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# Dealing with Uncertainties

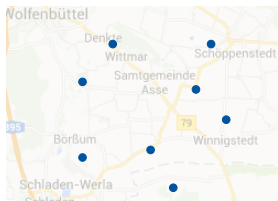
a Recoverable Robust Approach

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Christina Büsing



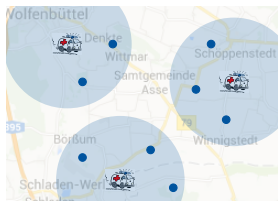
## Normal



## Deterministic Optimization

- ▶ find the best solution

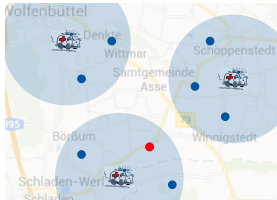
## Normal



## Deterministic Optimization

- ▶ find the best solution

## Normal

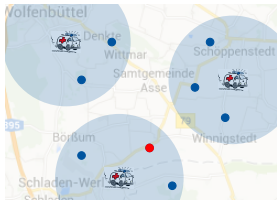


## Deterministic Optimization

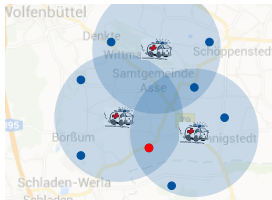
- ▶ find the best solution

# Recoverable Robustness

## Normal



## Robust



## Deterministic Optimization

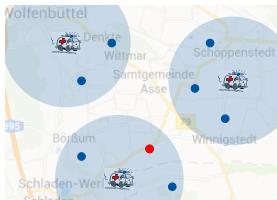
- ▶ find the best solution

## Robust Optimization

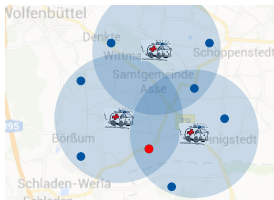
- ▶ find the best solution
- ▶ feasible in any **reasonable situation**

# Recoverable Robustness

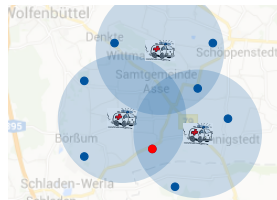
## Normal



## Robust



## Recoverable Robust



## Deterministic Optimization

- ▶ find the best solution

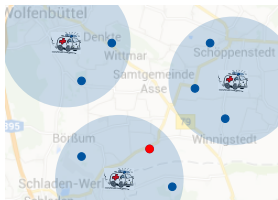
## Robust Optimization

- ▶ find the best solution
- ▶ feasible in any **reasonable situation**

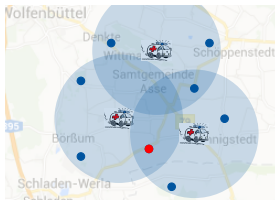
## Recoverable Robust Optimization

- ▶ find the best solution
- ▶ rapidly **adaptable** to any reasonable situation

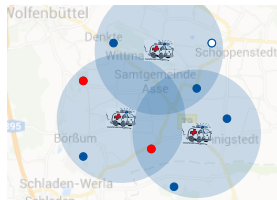
## Normal



## Robust



## Recoverable Robust



## Deterministic Optimization

- ▶ find the best solution

## Robust Optimization

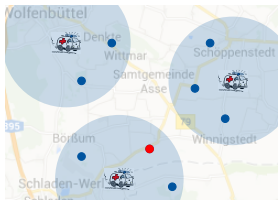
- ▶ find the best solution
- ▶ feasible in any **reasonable situation**

## Recoverable Robust Optimization

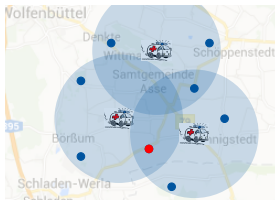
- ▶ find the best solution
- ▶ rapidly **adaptable** to any reasonable situation

