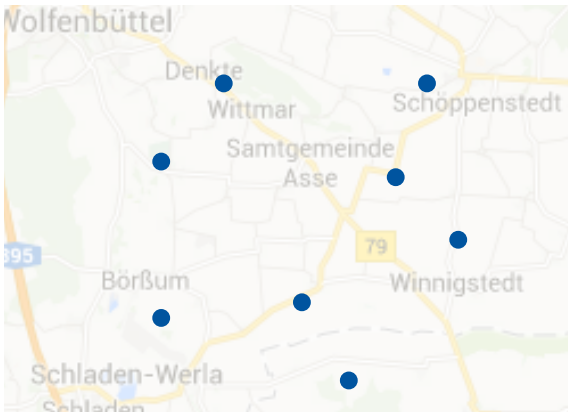
Dealing with Uncertainties

a Recoverable Robust Approach

Christina Büsing

Recoverable Robustness

Normal



Deterministic Optimization

- find the best solution

Recoverable Robustness

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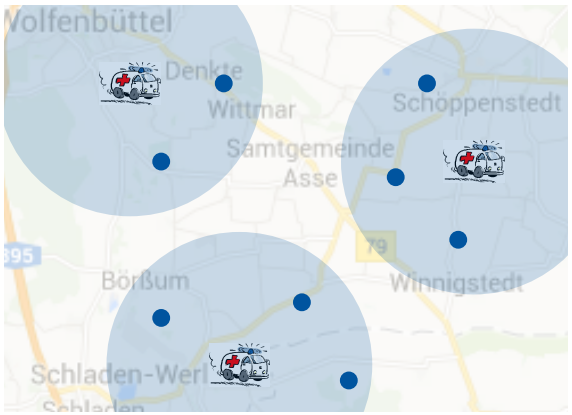

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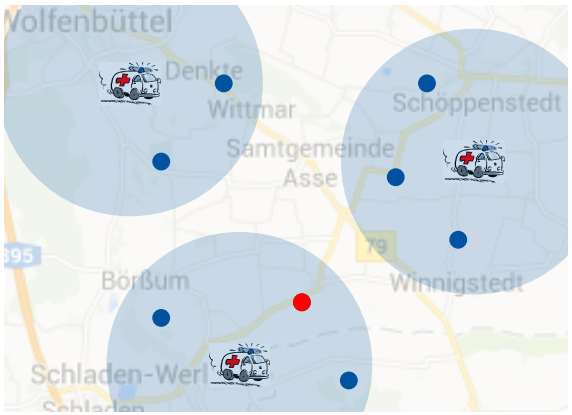

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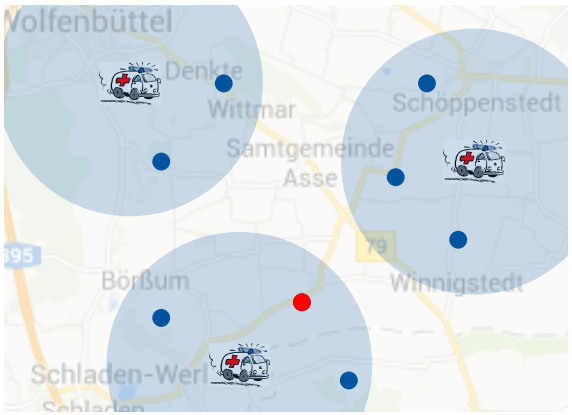
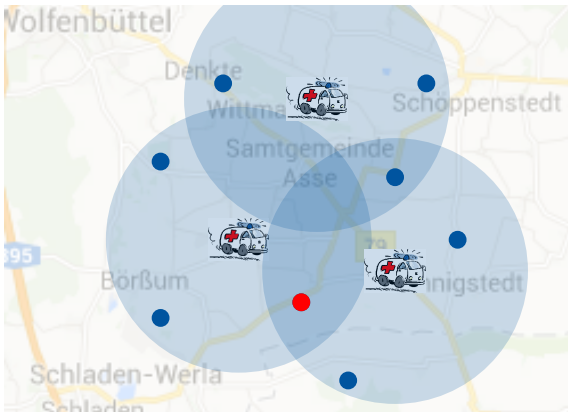


Robust



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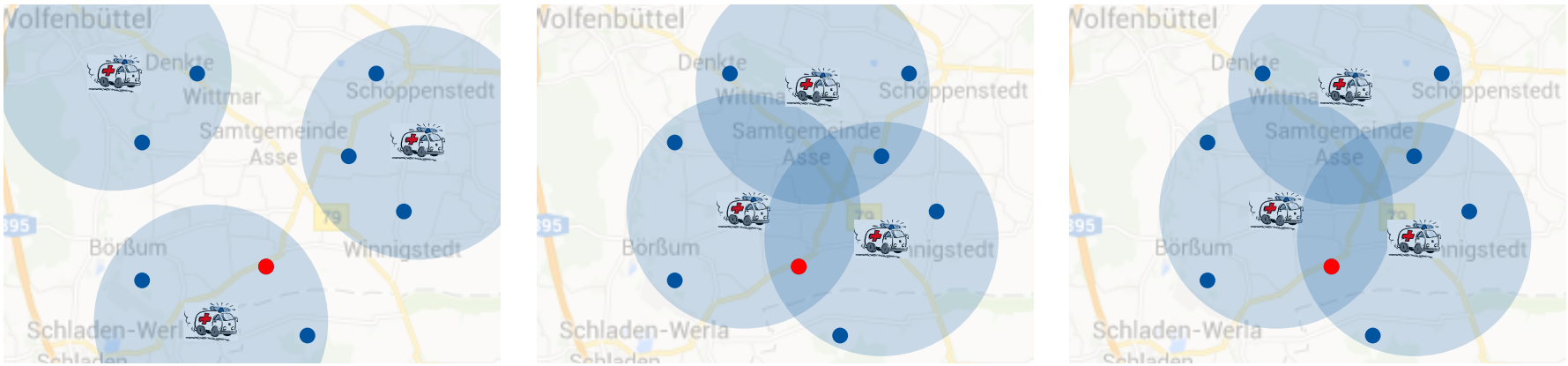

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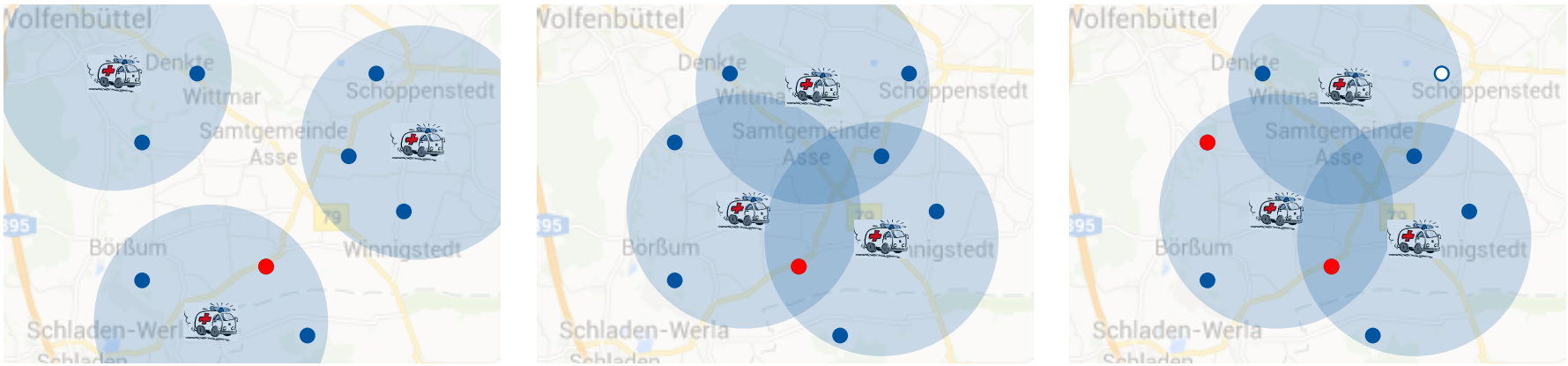

