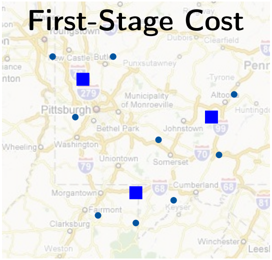
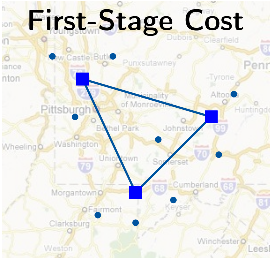
k-Delete Recoverable Robustness

Christina Büsing, Ivana Ljubic



Deterministic Problem

- 0-1 optimization problem



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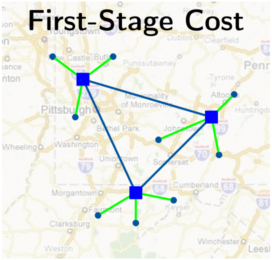

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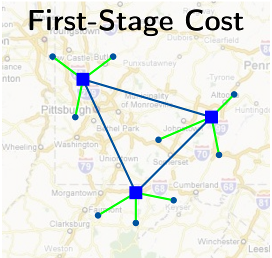

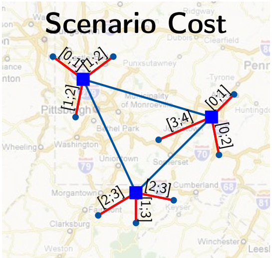

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Uncertainties

- Γ-scenarios: $c^S \in [\underline{c}, \underline{c} + \hat{c}]$, Γ-values deviate

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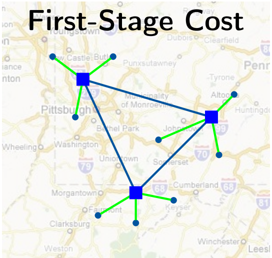

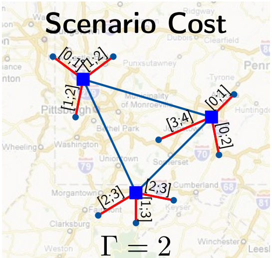

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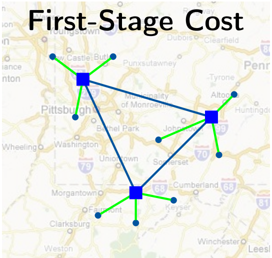

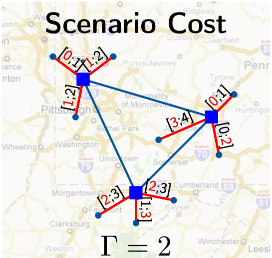

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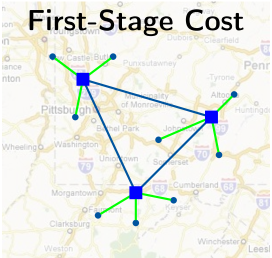

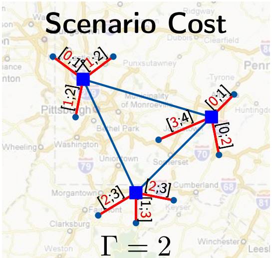

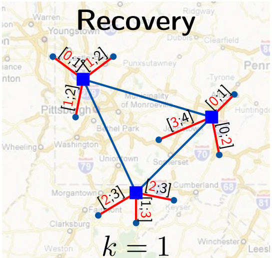

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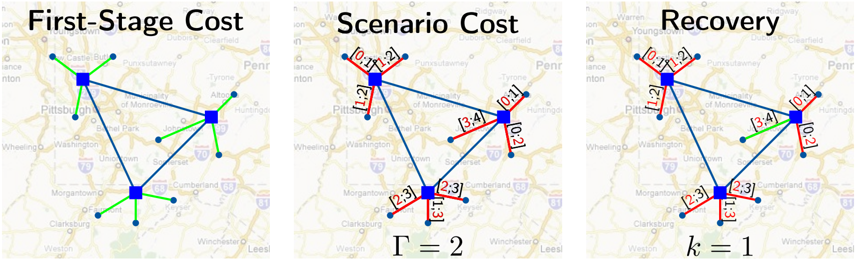
Recovery

- k elements deleted



Given: Feasible solutions $\mathcal{X} \subseteq \{0,1\}^n$, first-stage cost c^D, Γ-scenarios $\mathcal{S}$, recovery parameter k, recovery

$$\mathcal{X}^k(x) = \{x' \mid x' \leq x, |x \backslash x'| \leq k\}$$

Find: Feasible solution x with minimal total cost

$$c(x) = c^D(x) + \max_{S \in \mathcal{S}} \min_{x' \in \mathcal{X}^k(x)} c^S(x')$$

Compact Formulation

Theorem (B. and Ljubic, 2014)

A k-Delete RR 0-1 problem with Γ-scenarios can be formulated as linear mixed integer programm.