# Recoverable Robustness

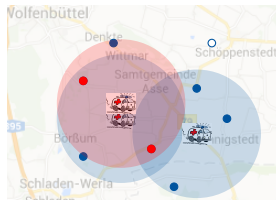
## Normal



## Robust



## Recoverable Robust



## Deterministic Optimization

- ▶ find the best solution

## Robust Optimization

- ▶ find the best solution
- ▶ feasible in any **reasonable situation**

## Recoverable Robust Optimization

- ▶ find the best solution
- ▶ rapidly **adaptable** to any reasonable situation



## Mixed Integer Program

$$\min c^\top x$$

$$Ax \geq b$$

$$x \geq 0$$

- ▶ shortest path, matching
- ▶ facility location, steiner tree, scheduling

## Mixed Integer Program

$$\min c^T x$$

$$Ax \geq b$$

$$x \geq 0$$

- ▶ shortest path, matching
- ▶ facility location, steiner tree, scheduling

## Data and Uncertainty

413096	ZTUG2011-11-01-0M/CD	KD-SKAMM-ZUGA2012-01-ZUGA2012-01	4,59939+12 11	11/ST	5 1m Kalender	ZUGA2011-11-ZUGA2011-11
414094	ZUGA2011-11-01-0M/CD	SM-26ANM-ZUGA2012-01-ZUGA2012-01	4,28935+12 11	11/ST	5 1m Kalender	ZUGA2011-11-ZUGA2011-11
414847	ZUGA2011-11-01-0M/CD	SM-16ANM-ZUGA2012-01-ZUGA2012-01	3,90315+12 11	11/ST	5 1m Kalender	ZUGA2011-11-ZUGA2011-11
413096	ZTUG2011-11-01-0M/CD	KD-SKAMM-ZUGA2012-01-ZUGA2012-01	4,59939+12 11	11/ST	5 1m Kalender	ZUGA2011-11-ZUGA2011-11
437207	0MAUG2011-11-01-0M/CD	KD-800NF-0MAUG2012-01-0MAUG2012-01	4,48248+12 11	11/ST	10 Erlaubung 6	ZUGA2011-11-ZUGA2011-11
425846	0MAUG2011-11-01-0M/CD	SM-26ANM-ZUGA2012-01-ZUGA2012-01	4,28934+12 11	11/ST	10 Erlaubung 6	ZUGA2011-11-ZUGA2011-11
426814	0MAUG2011-11-01-0M/CD	SM-16ANM-ZUGA2012-01-ZUGA2012-01	3,90315+12 11	11/ST	10 Erlaubung 6	ZUGA2011-11-ZUGA2011-11
481294	0MAUG2011-11-01-0M/CD	SM-26ANM-ZUGA2012-01-ZUGA2012-01	4,28934+12 11	11/ST	5 1m Kalender	ZUGA2011-11-ZUGA2011-11
476124	0MAUG2011-11-01-0M/CD	SM-16ANM-ZUGA2012-01-ZUGA2012-01	3,90315+12 11	11/ST	5 1m Kalender	ZUGA2011-11-ZUGA2011-11
511769	1ZAUG2011-11-01-UNTERS	WOM-18ANM-ZUGA2012-01-ZUGA2012-01	2,52551+12 11	11/ST	5 1m Kalender	ZUGA2011-11-ZUGA2011-11
564004	0MAUG2011-11-01-0M/CD	SM-26ANM-ZUGA2012-01-ZUGA2012-01	4,28934+12 11	11/ST	5 1m Kalender	ZUGA2011-11-ZUGA2011-11
569742	0MAUG2011-11-01-0M/CD	SM-16ANM-ZUGA2012-01-ZUGA2012-01	3,90315+12 11	11/ST	5 1m Kalender	ZUGA2011-11-ZUGA2011-11
569742	0MAUG2011-11-01-0M/CD	SM-26ANM-ZUGA2012-01-ZUGA2012-01	4,28934+12 11	11/ST	5 1m Kalender	ZUGA2011-11-ZUGA2011-11
589743	1ZAUG2011-11-01-UNTERS	WOM-18ANM-ZUGA2012-01-ZUGA2012-01	2,52551+12 11	11/ST	5 1m Kalender	ZUGA2011-11-ZUGA2011-11
644371	ZUGA2011-11-01-UNTERS	WOM-17SEP-ZUGA2012-01-ZUGA2012-01	3,4235+12 11	11/ST	5 1m Kalender	ZUGA2011-11-ZUGA2011-11
644371	ZUGA2011-11-01-UNTERS	SM-16ANM-ZUGA2012-01-ZUGA2012-01	3,90315+12 11	11/ST	5 1m Kalender	ZUGA2011-11-ZUGA2011-11
644371	ZUGA2011-11-01-UNTERS	SM-26ANM-ZUGA2012-01-ZUGA2012-01	4,28934+12 11	11/ST	5 1m Kalender	ZUGA2011-11-ZUGA2011-11
648323	ZUGA2011-11-01-0M/CD	SM-26ANM-ZUGA2012-01-ZUGA2012-01	4,28934+12 11	11/ST	5 1m Kalender	ZUGA2011-11-ZUGA2011-11
712134	11OCT2011-11-01-0M/CD	SM-26ANM-ZUGA2012-01-ZUGA2012-01	4,28934+12 11	11/ST	5 1m Kalender	11OCT2011-11-11OCT2011-11
764860	11OCT2011-11-01-0M/CD	SM-26ANM-ZUGA2012-01-ZUGA2012-01	4,28934+12 11	11/ST	5 1m Kalender	11OCT2011-11-11OCT2011-11
766872	14OCT2011-11-01-0M/CD	SM-16ANM-ZUGA2012-01-ZUGA2012-01	3,90315+12 11	11/ST	5 1m Kalender	14OCT2011-11-14OCT2011-11
819827	12DEC2011-11-01-0M/CD	KD-800NF-0MAUG2012-01-0MAUG2012-01	4,48248+12 11	11/ST	5 1m Kalender	12DEC2011-11-12DEC2011-11
826846	14DEC2011-11-01-0M/CD	KD-SKAMM-ZUGA2012-01-ZUGA2012-01	4,28934+12 11	11/ST	5 1m Kalender	14DEC2011-11-14DEC2011-11
848286	03NOV2011-11-01-0M/CD	SM-26ANM-ZUGA2012-01-ZUGA2012-01	4,28934+12 11	11/ST	5 1m Kalender	03NOV2011-11-03NOV2011-11
817976	ZFOCT2011-11-MEDI	MULTERTAG-ZUGA2012-01-ZUGA2012-01	3,5178+12 11	11/ST	10 1m Kalender	ZFOCT2011-11-ZFOCT2011-11
817976	ZFOCT2011-11-MEDI	MULTERTAG-ZUGA2012-01-ZUGA2012-01	3,5178+12 11	11/ST	10 Status über c	ZFOCT2011-11-ZFOCT2011-11
147490	09FEB2011-11-01-0M/CD	KD-SKAMM-ZUGA2012-01-ZUGA2012-01	3,9079+12 11	11/ST	5 1m Kalender	09FEB2011-11-09FEB2011-11
112889	06AUG2011-11-01-KONSTRTE	ZBMV-ZUGA2012-01-ZUGA2012-01	3,9624+12 11	11/ST	10 1m Kalender	06AUG2011-11-06AUG2011-11
112889	06AUG2011-11-01-KONSTRTE	ZBMV-ZUGA2012-01-ZUGA2012-01	3,9624+12 11	11/ST	10 Status über c	06AUG2011-11-06AUG2011-11
254841	09FEB2011-11-01-0M/CD	KD-SKAMM-ZUGA2012-01-ZUGA2012-01	3,9079+12 11	11/ST	5 1m Kalender	09FEB2011-11-09FEB2011-11
254841	09FEB2011-11-01-0M/CD	KD-SKAMM-ZUGA2012-01-ZUGA2012-01	3,9079+12 11	11/ST	5 1m Kalender	09FEB2011-11-09FEB2011-11

▶ errorous measurements

▶ future events, fluctuations

## Mixed Integer Program

$$\min c^T x$$

$$Ax \geq b$$

$$x \geq 0$$

- ▶ shortest path, matching
- ▶ facility location, steiner tree, scheduling

## Data and Uncertainty

41308	27JUL2011-11:0-0M/CD	KD-3KAMNH	25JUL2012:0	25JUL2012:0	4,59939+12 11	11:51	5 im Kalender	26JUL2011-11:27JUL2011
41309	26JUL2011-11:0-0M/CD	SM-2KAMNH	24JUL2012:0	24JUL2012:0	4,28305+12 11	11:51	5 im Kalender	26JUL2011-11:26JUL2011
41340	26JUL2011-11:0-0M/CD	SM-2KAMNH	24JUL2012:0	24JUL2012:0	3,90319+12 11	11:51	5 im Kalender	26JUL2011-11:26JUL2011
41309	27JUL2011-11:0-0M/CD	KD-3KAMNH	25JUL2012:0	25JUL2012:0	4,59939+12 11	11:51	5 im Kalender	26JUL2011-11:27JUL2011
43720	02AUG2011-11:0-0M/CD	KD-80WVF	30JUL2012:0	30JUL2012:0	4,40288+12 11	11:51	10 Erweiterung 6	25JUL2011-11:30AUG2011
43560	01AUG2011-11:0-0M/CD	SM-2KAMNH	27JUL2012:0	27JUL2012:0	1,20494+12 11	11:51	10 Erweiterung 6	25JUL2011-11:30AUG2011
43814	02AUG2011-11:0-0M/CD	SM-2KAMNH	28JUL2012:0	28JUL2012:0	1,20494+12 11	11:51	10 Erweiterung 6	25JUL2011-11:30AUG2011
48200	02AUG2011-11:0-0M/CD	SM-2KAMNH	28JUL2012:0	28JUL2012:0	1,20494+12 11	11:51	10 Erweiterung 6	25JUL2011-11:30AUG2011
47626	04AUG2011-11:0-0M/CD	SM-2KAMNH	29AUG2012:0	29AUG2012:0	1,20494+12 11	11:51	5 im Kalender	30JUL2011-11:09AUG2011
51769	12AUG2011-11:0-0M/CD	SM-2KAMNH	29AUG2012:0	29AUG2012:0	2,52021+12 11	11:51	5 im Kalender	30JUL2011-11:12AUG2011
56400	04AUG2011-11:0-0M/CD	SM-2KAMNH	29AUG2012:0	29AUG2012:0	1,20494+12 11	11:51	10 Erweiterung 6	25JUL2011-11:30AUG2011
56910	04AUG2011-11:0-0M/CD	SM-2KAMNH	29AUG2012:0	29AUG2012:0	1,20494+12 11	11:51	10 Erweiterung 6	25JUL2011-11:30AUG2011
56930	04AUG2011-11:0-0M/CD	SM-2KAMNH	29AUG2012:0	29AUG2012:0	1,20494+12 11	11:51	10 Erweiterung 6	25JUL2011-11:30AUG2011
56970	04AUG2011-11:0-0M/CD	SM-2KAMNH	29AUG2012:0	29AUG2012:0	1,20494+12 11	11:51	10 Erweiterung 6	25JUL2011-11:30AUG2011
64471	20SEP2011-11:0-0M/CD	SM-2KAMNH	17SEP2012:0	17SEP2012:0	3,4226+12 11	11:51	5 im Kalender	19SEP2011-11:20SEP2011
64472	20SEP2011-11:0-0M/CD	SM-2KAMNH	17SEP2012:0	17SEP2012:0	3,4226+12 11	11:51	5 im Kalender	19SEP2011-11:20SEP2011
71111	11OCT2011-11:0-0M/CD	SM-2KAMNH	09OCT2012:0	09OCT2012:0	1,20494+12 11	11:51	5 im Kalender	10OCT2011-11:09OCT2011
71112	11OCT2011-11:0-0M/CD	SM-2KAMNH	09OCT2012:0	09OCT2012:0	1,20494+12 11	11:51	5 im Kalender	11OCT2011-11:11OCT2011
76870	14OCT2011-11:0-0M/CD	SM-2KAMNH	11OCT2012:0	11OCT2012:0	3,98035+12 11	11:51	5 im Kalender	11OCT2011-11:14OCT2011
81967	12OCT2011-11:0-0M/CD	SM-2KAMNH	11OCT2012:0	11OCT2012:0	3,97912+12 11	11:51	5 im Kalender	11OCT2011-11:14OCT2011
84846	14OCT2011-11:0-0M/CD	SM-2KAMNH	11OCT2012:0	11OCT2012:0	3,97912+12 11	11:51	5 im Kalender	11OCT2011-11:14OCT2011
84820	03NOV2011-11:0-0M/CD	SM-2KAMNH	01NOV2012:0	01NOV2012:0	1,20494+12 11	11:51	5 im Kalender	03NOV2011-11:03NOV2011
81795	27OCT2011-11:MEDI	MULTERTAG	04AUG2012:1	04AUG2012:1	3,51776+12 11	11:51	10 im Kalender	27OCT2011-11:27OCT2011
81795	27OCT2011-11:MEDI	MULTERTAG	04AUG2012:0	04AUG2012:0	3,51776+12 11	11:51	10 Status über c	27OCT2011-11:27OCT2011
147400	09FEB2012-11:0-0M/CD	KD-3KAMNH	01AUG2012:0	01AUG2012:0	3,46792+12 11	11:51	5 im Kalender	09FEB2012-11:09FEB2012
112889	06AUG2012-11:0-0M/CD	SM-2KAMNH	23MAY2012:0	23MAY2012:0	3,9624+12 11	11:51	10 im Kalender	06AUG2012-11:06AUG2012
112889	06AUG2012-11:0-0M/CD	SM-2KAMNH	23MAY2012:0	23MAY2012:0	3,9624+12 11	11:51	10 Status über c	06AUG2012-11:06AUG2012
25441	04FEB2012-11:0-0M/CD	KD-3KAMNH	02AUG2012:0	02AUG2012:0	3,67725+12 11	11:51	5 im Kalender	04FEB2012-11:04FEB2012
25441	04FEB2012-11:0-0M/CD	KD-3KAMNH	02AUG2012:0	02AUG2012:0	3,67725+12 11	11:51	5 Erweiterung	04FEB2012-11:04FEB2012