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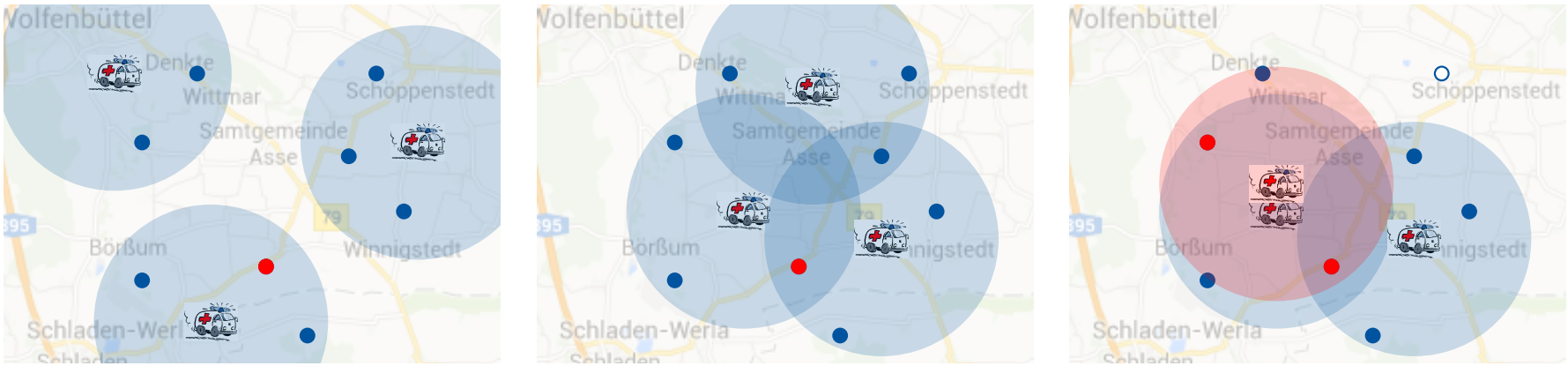

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$$\begin{aligned} \min c^\top x & \\ Ax &\geq b \\ x &\geq 0 \end{aligned}$$

- shortest path, matching
- facility location, steiner tree, scheduling

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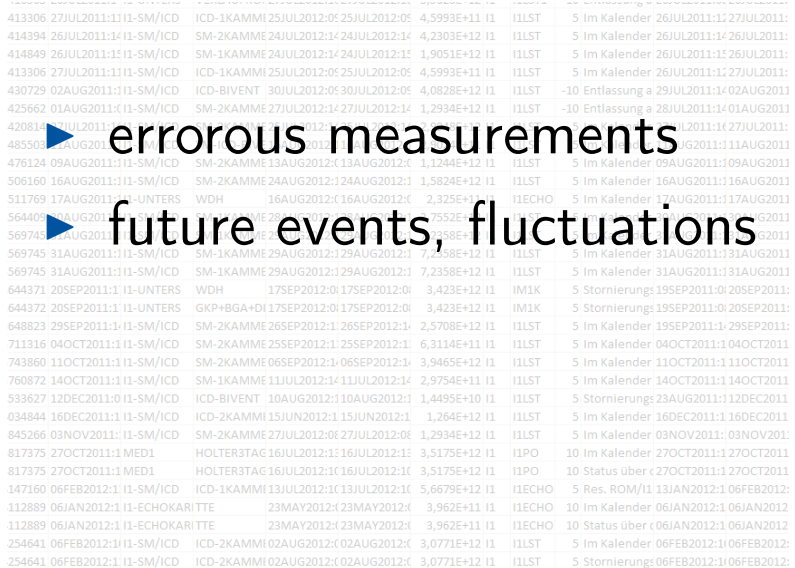

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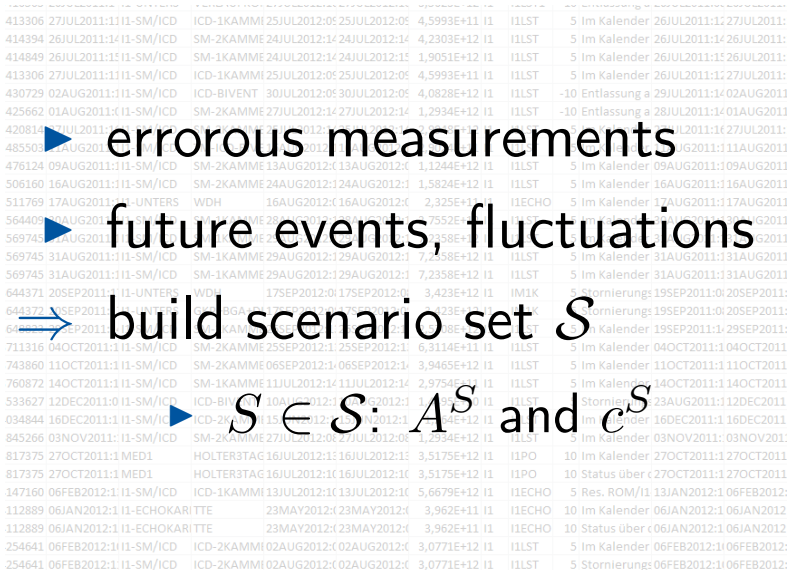

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$\Rightarrow$ build scenario set $\mathcal{S}$

- $S \in \mathcal{S}$: A^S and c^S

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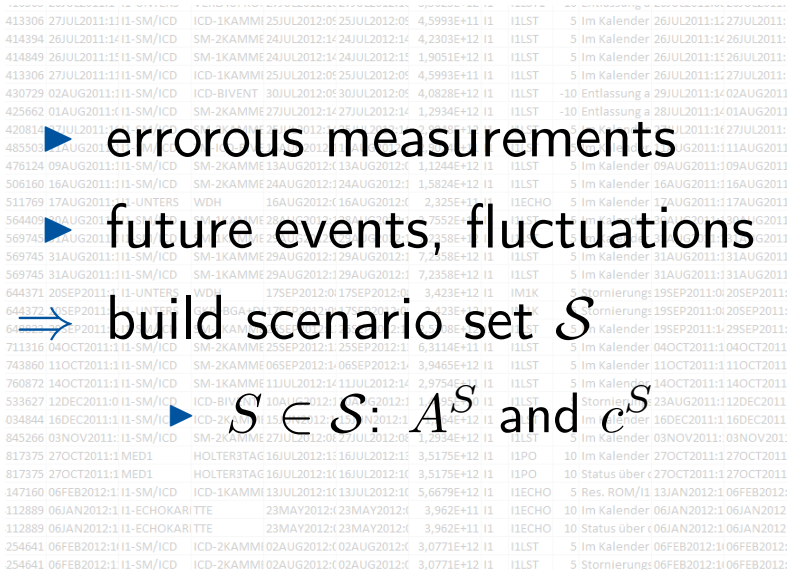

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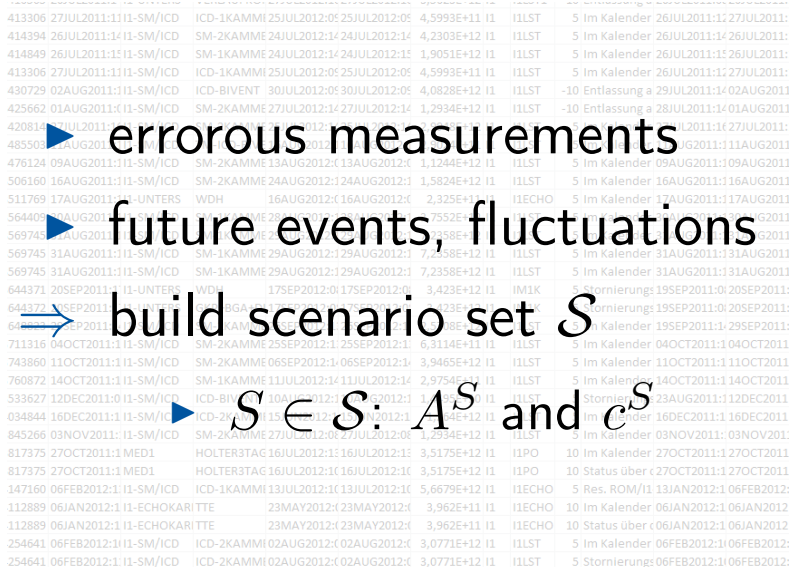

