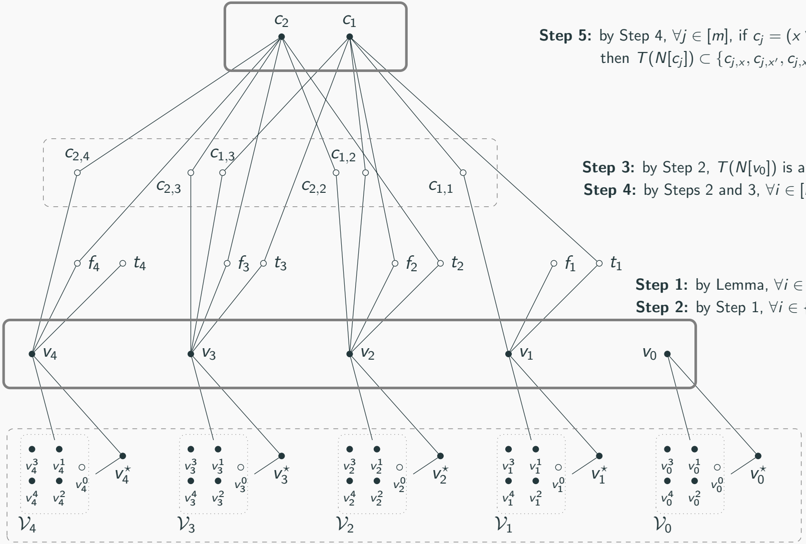
No NCTM of size 1 for $\mathcal{B} \to$ trivial.
So, let T be an NCTM of size 1 for $\mathcal{B}$.

For $N[u_1], \ldots, N[u_4]$ from $\mathcal{B}$, for any $i \in [4]$ and $v \in T(N[u_i])$:

$v = u_i$: v only distinguishes $N[u_i]$ and $N[u_j]$ for all $j \in [4] \setminus \{i\}$;
$v = u_j$ for $j \in [4] \setminus \{i\}$: v only distinguishes $N[u_i]$ and $N[u_j]$;
$v \notin \{u_1, \ldots, u_4\}$: v does not distinguish $N[u_i]$ and $N[u_j]$ for any $j \in [4] \setminus \{i\}$.

Suppose that $T(N[u_i]) \neq \{u_i\}$ for all $i \in [4]$. As there are 6 such pairs that need to be distinguished, T does not satisfy the non-clashing condition for some pair.

NCTD [CN & Balls]: NP-h. for solution size $k = 1$



Step 5: by Step 4, $\forall j \in [m]$, if $c_j = (x \vee x' \vee x'')$, then $T(N[c_j]) \subset \{c_{j,x}, c_{j,x'}, c_{j,x''}\}$

Step 3: by Step 2, $T(N[v_0])$ is a clause or variable vertex

Step 4: by Steps 2 and 3, $\forall i \in [n]$, $T(N[v_i]) \subset \{t_i, f_i\}$

Step 1: by Lemma, $\forall i \in \{0\} \cup [n]$, $T(N[v_i^p]) = \{v_i^p\}$ for some $p \in [4]$

Step 2: by Step 1, $\forall i \in \{0\} \cup [n]$, $T(N[v_i^\star]) \subset \mathcal{V}_i$

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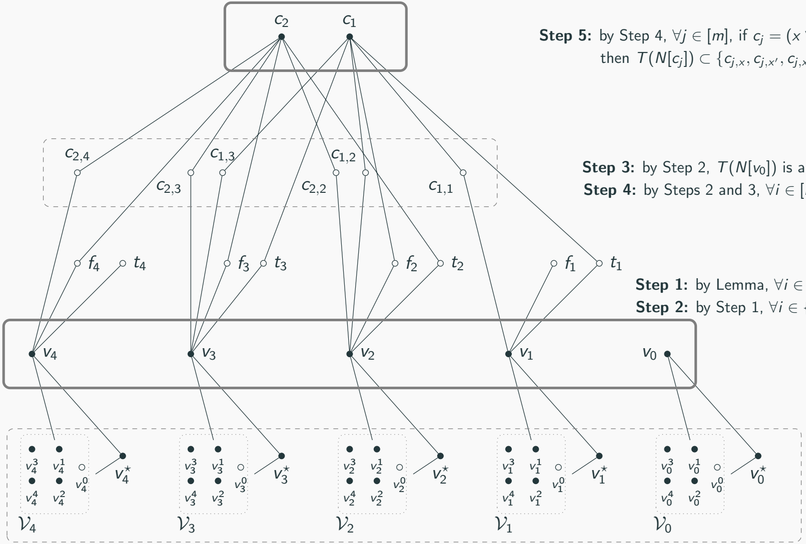

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For $v \in V(G)$, there are $\leq 2^{d(v)+1}$ choices for $T(N[v])$, so $\leq 2^{\sum_{v\in V(G)}(d(v)+1)} = 2^{\mathcal{O}(|E(G)|)}$ positive TMs for $\mathcal{B}$.