▶ erroneous measurements

▶ future events, fluctuations

→ build scenario set  $\mathcal{S}$

▶  $S \in \mathcal{S}: A^S$  and  $c^S$

## Mixed Integer Program

$$\min c^T x$$

$$Ax \geq b$$

$$x \geq 0$$

- ▶ shortest path, matching
- ▶ facility location, steiner tree, scheduling

## Objective

- ▶ stable solution
- ▶ small cost
- ▶ rapidly adaptable

## Data and Uncertainty

41308	27JUL2011:1:0:SM/CD	KD-3KAMNH	25AUG2012:0:25AUG2012:0:	4,59939+12:11	11:57	5 im Kalender	26AUG2011:1:27AUG2011
41309	26JUL2011:1:0:SM/CD	SM-2KAMNH	25AUG2012:0:25AUG2012:0:	4,28203+12:11	11:57	5 im Kalender	26AUG2011:1:26AUG2011
41309	27JUL2011:1:0:SM/CD	SM-3KAMNH	25AUG2012:0:25AUG2012:0:	3,90319+12:11	11:57	5 im Kalender	26AUG2011:1:26AUG2011
41309	27JUL2011:1:0:SM/CD	KD-3KAMNH	25AUG2012:0:25AUG2012:0:	4,59939+12:11	11:57	5 im Kalender	26AUG2011:1:27AUG2011
43702	05AUG2011:0:0:SM/CD	KD-80WVF	06AUG2012:0:06AUG2012:0:	4,00208+12:11	11:57	10 Erweiterung 6	25AUG2011:0:06AUG2011
43560	01AUG2011:0:0:SM/CD	SM-2KAMNH	27JUL2012:0:1:27AUG2012:0:	1,20939+12:11	11:57	10 Erweiterung 6	25AUG2011:0:06AUG2011
43814	01AUG2011:0:0:SM/CD	SM-2KAMNH	27JUL2012:0:1:27AUG2012:0:	1,20939+12:11	11:57	10 Erweiterung 6	25AUG2011:0:06AUG2011
48200	01AUG2011:0:0:SM/CD	SM-2KAMNH	27JUL2012:0:1:27AUG2012:0:	1,20939+12:11	11:57	10 Erweiterung 6	25AUG2011:0:06AUG2011
47626	01AUG2011:0:0:SM/CD	SM-2KAMNH	27JUL2012:0:1:27AUG2012:0:	1,20939+12:11	11:57	10 Erweiterung 6	25AUG2011:0:06AUG2011
50810	01AUG2011:0:0:SM/CD	SM-2KAMNH	27JUL2012:0:1:27AUG2012:0:	1,20939+12:11	11:57	5 im Kalender	18AUG2011:1:18AUG2011
51709	17AUG2011:0:0:UNTER	W0H	18AUG2012:0:18AUG2012:0:	2,32025+12:11	11:57	5 im Kalender	17AUG2011:1:17AUG2011
56400	01AUG2011:0:0:SM/CD	SM-2KAMNH	27JUL2012:0:1:27AUG2012:0:	1,20939+12:11	11:57	5 im Kalender	17AUG2011:1:17AUG2011
56910	01AUG2011:0:0:SM/CD	SM-2KAMNH	27JUL2012:0:1:27AUG2012:0:	1,20939+12:11	11:57	5 im Kalender	17AUG2011:1:17AUG2011
56910	01AUG2011:0:0:SM/CD	SM-2KAMNH	27JUL2012:0:1:27AUG2012:0:	1,20939+12:11	11:57	5 im Kalender	17AUG2011:1:17AUG2011
56910	01AUG2011:0:0:SM/CD	SM-2KAMNH	27JUL2012:0:1:27AUG2012:0:	1,20939+12:11	11:57	5 im Kalender	17AUG2011:1:17AUG2011
64471	20SEP2011:0:0:UNTER	17SEP2012:0:17SEP2012:0:	3,4226+12:11	11:57	5 im Kalender	19SEP2011:0:20SEP2011	
64471	20SEP2011:0:0:UNTER	17SEP2012:0:17SEP2012:0:	3,4226+12:11	11:57	5 im Kalender	19SEP2011:0:20SEP2011	
64471	20SEP2011:0:0:UNTER	17SEP2012:0:17SEP2012:0:	3,4226+12:11	11:57	5 im Kalender	19SEP2011:0:20SEP2011	
71111	11OCT2011:0:0:SM/CD	SM-2KAMNH	09OCT2012:0:09OCT2012:0:	3,99035+12:11	11:57	5 im Kalender	09OCT2011:1:09OCT2011
76880	11OCT2011:0:0:SM/CD	SM-2KAMNH	09OCT2012:0:09OCT2012:0:	3,99035+12:11	11:57	5 im Kalender	11OCT2011:1:11OCT2011
76870	14OCT2011:0:0:SM/CD	SM-3KAMNH	11AUG2012:0:11AUG2012:0:	3,57702+12:11	11:57	5 im Kalender	14OCT2011:1:14OCT2011
81907	12OCT2011:0:0:SM/CD	SM-3KAMNH	11AUG2012:0:11AUG2012:0:	3,57702+12:11	11:57	5 im Kalender	12OCT2011:1:12OCT2011
84886	14OCT2011:0:0:SM/CD	SM-2KAMNH	11AUG2012:0:11AUG2012:0:	3,57702+12:11	11:57	5 im Kalender	14OCT2011:1:14OCT2011
84886	14OCT2011:0:0:SM/CD	SM-2KAMNH	11AUG2012:0:11AUG2012:0:	3,57702+12:11	11:57	5 im Kalender	14OCT2011:1:14OCT2011
84886	14OCT2011:0:0:SM/CD	SM-2KAMNH	11AUG2012:0:11AUG2012:0:	3,57702+12:11	11:57	5 im Kalender	14OCT2011:1:14OCT2011
81370	17OCT2011:1:MEDI	MULTERTAG	06AUG2012:1:06AUG2012:1:	3,51716+12:11	11:57	10 im Kalender	17OCT2011:1:17OCT2011
81370	17OCT2011:1:MEDI	MULTERTAG	06AUG2012:1:06AUG2012:1:	3,51716+12:11	11:57	10 Status über 17OCT2011:1:17OCT2011	
84700	05FEB2012:1:0:SM/CD	KD-3KAMNH	01AUG2012:0:1:01AUG2012:0:	3,60792+12:11	11:57	5 im Kalender	05FEB2012:1:05FEB2012
11289	06AUG2012:1:0:SCHNARTT	23MAY2012:1:23MAY2012:1:	3,9623+12:11	11:57	10 im Kalender	06AUG2012:1:06AUG2012	
11289	06AUG2012:1:0:SCHNARTT	23MAY2012:1:23MAY2012:1:	3,9623+12:11	11:57	10 Status über 06AUG2012:1:06AUG2012		
25441	04FEB2012:1:0:SM/CD	KD-3KAMNH	01AUG2012:1:01AUG2012:1:	3,67712+12:11	11:57	5 im Kalender	04FEB2012:1:04FEB2012
25441	04FEB2012:1:0:SM/CD	KD-3KAMNH	01AUG2012:1:01AUG2012:1:	3,67712+12:11	11:57	5 Erweiterung	04FEB2012:1:04FEB2012