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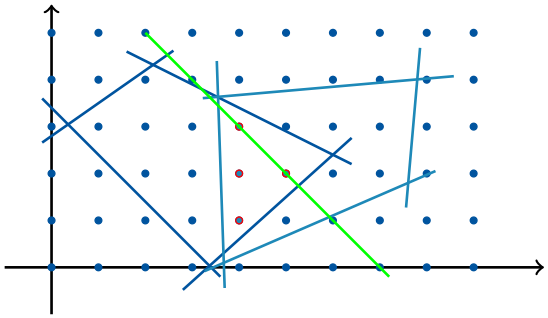
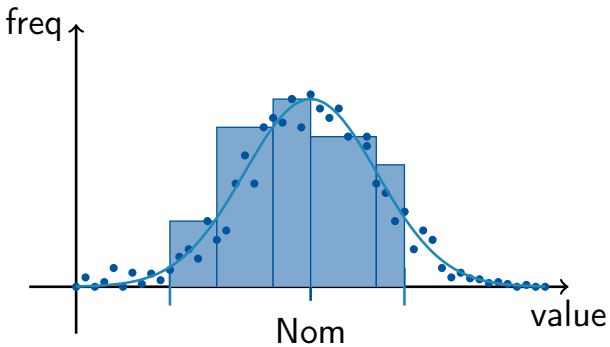
(Recoverable) Robustness

$$\begin{aligned} \min\, & c^\top x \\ & A^S x + B y^S \geq b \quad \forall S \in \mathcal{S} \\ & x, y^S \geq 0 \quad \forall S \in \mathcal{S} \end{aligned}$$

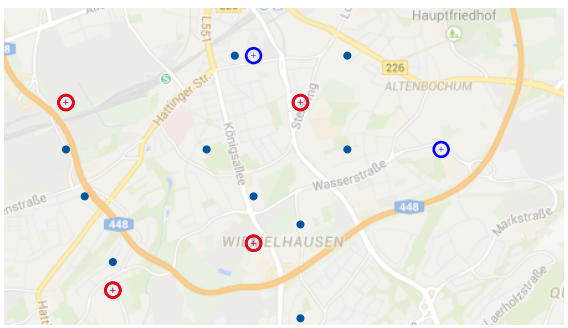
Research Program



Exact Algorithms and Heuristics

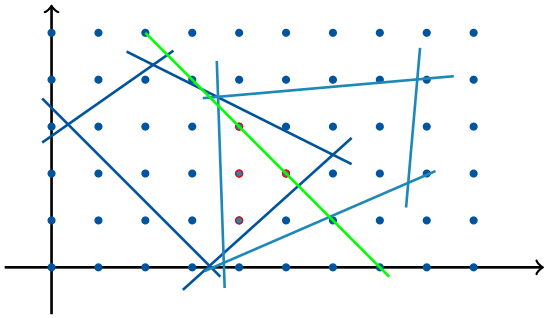


Generation of Scenario Sets



Applications

Research Program



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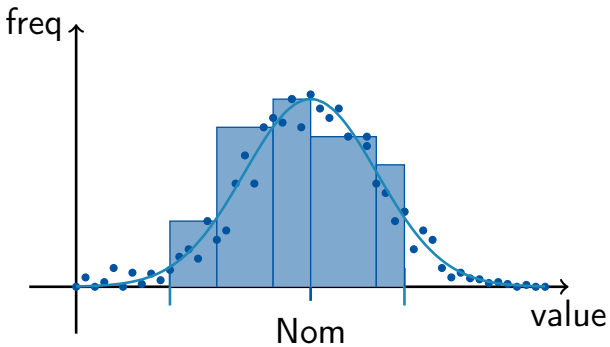

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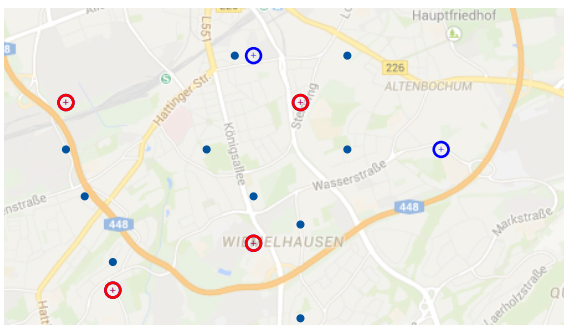

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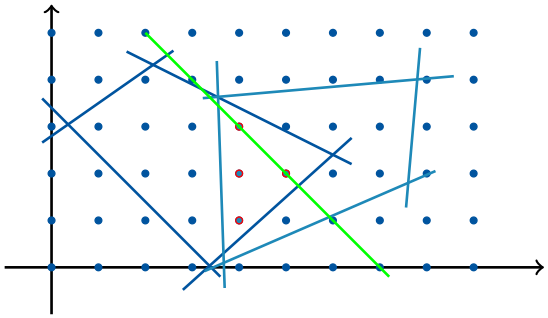

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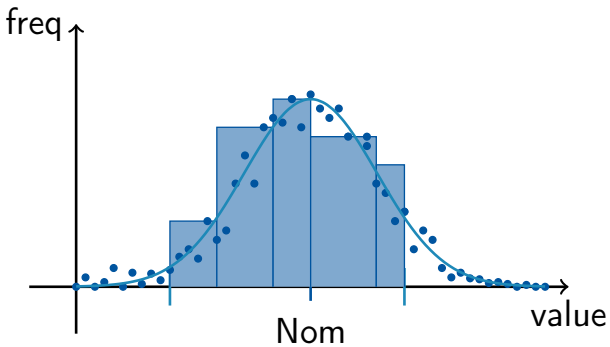

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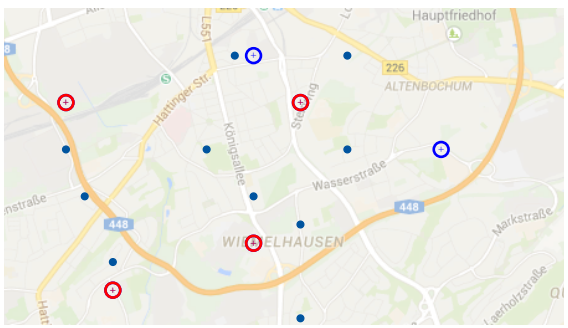

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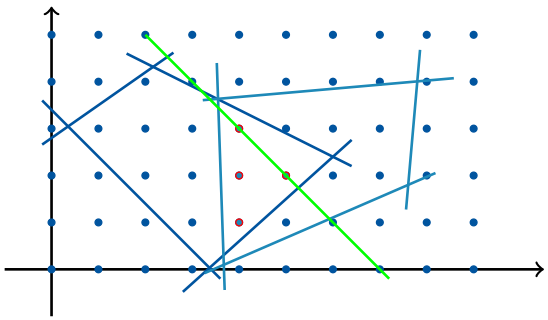

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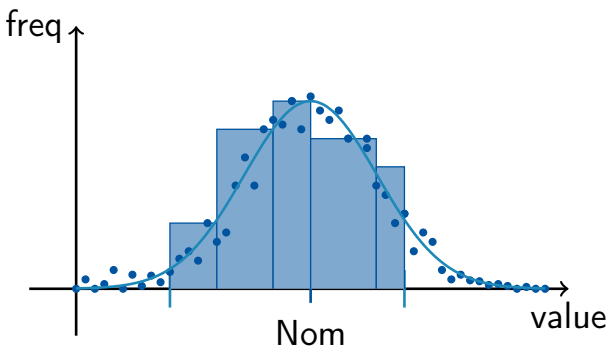