NCTD CN: FPT parameterized by vcn

Let $X \subseteq V(G)$ be a vertex cover of G, and $I := V(G) \setminus X$. Any two distinct vertices $u, v \in I$ are in the same *equivalence class* $\mathcal{S} \subseteq I$ if and only if u and v are false twins.

Lemma. Given a graph G and a vertex cover $X \subseteq V(G)$ of G, for any $\mathcal{B}$ consisting of closed neighborhoods of G, it holds that $\text{NCTD}(\mathcal{B}) \leq 2^{|X|+1} + |X|$.

Proof. Let $\mathcal{S}_1, \ldots, \mathcal{S}_p$ be the distinct equivalence classes.

For $x \in X$: $T(N[x]) := X \cup \{s_1, \ldots, s_p\} \cup \{t_1, \ldots, t_p\}$
For $y \in I$: $T(N[y]) := X \cup \{y\}$

Consider each pair $N[u]$ and $N[v]$.

Kernelization Procedure

Given a graph G and a vertex cover $X \subseteq V(G)$ of G; and let $q = 2^{2^{|X|}+|X|} + 1$.

Reduction Rule 1. If $k < 2^{2|X|+1} + |X|$ and there exist $q + 2k + 1$ vertices in $I := V(G) \setminus X$ that are pairwise false twins whose closed neighborhoods are in $\mathcal{B}$, then delete one of them.

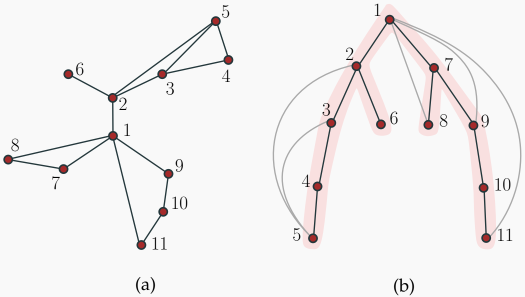
Reduction Rule 2. If there exists a pair $u, v \in V(G)$ of false twins in $I := V(G) \setminus X$ such that $N[u], N[v] \notin \mathcal{B}$, then delete v.

If $k \geq 2^{|X|+1} + |X|$, then it is a YES-instance.
Otherwise, Reduction rules take polynomial time and the resultant graph has at most $2^{|X|}(q + 2k) + 2^{|X|} + |X| \leq 2^{|X|}(2^{2^{|X|}+|X|} + 2^{|X|+2} + 2|X|) + |X|$ vertices.

Treedepth

The treedepth $\mathtt{td}(G)$ of a connected graph G is the minimum height of a rooted tree $\mathcal{T}$ such that $V(\mathcal{T}) = V(G)$ and, for all $uv \in E(G)$, either u is an ancestor of v or v is an ancestor of u in $\mathcal{T}$.

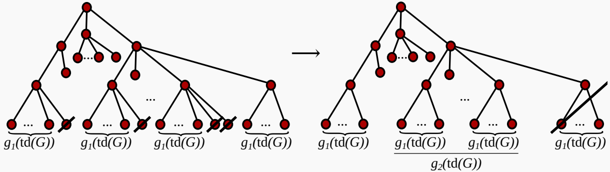
The tree $\mathcal{T}$ is a treedepth decomposition of G.



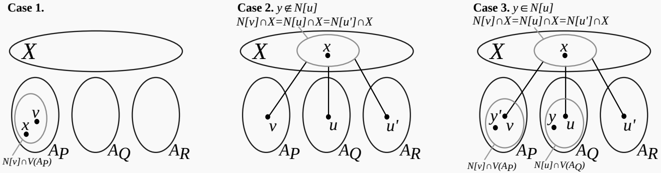
(a) A graph G. (b) A treedepth decomposition $\mathcal{T}$ of G witnessing $\mathtt{td}(G) \leq 4$.

NCTD$^+$ [CN]: FPT parameterized by td

Pruning of a treedepth decomposition.



Reduction Rule. Let G be a graph, $X \subseteq V(G)$ a subset of its vertices, and $A = \{A_1, \ldots, A_\ell\}$ a subset of the connected components of $G - X$ such that $\max_{i \in [\ell]} |A_i| = t$. If $\ell > (|X| + t) \cdot 2^{(|X|+t)^2} \cdot 2^{2t+|X|+1}$, then delete a particular connected component in A.



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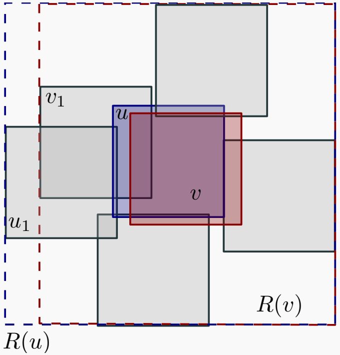
NCTD$^+$ for CN of Unit Square Graphs

Theorem. For any unit square graph G, if $\mathcal{B} = \{N[v] \mid v \in V(G)\}$, then $\mathrm{NCTD}^+(\mathcal{B}) \leq 4$.