▶ erroneous measurements

▶ future events, fluctuations

→ build scenario set  $\mathcal{S}$

▶  $S \in \mathcal{S}: A^S$  and  $c^S$

## Mixed Integer Program

$$\min c^T x$$

$$Ax \geq b$$

$$x \geq 0$$

- ▶ shortest path, matching
- ▶ facility location, steiner tree, scheduling

## Objective

- ▶ stable solution
- ▶ small cost
- ▶ rapidly adaptable

## Data and Uncertainty

41308	2704201211:0:0M/CD	KD-SAMMR20A12012:0:20A12012:0	4,59939+12:01	10:ST	5 Im Kalender 26042012:1:27042012
41309	2604201211:0:0M/CD	SM-SAMMR20A12012:0:26A12012:0	4,28205+12:01	10:ST	5 Im Kalender 26042012:1:26042012
41309	2704201211:0:0M/CD	KD-SAMMR20A12012:0:20A12012:0	4,59939+12:01	10:ST	5 Im Kalender 26042012:1:27042012
43700	05AUG201211:0:0M/CD	KD-BRUPF 05AUG2012:0:05AUG2012:0	4,40820+12:01	10:ST	10 Erhebung 6 26042012:1:05AUG2012
43700	05AUG201211:0:0M/CD	SM-SAMMR27042012:0:07AUG2012:0	4,29434+12:01	10:ST	10 Erhebung 6 26042012:1:05AUG2012
43804	05AUG201211:0:0M/CD	KD-SAMMR27042012:0:07AUG2012:0	4,29434+12:01	10:ST	10 Im Kalender 05AUG2012:1:07AUG2012
48200	05AUG201211:0:0M/CD	KD-SAMMR27042012:0:07AUG2012:0	4,29434+12:01	10:ST	10 Im Kalender 05AUG2012:1:07AUG2012
48226	05AUG201211:0:0M/CD	KD-SAMMR27042012:0:07AUG2012:0	4,29434+12:01	10:ST	10 Im Kalender 05AUG2012:1:09AUG2012
50800	05AUG201211:0:0M/CD	SM-SAMMR24AUG2012:0:24AUG2012:0	5,36043+12:01	10:ST	5 Im Kalender 05AUG2012:1:19AUG2012
51700	12AUG201211:UNTER	SM-SAMMR201211:0AUG2012:0	2,30255+12:01	10:CD	5 Im Kalender 05AUG2012:1:12AUG2012
56400	05AUG201211:0:0M/CD	KD-SAMMR27042012:0:07AUG2012:0	4,29434+12:01	10:ST	10 Im Kalender 05AUG2012:1:05AUG2012
56900	05AUG201211:0:0M/CD	KD-SAMMR27042012:0:07AUG2012:0	4,29434+12:01	10:ST	10 Im Kalender 05AUG2012:1:09AUG2012
56900	05AUG201211:0:0M/CD	SM-SAMMR24AUG2012:0:24AUG2012:0	5,22038+12:01	10:ST	5 Im Kalender 05AUG2012:1:19AUG2012
64471	20SEP201211:UNTER	17SEP2012:0:17SEP2012:0	3,4226+12:01	10:CD	Erhebung 19SEP2012:0:20SEP2012
64471	20SEP201211:UNTER	17SEP2012:0:17SEP2012:0	3,4226+12:01	10:CD	Erhebung 19SEP2012:0:20SEP2012
71111	11OCT201211:0:0M/CD	SM-SAMMR09OCT2012:0:09OCT2012:0	4,11911+12:01	10:ST	5 Im Kalender 09OCT2012:1:09OCT2012
71111	11OCT201211:0:0M/CD	SM-SAMMR09OCT2012:0:09OCT2012:0	4,11911+12:01	10:ST	5 Im Kalender 11OCT2012:1:11OCT2012
76870	14OCT201211:0:0M/CD	SM-SAMMR11AUG2012:0:11AUG2012:0	3,57702+12:01	10:ST	5 Im Kalender 11OCT2012:1:14OCT2012
81807	27OCT201211:0:0M/CD	SM-SAMMR11AUG2012:0:11AUG2012:0	3,57702+12:01	10:ST	5 Im Kalender 27OCT2012:1:14OCT2012
82886	14OCT201211:0:0M/CD	SM-SAMMR11AUG2012:0:11AUG2012:0	3,57702+12:01	10:ST	5 Im Kalender 14OCT2012:1:14OCT2012
84500	03NOV201211:0:0M/CD	SM-SAMMR03NOV2012:0:03NOV2012:0	4,29434+12:01	10:ST	5 Im Kalender 03NOV2012:1:03NOV2012
81370	27OCT201211:MEDI	MULTERTAG 05AUG2012:1:05AUG2012:1	3,51704+12:01	10:PO	10 Im Kalender 27OCT2012:1:27OCT2012
81370	27OCT201211:MEDI	MULTERTAG 05AUG2012:1:05AUG2012:1	3,51704+12:01	10:PO	10 Status über c 27OCT2012:1:27OCT2012
84700	05FEB201211:0:0M/CD	KD-SAMMR05FEB2012:0:05FEB2012:0	3,60770+12:01	10:CD	5 Im Kalender 05FEB2012:1:05FEB2012
112800	05AUG201211:0:SCHNARTT	20MAY2012:1:20MAY2012:1	3,9623+12:01	10:CD	10 Im Kalender 05AUG2012:1:05AUG2012
112800	05AUG201211:0:SCHNARTT	20MAY2012:1:20MAY2012:1	3,9623+12:01	10:CD	10 Status über c 05AUG2012:1:05AUG2012
25444	09FEB201211:0:0M/CD	KD-SAMMR09FEB2012:0:09FEB2012:0	3,67775+12:01	10:ST	5 Im Kalender 09FEB2012:1:09FEB2012
25444	09FEB201211:0:0M/CD	KD-SAMMR09AUG2012:1:09AUG2012:1	3,67775+12:01	10:ST	5 Sternung 09FEB2012:1:09FEB2012