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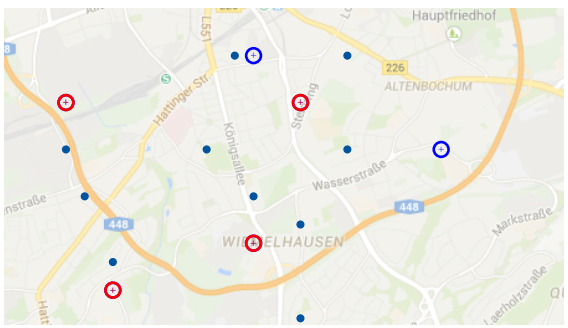

Applications

- robust ambulance optimization
- revenue management

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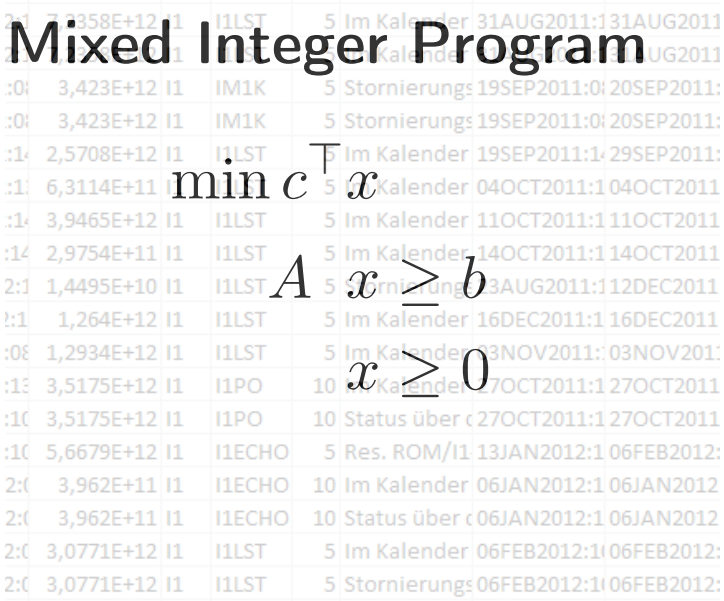

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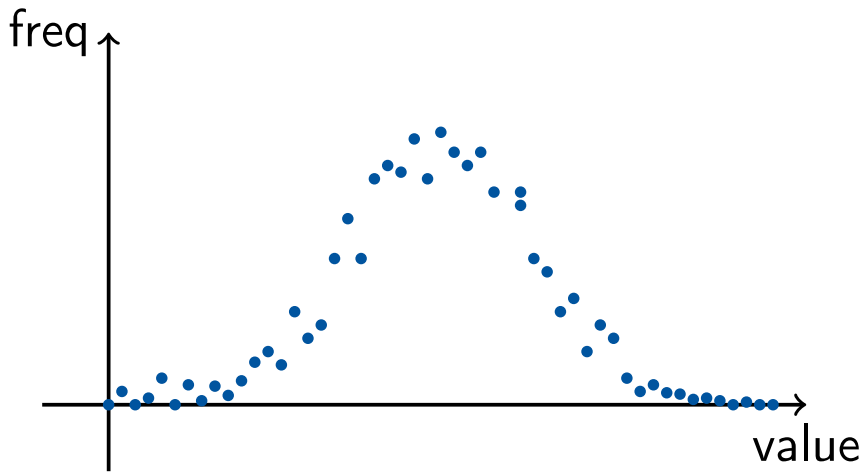

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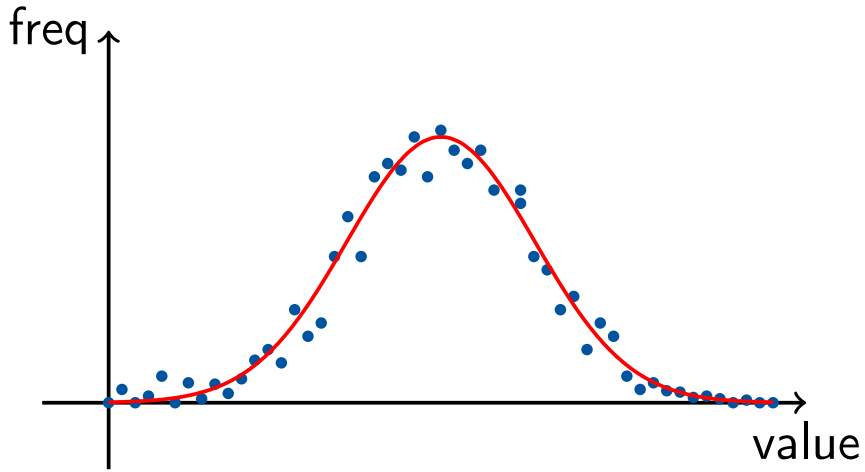

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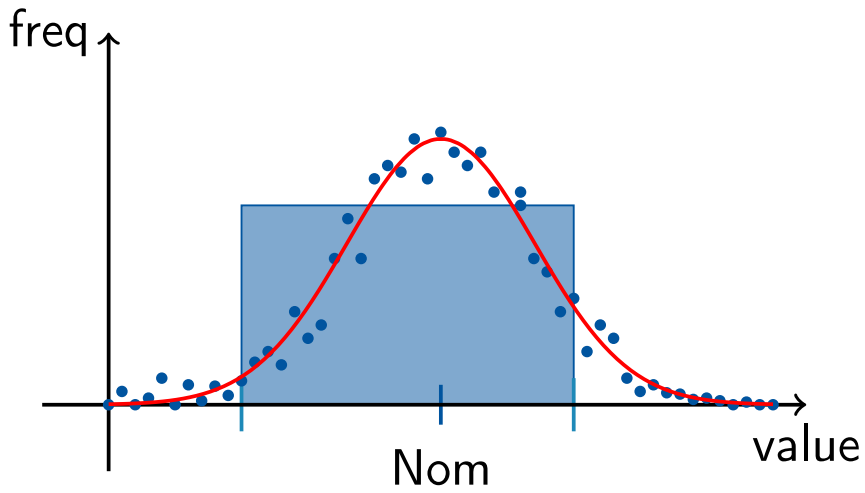

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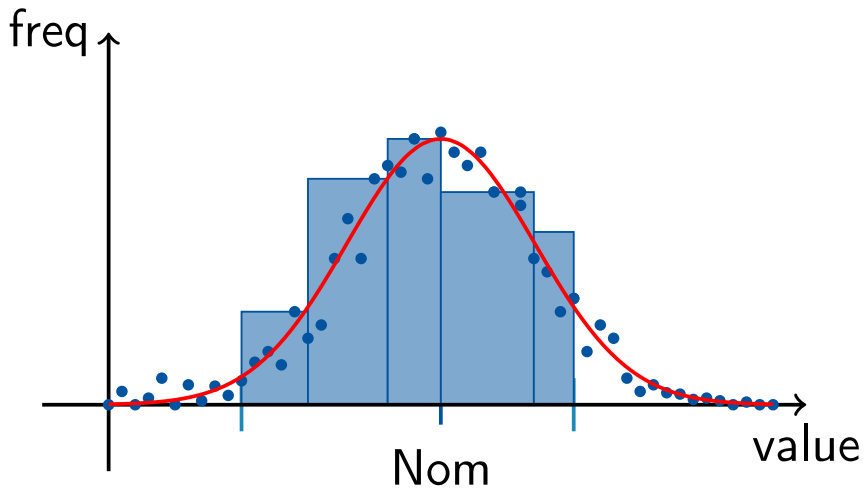