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- Original formulation

$$\min_{x \in \mathcal{X}} c^D(x) + \max_{y \in \{0,1\}^n} \left\{ \sum_{i=1}^{n} (\underline{c}_i x_i + \hat{c}_i x_i y_i) - \max_{z \in \{0,1\}^n} \left\{ \sum_{i=1}^{n} (\underline{c}_i x_i + \hat{c}_i x_i y_i) \, z_i \mid \sum_{i=1}^{n} z_i \leq k \right\} \mid \sum_{i=1}^{n} y_i \leq \Gamma \right\}$$

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- Dualize inner maximization problem

$$\begin{aligned} \max \quad & \sum_{i=1}^n (\underline{c}_i x_i + \hat{c}_i x_i y_i)\, z_i \\ & \sum_{i=1}^n z_i \le k \\ & 0 \le z_i \le 1 \end{aligned} \qquad\qquad \begin{aligned} \min \quad & uk + \sum_{i=1}^n v_i \\ & u + v_i \ge (\underline{c}_i x_i + \hat{c}_i x_i y_i) \\ & u, v_i \ge 0 \end{aligned}$$

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- New Formulation

$$\min_{x\in\mathcal{X}} c^D(x) + \max_{y\in\{0,1\}^n} \max_{u\ge 0}\left\{\sum_{i=1}^n \underline{c}_i(u) x_i + \sum_{i=1}^n \hat{c}_i(u) x_i y_i - ku \mid \sum_{i=1}^n y_i \le \Gamma\right\}$$

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- $\exists$ optimal $u \in U = \{\underline{c}_i, \underline{c}_i + \hat{c}_i \mid i \in [n]\} \cup \{0\}$.

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- Replace u by a set of constraints

$$\min \omega$$

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$$x \in \mathcal{X}$$

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$$x \in \mathcal{X}$$

- Dualization

$$\min \omega$$

$$\sum_{i=1}^n \tilde{c}_i(u)x_i + \Gamma\xi^u + \sum_{i=1}^n \theta_i^u - ku \leq \omega \quad \forall u \in U$$

$$c_i(u)x_i - \xi^u + \theta_i^u \leq 0 \quad \forall i \in [n], u \in U$$

$$\xi^u, \theta_i^u \geq 0$$

$$x \in \mathcal{X}$$

Compact Formulation (Comp)

$$
\begin{aligned}
\min \omega & \\
x &\in \mathcal{X} \\
\sum_{i=1}^{n} \tilde{c}_i(u) x_i + \Gamma \xi^u + \sum_{i=1}^{n} \theta_i^u - ku &\leq \omega \quad \forall u \in U \\
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$$

Compact Formulation (Comp)

$$\min_{x \in \mathcal{X}} \omega$$

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Separation of U (SepU): Add in case of violation in $u \in U$

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Solution Approaches

Compact Formulation (Comp)

$$\min \omega$$
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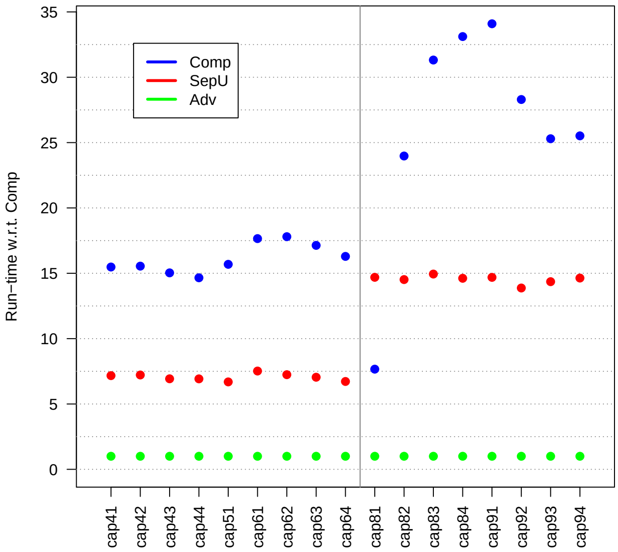
$$c_i(u)x_i - \xi^u + \theta_i^u \leq 0 \quad \forall i \in [n]$$
$$\xi^u, \theta_i^u \geq 0$$

Advanced Separation (Adv): Add in case of violation in $u \in U$

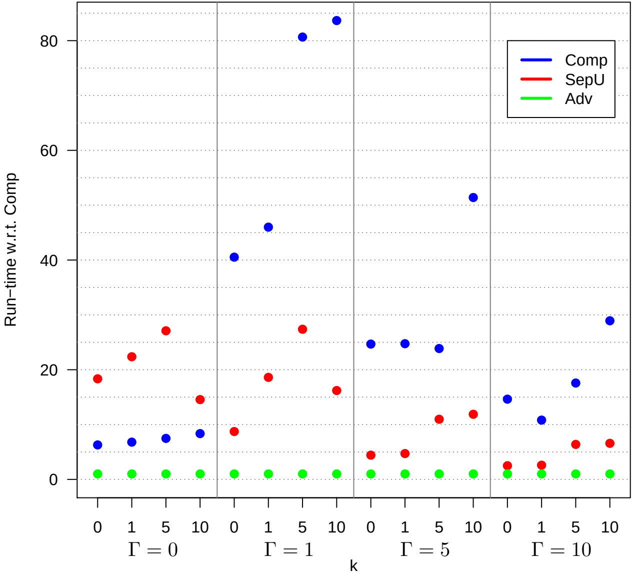
$$\sum_{i=1}^{n}(c_i^0 + c'(u))x_i + \sum_{i \in X(u)} \hat{c}_i(u)x_i - ku \leq \omega$$

- Modification of capacitated facility location instances (ORLIP)
- 17 instances
- $\{16, 25\}$ facilities, 50 customers
- Costs:
  - first stage cost c_i^0
  - $\underline{c}_i \in [0.9, 1.1] \cdot 0.05 \cdot c_i^0$
  - $\hat{c}_i \in [0.9, 1.1] \cdot 0.1 \cdot c_i^0$
- $\Gamma, k \in \{0, 1, 5, 10\}$

Mean Run-time per Instance



Mean Run-time per (Γ, k)



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- Introduction to k-delete recoverable robustness
- $\mathbf{NP}$-hardness for shortest path, perfect matching, (s,t)-cut
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Thank you for your attention!