▶ erroneous measurements

▶ future events, fluctuations

⇒ build scenario set  $\mathcal{S}$

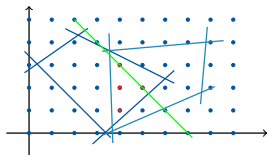
▶  $S \in \mathcal{S}: A^S$  and  $c^S$

## (Recoverable) Robustness

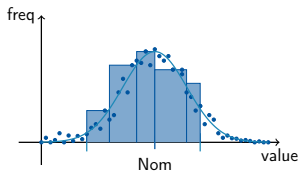
$$\min c^T x$$

$$A^S x + B y^S \geq b \quad \forall S \in \mathcal{S}$$

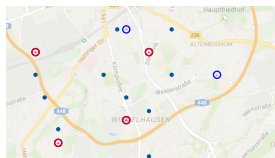
$$x, y^S \geq 0 \quad \forall S \in \mathcal{S}$$



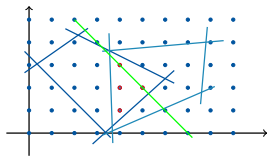
## Exact Algorithms and Heuristics



## Generation of Scenario Sets

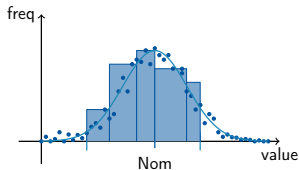


## Applications

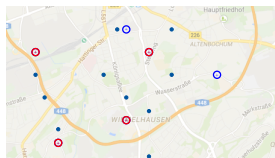


## Exact Algorithms and Heuristics

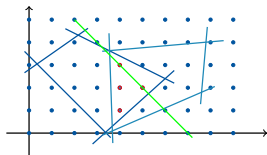
- ▶ compact formulations & cutting planes
- ▶ local search, ...



## Generation of Scenario Sets

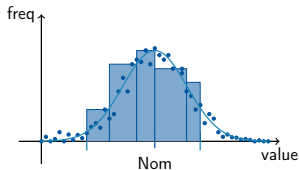


## Applications



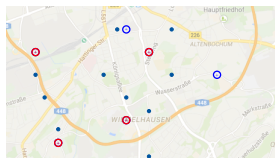
## Exact Algorithms and Heuristics

- ▶ compact formulations & cutting planes
- ▶ local search, ...



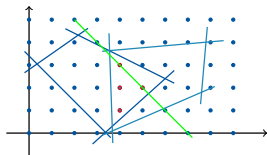
## Generation of Scenario Sets

- ▶ automated generation
- ▶ robustness measurements



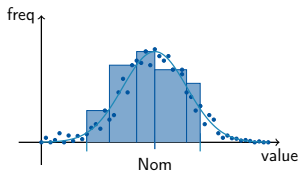
## Applications





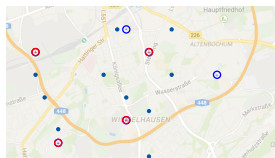
## Exact Algorithms and Heuristics

- ▶ compact formulations & cutting planes
- ▶ local search, ...



## Generation of Scenario Sets

- ▶ automated generation
- ▶ robustness measurements



## Applications

- ▶ robust ambulance optimization
- ▶ revenue management

# Generation of Scenario Sets

id	cost	type	category	description	start	end
1	358E+12	I	I1LST	5 Im Kalender	31AUG2011	31AUG2011
2	358E+12	I	I1LST	5 Im Kalender	01AUG2011	01AUG2011
3	423E+12	I	IM1K	5 Stornierungs	19SEP2011:0	20SEP2011:
4	423E+12	I	IM1K	5 Stornierungs	19SEP2011:0	20SEP2011:
5	5708E+12	I	I1LST	5 Im Kalender	19SEP2011:1	29SEP2011:
6	3114E+11	T	I1LST	5 Kalender	04OCT2011:1	04OCT2011
7	3465E+12	I	I1LST	5 Im Kalender	11OCT2011:1	11OCT2011
8	29754E+11	I	I1LST	5 Im Kalender	14OCT2011:1	14OCT2011
9	4495E+10	I	I1LST	5 Stornierungs	03AUG2011:1	12DEC2011
10	1264E+12	I	I1LST	5 Im Kalender	16DEC2011:1	16DEC2011
11	2934E+12	I	I1LST	5 Im Kalender	03NOV2011:0	03NOV2011:
12	35175E+12	I	I1PO	10 Kalender	07OCT2011:1	27OCT2011
13	35175E+12	I	I1PO	10 Status über c	27OCT2011:1	27OCT2011
14	56679E+12	I	I1ECHO	5 Res. ROM/i1	13JAN2012:1	06FEB2012:
15	3962E+11	I	I1ECHO	10 Im Kalender	06JAN2012:1	06JAN2012
16	3962E+11	I	I1ECHO	10 Status über c	06JAN2012:1	06JAN2012
17	30771E+12	I	I1LST	5 Im Kalender	06FEB2012:1	06FEB2012:
18	30771E+12	I	I1LST	5 Stornierungs	06FEB2012:1	06FEB2012:

## Mixed Integer Program

$$\min c^T x$$

$$Ax \geq b$$

$$x \geq 0$$

# Generation of Scenario Sets

id	value	unit	type	description	start	end
:0	3,358E+12	I	I1LST	5 Im Kalender	31AUG2011	31AUG2011
:0	3,423E+12	I	IM1K	5 Stornierungs	19SEP2011:0	20SEP2011
:0	3,423E+12	I	IM1K	5 Stornierungs	19SEP2011:0	20SEP2011
:1	2,5708E+12	I	I1LST	5 Im Kalender	19SEP2011:1	29SEP2011
:1	6,3114E+11	I	I1LST	5 Im Kalender	04OCT2011:1	04OCT2011
:1	3,9465E+12	I	I1LST	5 Im Kalender	11OCT2011:1	11OCT2011
:1	2,9754E+11	I	I1LST	5 Im Kalender	14OCT2011:1	14OCT2011
:2	1,4495E+10	I	I1LST	5 Stornierungs	03AUG2011:1	03SEP2011
:1	1,264E+12	I	I1LST	5 Im Kalender	16DEC2011:1	16DEC2011
:0	1,2934E+12	I	I1LST	5 Im Kalender	03NOV2011:0	03NOV2011
:1	3,5175E+12	I	I1PO	10 Im Kalender	07OCT2011:1	27OCT2011
:1	3,5175E+12	I	I1PO	10 Status über c	27OCT2011:1	27OCT2011
:1	5,6679E+12	I	I1ECHO	5 Res. ROM/i1	13JAN2012:1	06FEB2012
:2	3,962E+11	I	I1ECHO	10 Im Kalender	06JAN2012:1	06JAN2012
:2	3,962E+11	I	I1ECHO	10 Status über c	06JAN2012:1	06JAN2012
:2	3,0771E+12	I	I1LST	5 Im Kalender	06FEB2012:1	06FEB2012
:2	3,0771E+12	I	I1LST	5 Stornierungs	06FEB2012:1	06FEB2012