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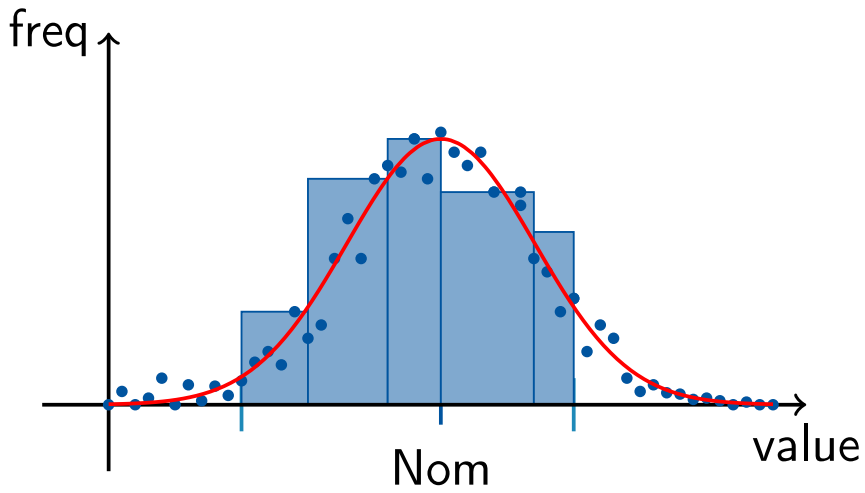

Results

- compact formulation for robust MIP
- robust 0-1 problems remain tractable
- efficient scenario separation
- first applications

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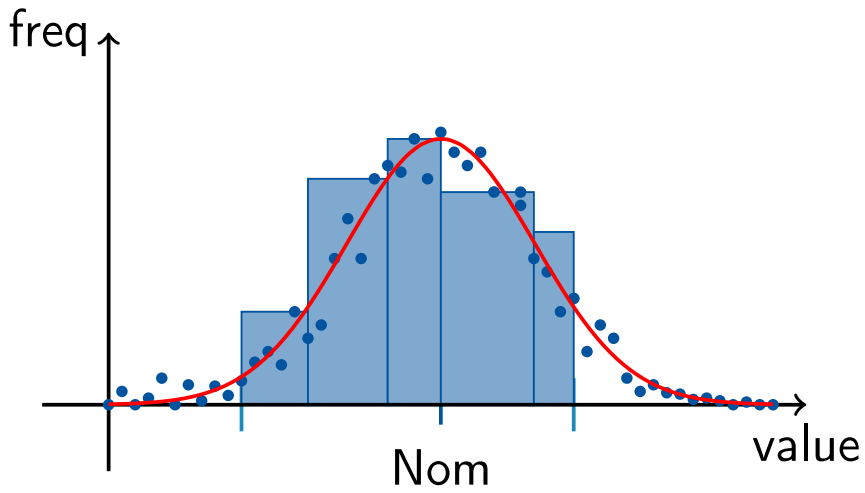

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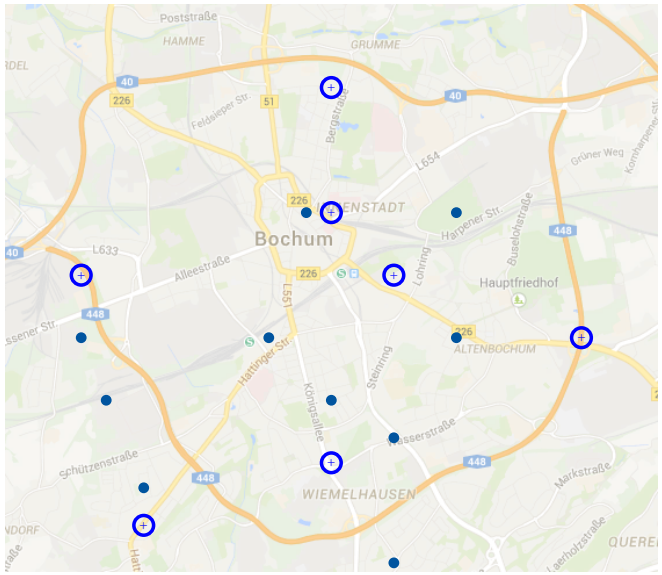
Ongoing Work

- rules for generation
- influence on solution
- measurements for robustness
  - price of robustness
  - radius of stability

Robust Ambulance Optimization

Degel, Lutter



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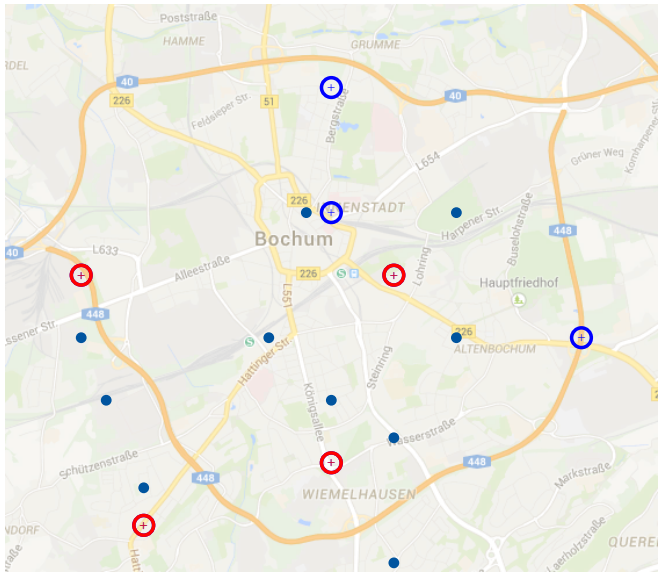

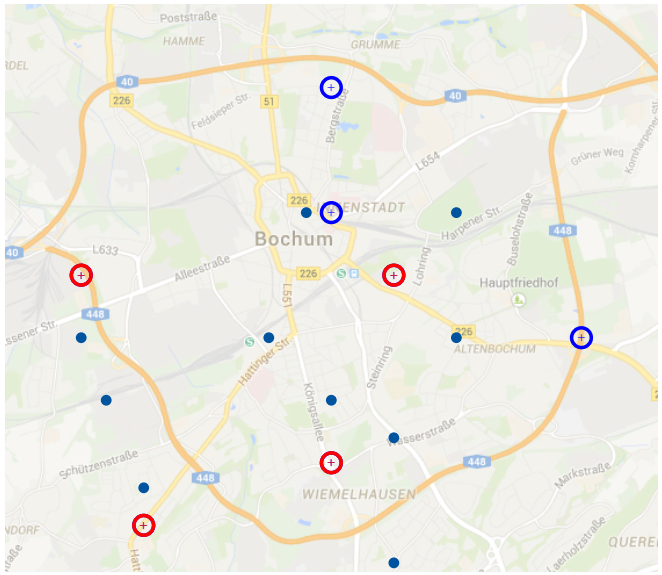

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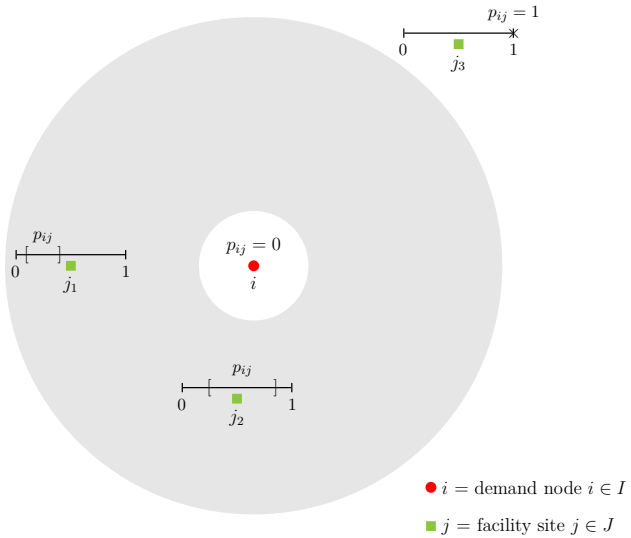

