

# *Backdoors and modulators*

*Eduard Eiben and Robert Ganian*

*Retreat*



ALGORITHMS AND  
COMPLEXITY GROUP

logics



Logical Methods in  
Computer Science

# *The islands of tractability*

*3-colorability*

bipartite

cograph

threshold

acyclic

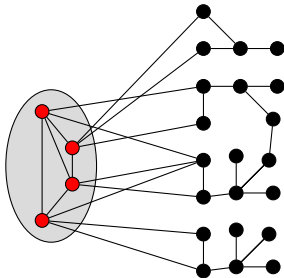


chordal

outer-  
planar

# Modulators

to a graph class  $\mathcal{H}$



A graph with a small modulator to forests

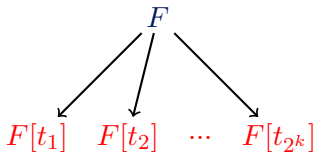
- Finding size  $k$  modulators ( $XP(n^k)$  if  $\mathcal{H}$  is polynomially recognizable; aim for  $FPT$ )
- Using modulators (aim for  $FPT$ )

# Backdoors

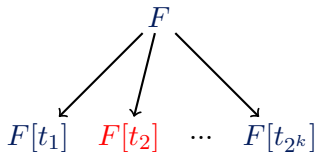
into a tractable class  $\mathcal{H}$

Given a CNF formula  $F$  and set  $X$  of  $k$  variables.

Let  $t_1, \dots, t_{2^k}$  be the truth assignments on  $X$ .



$X$  is a **strong backdoor** if all the  $F[t_i]$  belong to  $\mathcal{H}$ .



$X$  is a **weak backdoor** if some  $F[t_i]$  belongs to  $\mathcal{H}$  and is **satisfiable**.

If we know a backdoor of size  $k$ , then we can decide  $F$  in time  $2^k \text{ poly}$ .

# Backdoors

complexity of *BD* detection

Base class	strong bd	weak bd	weak bd on $r$ - <i>CNF</i>
Horn	<i>FPT</i>	$W[2]$ - <i>h</i>	<i>FPT</i>
2CNF	<i>FPT</i>	$W[2]$ - <i>h</i>	<i>FPT</i>
UP	$W[P]$ - <i>c</i>	$W[P]$ - <i>c</i>	$W[P]$ - <i>c</i>
renamable Horn	$W[2]$ - <i>h</i>	$W[2]$ - <i>h</i>	$W[2]$ - <i>h</i>
Acyclic	<i>FPT-apx</i>	$W[2]$ - <i>h</i>	<i>FPT</i>
Treewidth[ $t$ ]	<i>FPT-apx</i>	$W[2]$ - <i>h</i>	<i>FPT</i>

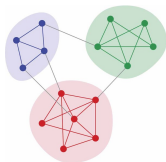
## *Relation to our work*

*FWF project “X-TRACT”*

**Backdoors which “disconnect” instance into several islands of tractability**

- great for CSPs, but also works well on SAT and graphs

**Using “community structure” to solve SAT instances**

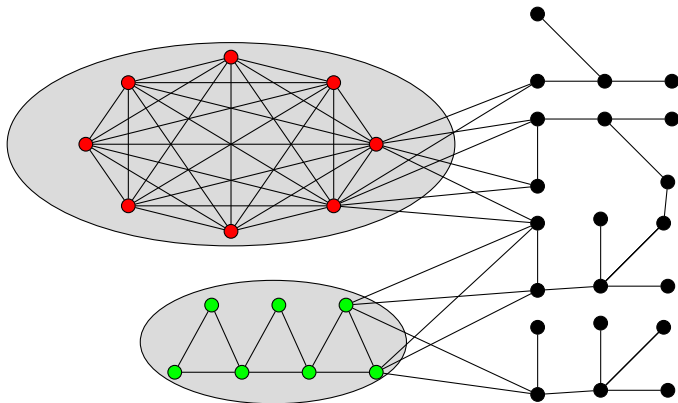


- real-world SAT instances seem to have community structure
- a more restricted notion is required to have rigorous algorithmic results

# Well-structured modulators

FWF project “X-TRACT”

Basic idea: what if the graph has a large but **well-structured** modulator to  $\mathcal{H}$ ?



A graph with a 2-well-structured modulator to forests