## Mixed Integer Program

$$\min c^T x$$

$$Ax \geq b \quad S \in \mathcal{S}$$

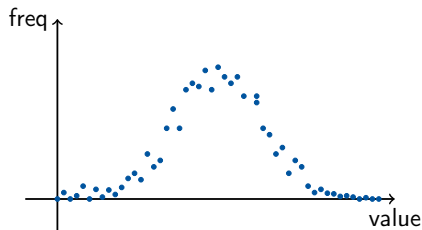
$$x \geq 0$$

# Generation of Scenario Sets

Mixed Integer Program				
1:1	3,358E+12	11	I1LST	5 Im Kalender 31AUG2011:131AUG2011
1:0	3,423E+12	11	IM1K	5 Stornierungs 19SEP2011:0120SEP2011
1:0	3,423E+12	11	IM1K	5 Stornierungs 19SEP2011:0120SEP2011
1:4	2,5708E+12	11	I1LST	5 Im Kalender 19SEP2011:129SEP2011
1:1	6,3114E+11	11	I1LST	5 Im Kalender 04OCT2011:104OCT2011
1:4	3,9465E+12	11	I1LST	5 Im Kalender 11OCT2011:11OCT2011
1:4	2,9754E+11	11	I1LST	5 Im Kalender 14OCT2011:114OCT2011
2:1	1,4495E+10	11	I1LST	5 Stornierungs 03AUG2011:03SEP2011
1:1	1,264E+12	11	I1LST	5 Im Kalender 16DEC2011:16DEC2011
1:0	1,2934E+12	11	I1LST	5 Im Kalender 03NOV2011:03NOV2011
1:1	3,5175E+12	11	I1PO	10 Im Kalender 07OCT2011:127OCT2011
1:0	3,5175E+12	11	I1PO	10 Status über c 27OCT2011:127OCT2011
1:0	5,6679E+12	11	I1ECHO	5 Res. ROM/1 13JAN2012:106FEB2012
2:0	3,962E+11	11	I1ECHO	10 Im Kalender 06JAN2012:106JAN2012
2:4	3,962E+11	11	I1ECHO	10 Status über c 06JAN2012:106JAN2012
2:0	3,0771E+12	11	I1LST	5 Im Kalender 06FEB2012:106FEB2012
2:0	3,0771E+12	11	I1LST	5 Stornierungs 06FEB2012:106FEB2012

$$\min c^T x$$
$$ASx \geq b, S \in \mathcal{S}$$
$$x \geq 0$$

## Scenario Set

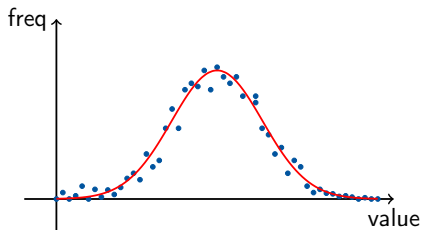


# Generation of Scenario Sets

Mixed Integer Program						
1:1	3,358E+12	11	I1LST	5	Im Kalender	31AUG2011:131AUG2011
1:0	3,423E+12	11	IM1K	5	Stornierungs	19SEP2011:0120SEP2011
1:4	2,5708E+12	11	I1LST	5	Im Kalender	19SEP2011:129SEP2011
1:1	6,3114E+11	11	I1LST	5	Im Kalender	04OCT2011:104OCT2011
1:4	3,9465E+12	11	I1LST	5	Im Kalender	11OCT2011:111OCT2011
1:4	2,9754E+11	11	I1LST	5	Im Kalender	14OCT2011:114OCT2011
2:1	1,4495E+10	11	I1LST	5	Stornierungs	03AUG2011:03SEP2011
1:1	1,264E+12	11	I1LST	5	Im Kalender	16DEC2011:116DEC2011
1:0	1,2934E+12	11	I1LST	5	Im Kalender	03NOV2011:03NOV2011
1:1	3,5175E+12	11	I1PO	10	Im Kalender	07OCT2011:127OCT2011
1:0	3,5175E+12	11	I1PO	10	Status über c	27OCT2011:127OCT2011
1:0	5,6679E+12	11	I1ECHO	5	Res. ROM/1	13JAN2012:106FEB2012
2:0	3,962E+11	11	I1ECHO	10	Im Kalender	06JAN2012:106JAN2012
2:4	3,962E+11	11	I1ECHO	10	Status über c	06JAN2012:106JAN2012
2:0	3,0771E+12	11	I1LST	5	Im Kalender	06FEB2012:106FEB2012
2:0	3,0771E+12	11	I1LST	5	Stornierungs	06FEB2012:106FEB2012

$\min c^T x$   
 $ASx \geq b, S \in S$   
 $x \geq 0$

## Scenario Set

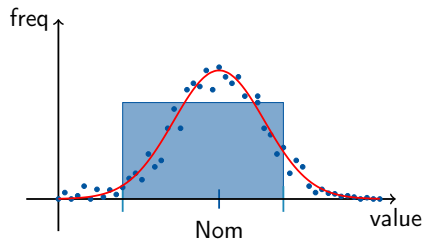


# Generation of Scenario Sets

Mixed Integer Program			
1:1	3,358E+12	11	I1LST 5 Im Kalender 31AUG2011:131AUG2011
1:0	3,423E+12	11	IM1K 5 Stornierungs 19SEP2011:0120SEP2011
1:4	2,5708E+12	11	I1LST 5 Im Kalender 19SEP2011:129SEP2011
1:1	6,3114E+11	11	I1LST 5 Im Kalender 04OCT2011:1104OCT2011
1:4	3,9465E+12	11	I1LST 5 Im Kalender 11OCT2011:111OCT2011
1:4	2,9754E+11	11	I1LST 5 Im Kalender 14OCT2011:114OCT2011
2:1	1,4495E+10	11	I1LST 5 Stornierungs 13AUG2011:110SEP2011
1:1	1,264E+12	11	I1LST 5 Im Kalender 16DEC2011:116DEC2011
1:0	1,2934E+12	11	I1LST 5 Im Kalender 03NOV2011:03NOV2011
1:1	3,5175E+12	11	I1PO 10 Im Kalender 07OCT2011:1127OCT2011
1:0	3,5175E+12	11	I1PO 10 Status Über c 27OCT2011:1127OCT2011
1:0	5,6679E+12	11	I1ECHO 5 Res. ROM/1 13JAN2012:1106FEB2012
2:0	3,962E+11	11	I1ECHO 10 Im Kalender 06JAN2012:1106JAN2012
2:4	3,962E+11	11	I1ECHO 10 Status Über c 06JAN2012:1106JAN2012
2:0	3,0771E+12	11	I1LST 5 Im Kalender 06FEB2012:1106FEB2012
2:0	3,0771E+12	11	I1LST 5 Stornierungs 06FEB2012:1106FEB2012