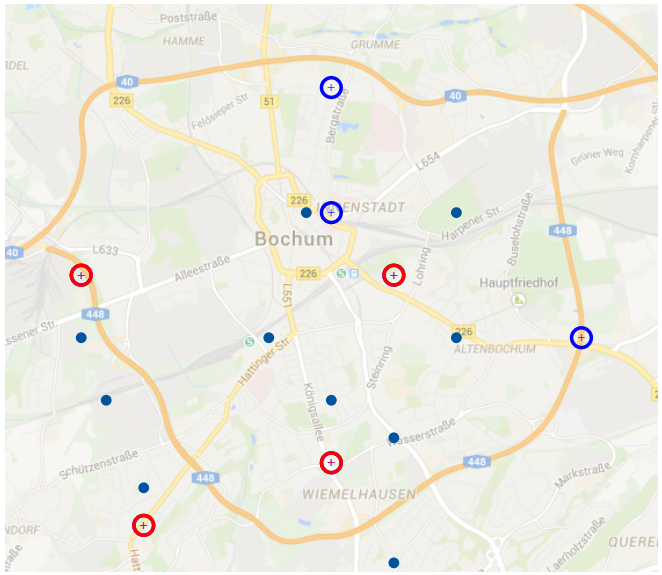

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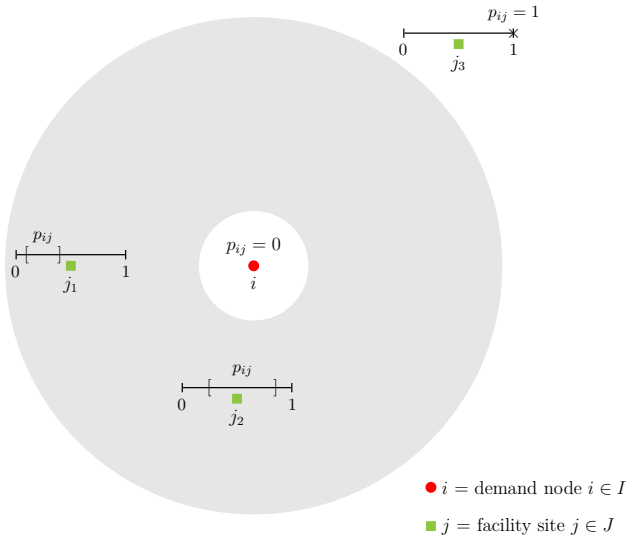

Robust Set Cover Problem

$$\min \sum_{j \in J} y_j$$

$$\mathbb{P}\left[\sum_{j \in J} a_{ij} y_j \geq 1\right] \geq \alpha \qquad i \in I$$

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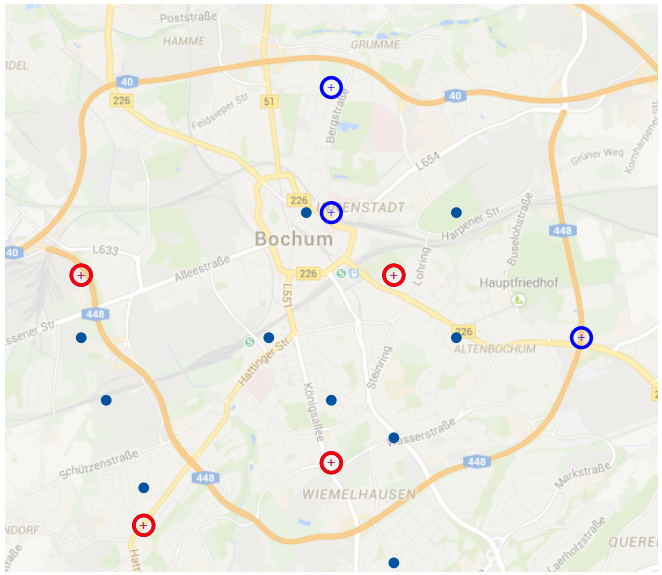

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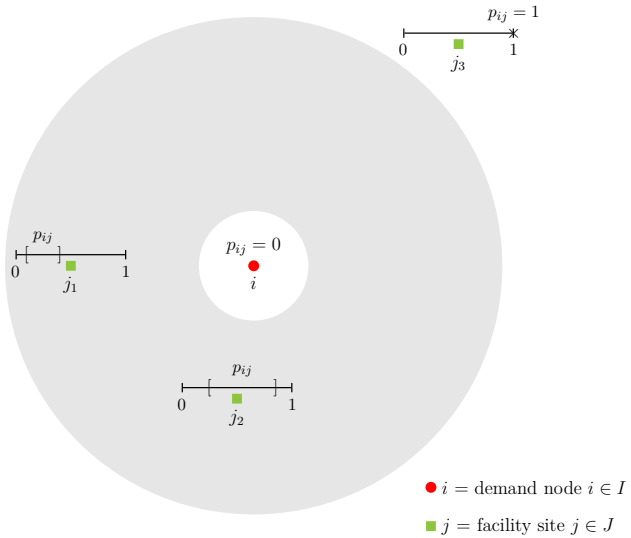

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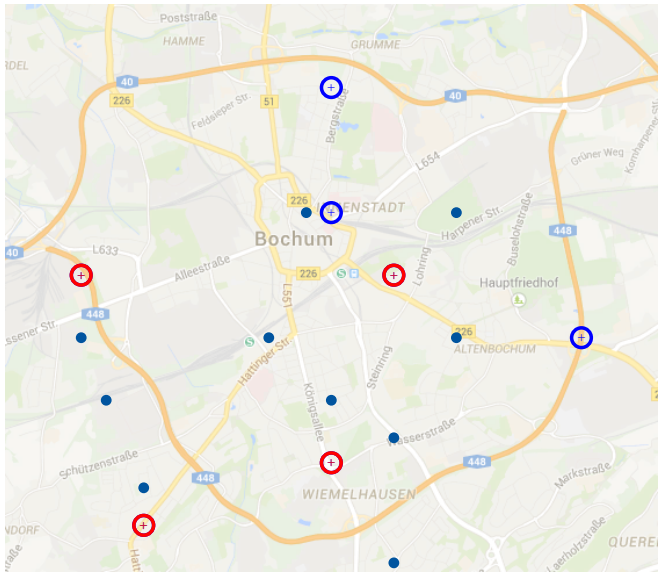

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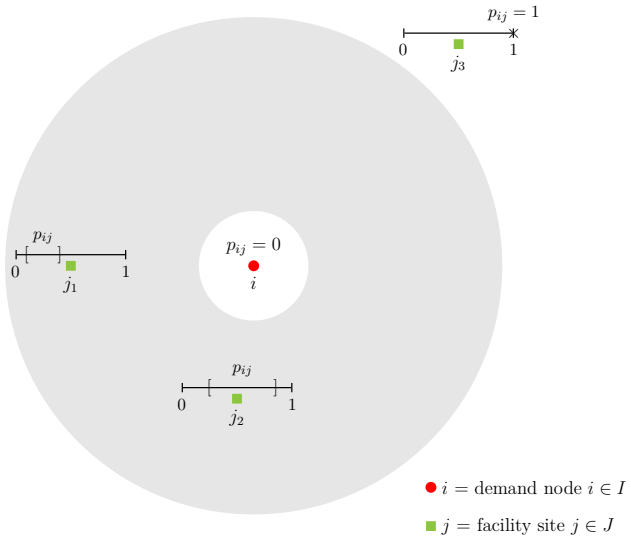