Proof.



$T(N[v])$ is the vertices represented by the leftmost, rightmost, topmost, and bottommost squares in $R(v)$ (they may not be unique).

$R(u) \neq R(v)$: $\exists x \in T(N[u]) \cup T(N[v])$ that distinguishes $N[u]$ and $N[v]$;

$R(u) = R(v)$: $N[u] = N[v]$ as each square in $R(v)$ ($R(u)$, resp.) intersects v (u, resp.).

In the proof, the minimum rectangles $R(u)$ (blue) and $R(v)$ (red) enclosing $N[u]$ and $N[v]$, respectively. The leftmost square in $R(u)$ ($u_1 \in T(N[u])$) is not contained in $R(v)$.

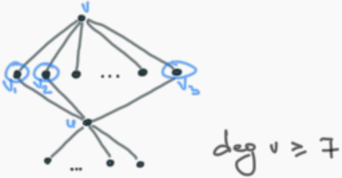
NCTD$^+$ for CN of Planar Graphs

Theorem. For any planar graph G, if $\mathcal{B} = \{N[v] \mid v \in V(G)\}$, then $\mathrm{NCTD}^+(\mathcal{B}) \leq 7$.

Proof. For all $v \in V(G)$, if $d(v) \leq 6$, then set $T(N[v]) := N[v]$.
Otherwise, for each $v \in V(G)$, place any 3 neighbors (v_1, v_2, v_3) of v in $T(N[v])$.

There is at most one other vertex $u \in V(G)$ such that $v_1, v_2, v_3 \in N(u)$

If exists $N[v], N[y] \in \mathcal{B}$ such that $d(v) \geq 7$, $N[v] \neq N[y]$, and T does not currently satisfy the non-clashing condition, then there exists $w \in (N[v] \setminus N[y]) \cup (N[y] \setminus N[v])$ that will be added to $T(N[v])$ or $T(N[y])$ to distinguish the pair.



NCTD for CN of Planar Graphs

Theorem. For any planar graph G, if $\mathcal{B} = \{N[v] \mid v \in V(G)\}$, then $\text{NCTD}(\mathcal{B}) \leq 5$.

Proof. For all $v \in V(G)$, if $d(v) \leq 4$, then set $T(N[v]) := N[v]$.
Otherwise, for each $v \in V(G)$, place any 3 neighbors (v_1, v_2, v_3) of v in $T(N[v])$.

There is at most one other vertex $u \in V(G)$ such that $v_1, v_2, v_3 \in N(u)$

If exists $N[v], N[y] \in \mathcal{B}$ such that $d(v) \geq 5$, $N[v] \neq N[y]$, and T does not currently satisfy the non-clashing condition, then there exists $w \in (N[v] \setminus N[y]) \cup (N[y] \setminus N[v])$ that will be added to $T(N[v])$ or $T(N[y])$ to distinguish the pair.

Now, T is a positive NCTM for $\mathcal{B}$ and for each $v \in V(G)$ with $d(v) \geq 5$. But at most 2 vertices were added to any teaching set during the above process as the vertices in $\{u, v, v_1, v_2, v_3\}$ cannot form a K_5.

if $v_2 v_3 \notin E(G)$, then $y \in \{u, v_1\}$

Thank you for your attention!

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Treedepth	?	FPT	?	?
Treewidth	$W[1]$-hard [1]	?	?	?
Alg. up. bound	$2^{\mathcal{O}(k \cdot d \cdot \lvert V \rvert \cdot \log \lvert V \rvert)}$ [1]	$2^{\mathcal{O}(\lvert E \rvert)}$	$2^{\mathcal{O}(k \cdot d \cdot \lvert V \rvert \cdot \log \lvert V \rvert)}$ [1]	$2^{\mathcal{O}(k \cdot \lvert V \rvert \cdot \log \lvert V \rvert)}$ [1]
Alg. low. bound	$2^{o(k \cdot d \cdot \lvert V \rvert)}$ [1]	$2^{o(f(k) \cdot (\lvert V \rvert + \lvert E \rvert))}$	$2^{o(f(k) \cdot \lvert V \rvert)}$	$2^{o(f(k) \cdot \lvert V \rvert)}$
Planar	Unbounded by cycle [2]	≤ 7	≤ 615 [2]	≤ 5
Unit square	Unbounded by cycle [2]	≤ 4	≤ 615 [2]	≤ 4

A key open question of Kirkpatrick et al. is whether there is a concept class $\mathcal{C}$ such that $\text{NCTD}(\mathcal{C}) > \text{VCD}(\mathcal{C})$.