$\min c^T x$   
 $ASx \geq b, S \in S$   
 $x \geq 0$

## Scenario Set

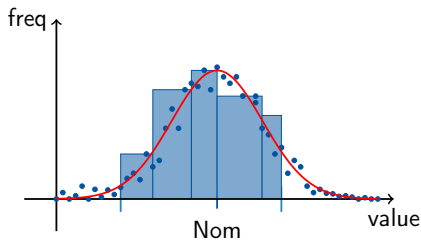


# Generation of Scenario Sets

Mixed Integer Program							
1:1	3,358E+12	1	I1LST	5	Im Kalender	31AUG2011:1	31AUG2011
1:0	3,423E+12	1	IM1K	5	Stornierungs	19SEP2011:0	20SEP2011
1:4	2,5708E+12	1	I1LST	5	Im Kalender	19SEP2011:1	29SEP2011
1:1	6,3114E+11	1	I1LST	5	Im Kalender	04OCT2011:1	04OCT2011
1:4	3,9465E+12	1	I1LST	5	Im Kalender	11OCT2011:1	11OCT2011
1:4	2,9754E+11	1	I1LST	5	Im Kalender	14OCT2011:1	14OCT2011
2:1	1,4495E+10	1	I1LST	5	Stornierungs	03AUG2011:1	03AUG2011
1:1	1,264E+12	1	I1LST	5	Im Kalender	16DEC2011:1	16DEC2011
1:0	1,2934E+12	1	I1LST	5	Im Kalender	03NOV2011:1	03NOV2011
1:1	3,5175E+12	1	I1PO	10	Im Kalender	27OCT2011:1	27OCT2011
1:0	3,5175E+12	1	I1PO	10	Status über c	27OCT2011:1	27OCT2011
1:0	5,6679E+12	1	I1ECHO	5	Res. ROM/i1	13JAN2012:1	06FEB2012
2:0	3,962E+11	1	I1ECHO	10	Im Kalender	06JAN2012:1	06JAN2012
2:4	3,962E+11	1	I1ECHO	10	Status über c	06JAN2012:1	06JAN2012
2:0	3,0771E+12	1	I1LST	5	Im Kalender	06FEB2012:1	06FEB2012
2:0	3,0771E+12	1	I1LST	5	Stornierungs	06FEB2012:1	06FEB2012

$\min c^T x$   
 $ASx \geq b, S \in S$   
 $x \geq 0$

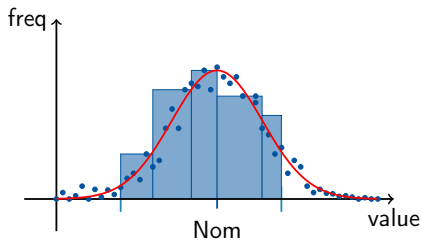
## Scenario Set



```
3: 3,358E+12 I1 I1LST 5 Im Kalender 31AUG2011:31AUG2011
:0: 3,423E+12 I1 IM1K 5 Stornierungs 19SEP2011:0:20SEP2011:
:0: 3,423E+12 I1 IM1K 5 Stornierungs 19SEP2011:0:20SEP2011:
:1: 2,5708E+12 I1 I1LST 5 Im Kalender 19SEP2011:1:29SEP2011:
:1: 6,3114E+11 I1 I1LST 5 Im Kalender 04OCT2011:1:04OCT2011
:1: 3,9465E+12 I1 I1LST 5 Im Kalender 11OCT2011:1:11OCT2011
:14: 2,9754E+11 I1 I1LST 5 Im Kalender 14OCT2011:1:14OCT2011
2:1: 1,4495E+10 I1 I1LST 5 Stornierungs 03AUG2011:0:03AUG2011
!1: 1,264E+12 I1 I1LST 5 Im Kalender 16DEC2011:1:16DEC2011
:0: 1,2934E+12 I1 I1LST 5 Im Kalender 03NOV2011:0:03NOV2011
:13: 3,5175E+12 I1 I1PO 10 Im Kalender 07OCT2011:1:27OCT2011
:10: 3,5175E+12 I1 I1PO 10 Status Über c 27OCT2011:1:27OCT2011
:10: 5,6679E+12 I1 I1ECHO 5 Res. ROM/1 13JAN2012:1:06FEB2012:
2:0: 3,962E+11 I1 I1ECHO 10 Im Kalender 06JAN2012:1:06JAN2012
2:4: 3,962E+11 I1 I1ECHO 10 Status Über c 06JAN2012:1:06JAN2012
2:0: 3,0771E+12 I1 I1LST 5 Im Kalender 06FEB2012:1:06FEB2012:
2:0: 3,0771E+12 I1 I1LST 5 Stornierungs 06FEB2012:1:06FEB2012:
```

$\min c^T x$   
 $ASx \geq b, S \in S$   
 $x \geq 0$

## Scenario Set



## Results

- ▶ compact formulation for robust MIP
- ▶ robust 0-1 problems remain tractable
- ▶ efficient scenario separation
- ▶ first applications



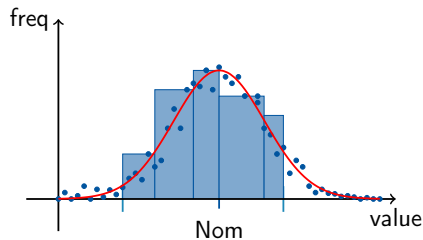
Mixed Integer Program	
1:1	3,358E+12 II I1LST 5 Im Kalender 31AUG2011:31AUG2011
1:0	3,423E+12 II I1M1K 5 Stornierungs 19SEP2011:020SEP2011
1:1	2,5708E+12 II I1LST 5 Im Kalender 19SEP2011:129SEP2011
1:1	6,3114E+11 I1LST 5 Im Kalender 04OCT2011:104OCT2011
1:4	3,9465E+12 II I1LST 5 Im Kalender 11OCT2011:11OCT2011
1:4	2,9754E+11 II I1LST 5 Im Kalender 14OCT2011:14OCT2011
2:1	1,4495E+10 II I1LST 5 Stornierungs 03AUG2011:03AUG2011
1:1	1,264E+12 II I1LST 5 Im Kalender 16DEC2011:16DEC2011
1:0	1,2934E+12 II I1LST 5 Im Kalender 03NOV2011:03NOV2011
1:1	3,5175E+12 II I1PO 10 Im Kalender 07OCT2011:127OCT2011
1:0	3,5175E+12 II I1PO 10 Status Über c 27OCT2011:127OCT2011
1:0	5,6679E+12 II I1ECHO 5 Res. ROM/1 13JAN2012:106FEB2012
2:0	3,962E+11 II I1ECHO 10 Im Kalender 06JAN2012:106JAN2012
2:4	3,962E+11 II I1ECHO 10 Status Über c 06JAN2012:106JAN2012
2:0	3,0771E+12 II I1LST 5 Im Kalender 06FEB2012:106FEB2012
2:0	3,0771E+12 II I1LST 5 Stornierungs 06FEB2012:106FEB2012

$\min c^T x$   
 $ASx \geq b, S \in S$   
 $x \geq 0$

## Results

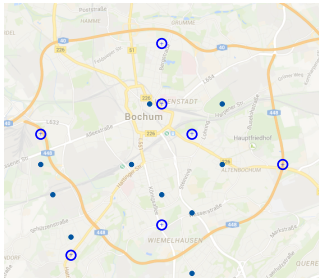
- ▶ compact formulation for robust MIP
- ▶ robust 0-1 problems remain tractable
- ▶ efficient scenario separation
- ▶ first applications

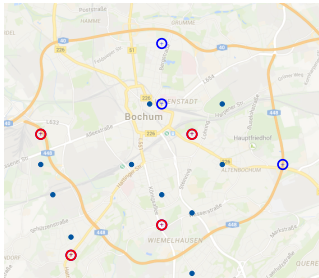
## Scenario Set

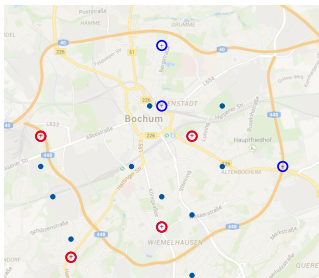


## Ongoing Work

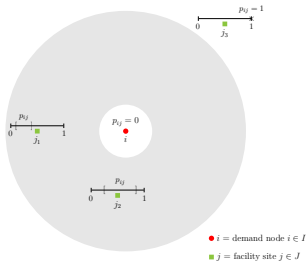
- ▶ rules for generation
- ▶ influence on solution
- ▶ measurements for robustness
  - ▶ price of robustness
  - ▶ radius of stability