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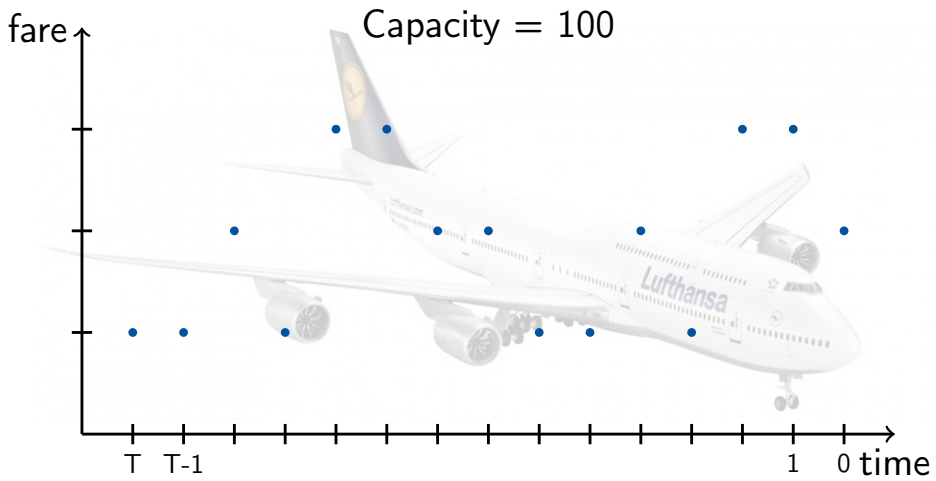
Results

- compact MIP-formulation
- separation algorithms
- computational study

Revenue Management

Cleophas, Kadatz



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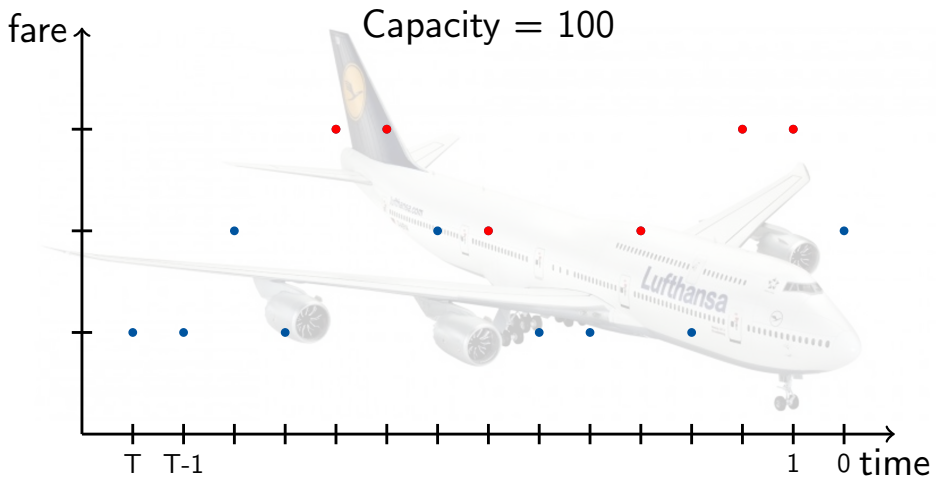
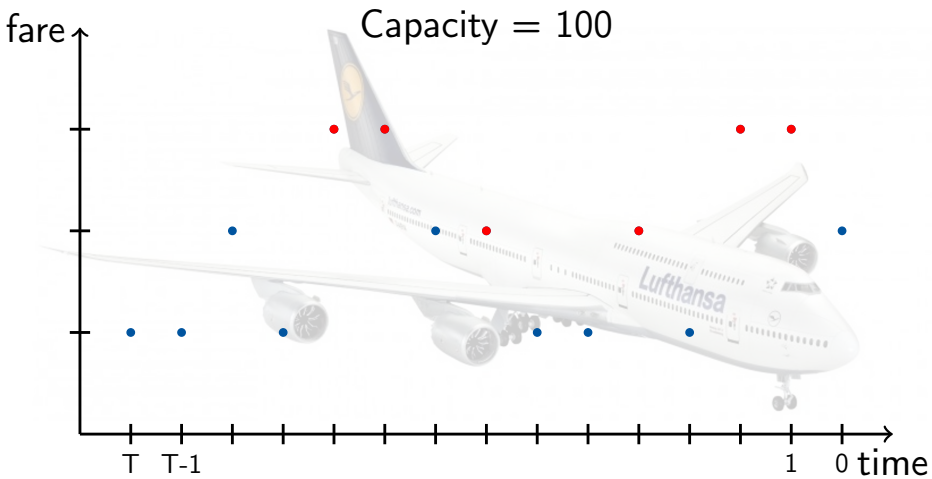
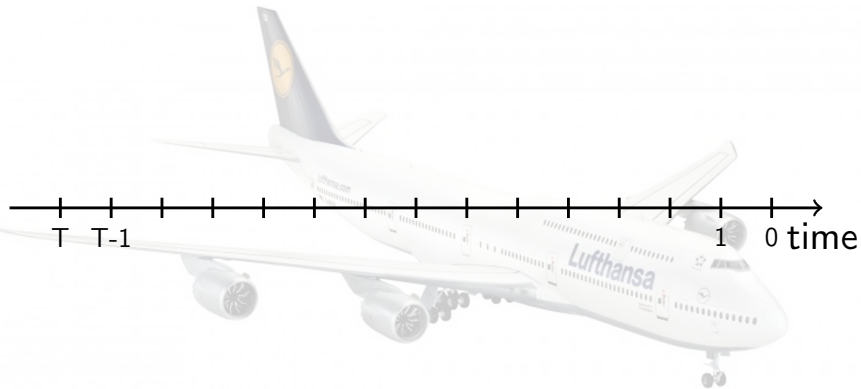
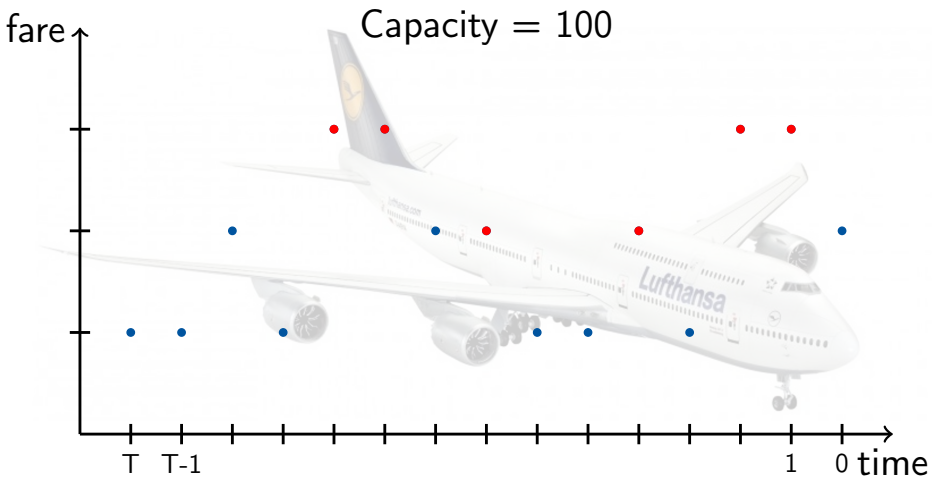




Change of Capacity





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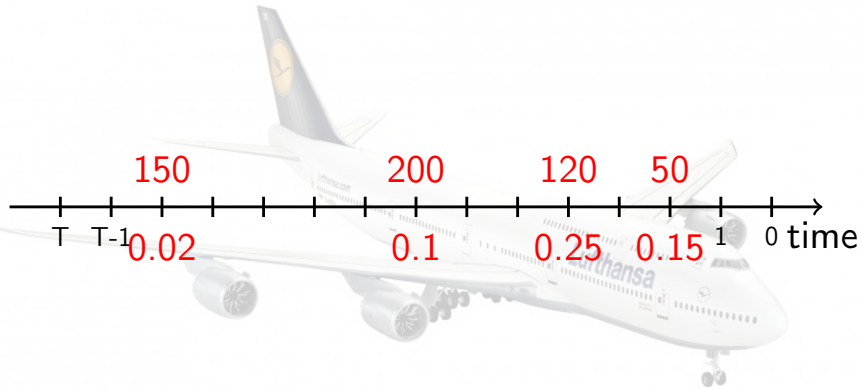