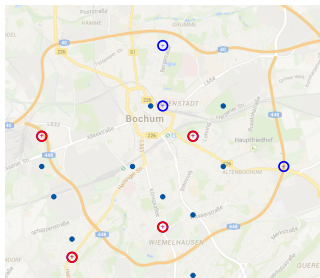




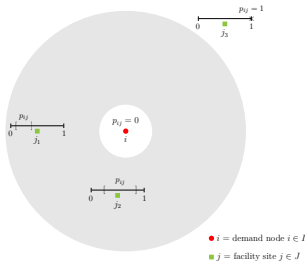


## Uncertainties





## Uncertainties

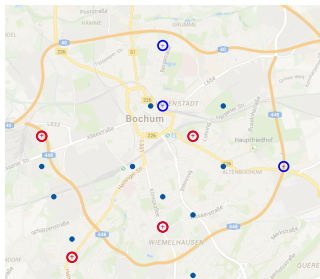


## Robust Set Cover Problem

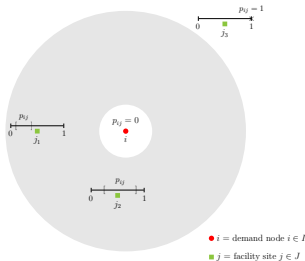
$$\min \sum_{j \in J} y_j$$

$$\mathbb{P} \left[ \sum_{j \in J} a_{ij} y_j \geq 1 \right] \geq \alpha \quad i \in I$$

$$y_j \in \{0, 1\} \quad j \in J$$



## Uncertainties

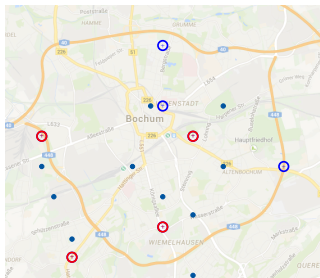


## Robust Set Cover Problem

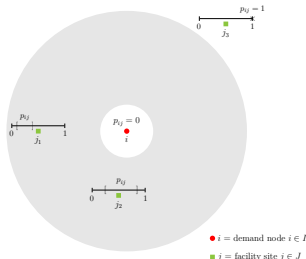
$$\min \sum_{j \in J} y_j$$

$$\min_{p \in P} \mathbb{P}_p \left[ \sum_{j \in J} a_{ij} y_j \geq 1 \right] \geq \alpha \quad i \in I$$

$$y_j \in \{0, 1\} \quad j \in J$$



## Uncertainties



## Robust Set Cover Problem

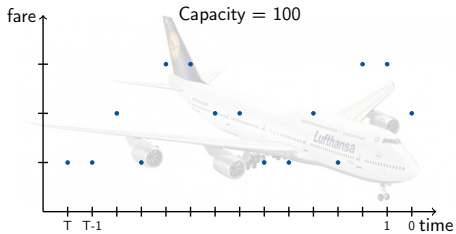
$$\min \sum_{j \in J} y_j$$

$$\min_{p \in P} \mathbb{P}_p \left[ \sum_{j \in J} a_{ij} y_j \geq 1 \right] \geq \alpha \quad i \in I$$

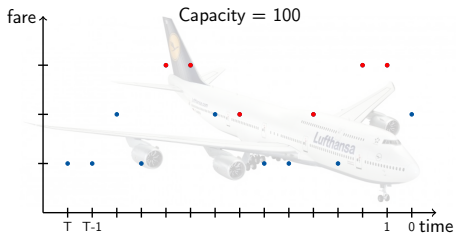
$$y_j \in \{0, 1\} \quad j \in J$$

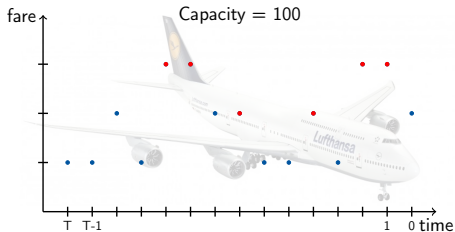
## Results

- ▶ compact MIP-formulation
- ▶ separation algorithms
- ▶ computational study

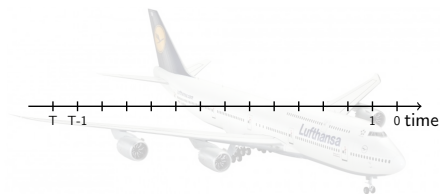


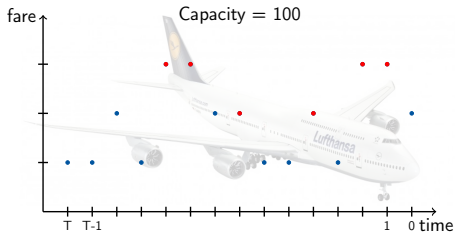




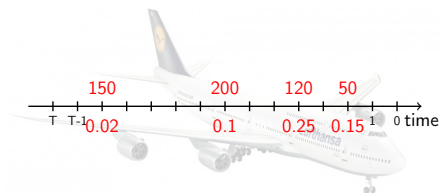


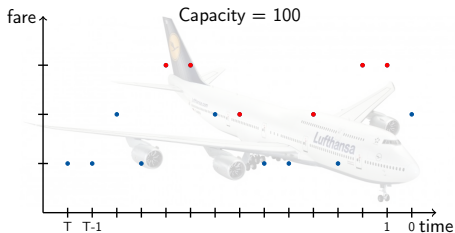
## Change of Capacity



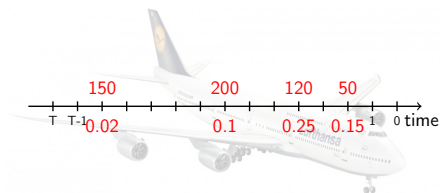


## Change of Capacity



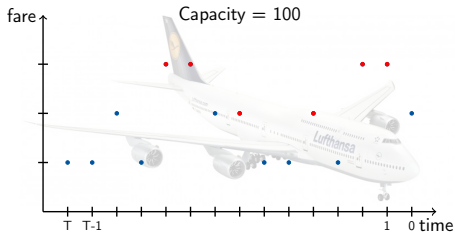


## Change of Capacity

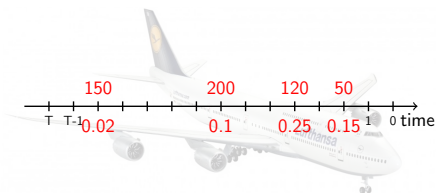


## Results

- ▶ IP-formulation
- ▶ Combinatorial algorithm
- ▶ Computational study

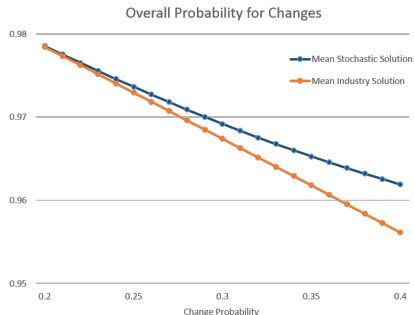


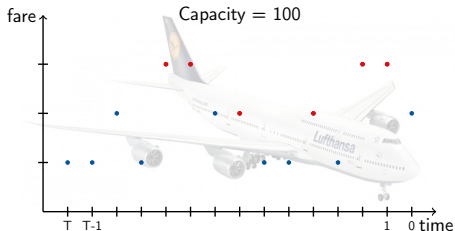
## Change of Capacity



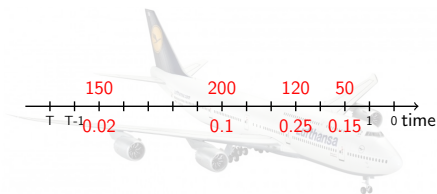
## Results

- ▶ IP-formulation
- ▶ Combinatorial algorithm
- ▶ Computational study





## Change of Capacity



## Results

- ▶ IP-formulation
- ▶ Combinatorial algorithm
- ▶ Computational study

## Future Work

- ▶ Network
- ▶ Uncertain probability