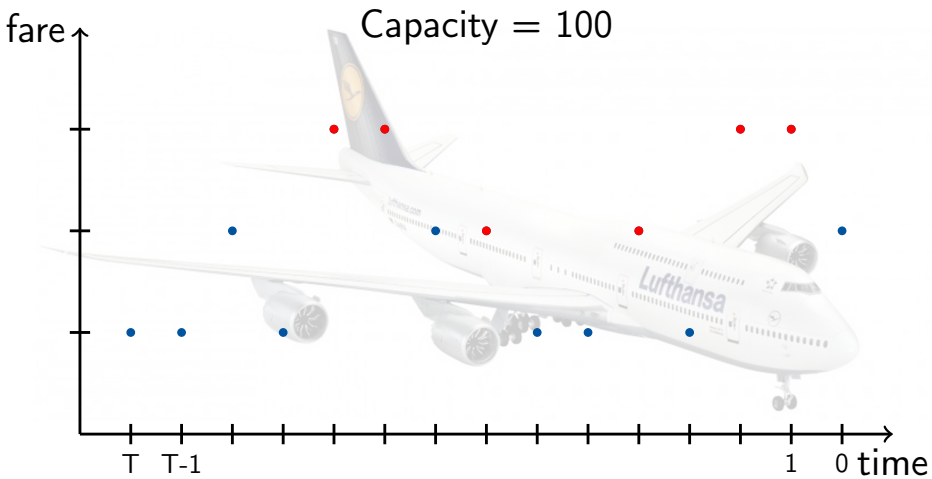

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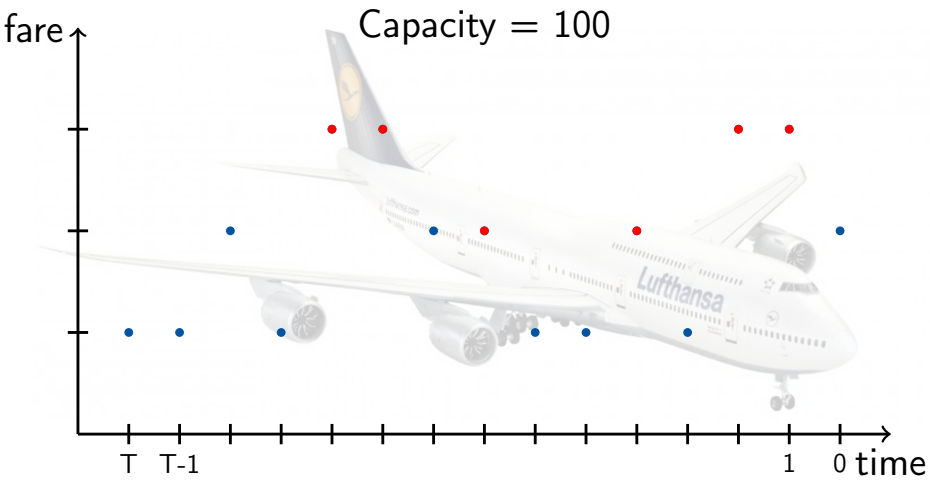
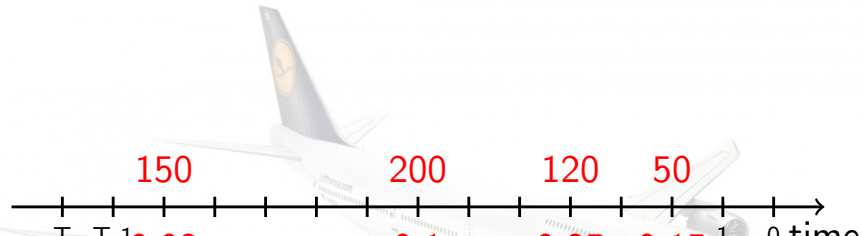
Results

- IP-formulation
- Combinatorial algorithm
- Computational study



Change of Capacity



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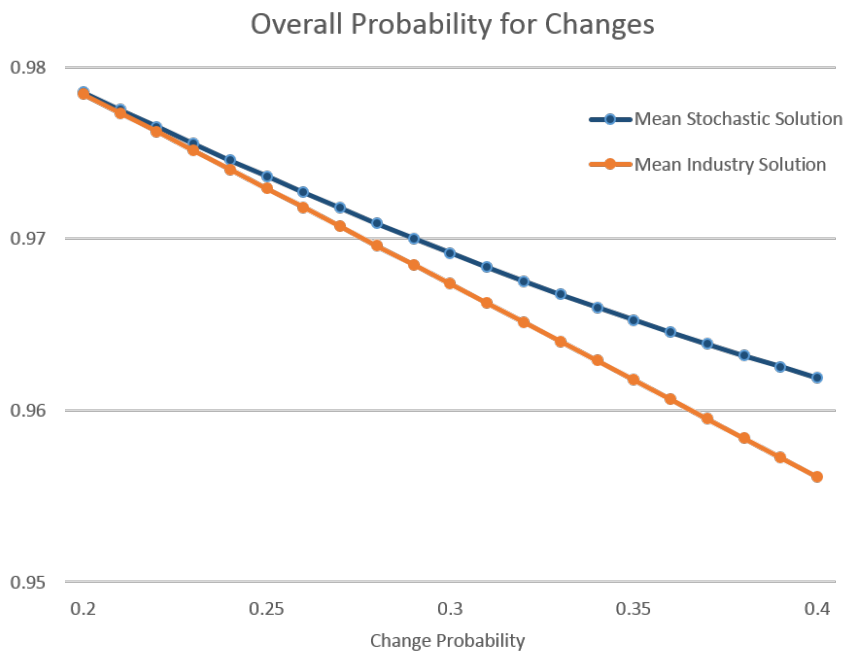
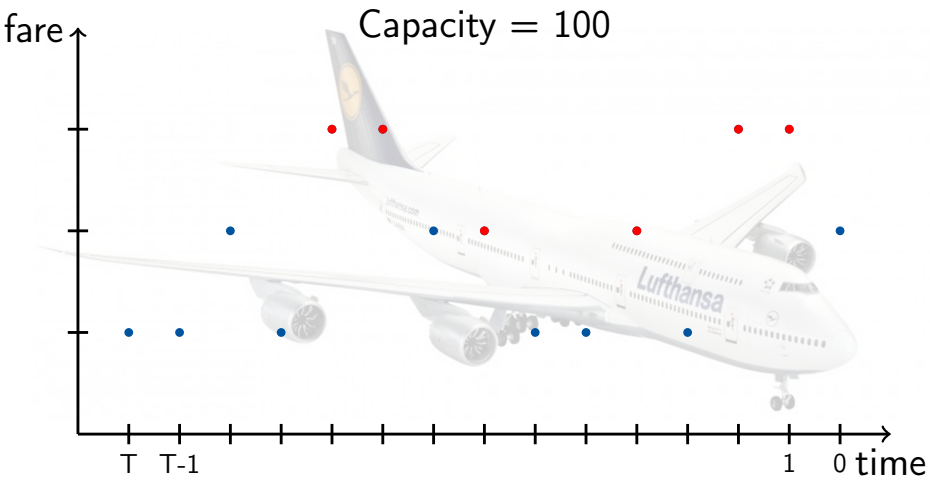
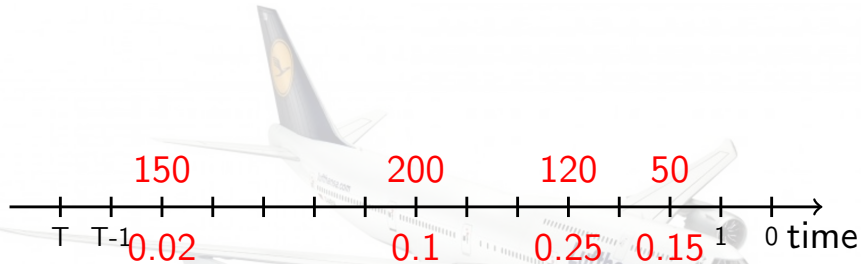




Change of Capacity



Results

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Future Work

- Network
- Uncertain probability