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Two new Mixed-Integer Linear Programming Models for Solving the Multidimensional Multi-Way Number Partitioning Problem

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Abstract

The Multidimensional Multi-Way Number Partitioning Problem considers a set of n vectors of an arbitrary dimension as an input. The goal is to find partitioning of the set into $k \geq 2$ partitions such that sums per each coordinate in each partition set are as nearly equal as possible. The problem has applications in key encryption and multiprocessor scheduling. Prior to this paper, there has been only one mixed-integer model proposed for $k \geq 2$. In this paper, we propose two new mixed-integer programming models for this general case. These models are competitive with the model from literature, and outperform it significantly when k grows.

1. Introduction

Let $S = \{v_1, v_2, \ldots, v_n\}$ be a set of n vectors, where each vector has m real coordinates. Let k, $2 \leq k \leq n$, be an integer. The *Multidimensional Multi-Way Number Partitioning Problem* (MDMWNPP) asks for a disjoint and collectively exhaustive partition $(S_1, S_2, \ldots, S_k)$ of set S, where sums per each coordinate in each partition set are as nearly equal as possible.

MDMWNPP is a generalization of the well known *Two-way Number Partitioning Problem* (TWNPP) which goal is to divide a set S of real numbers in two partitions, such that sums of the numbers in each subset are as close as possible. MDMWNPP generalizes TWNPP in two ways: (i) in MDMWNPP, instead of real numbers, the set S contains m-dimensional vectors and (ii) the number of subsets when partitioning set of vectors S can be an arbitrary number $k \geq 2$. TWNPP is one of Karp's twenty one basic $\mathcal{NP}$–hard problems [7] (it is also $\mathcal{NP}$ complete). Thus, MDMWNPP is $\mathcal{NP}$–hard in a general case, as a generalization of an $\mathcal{NP}$–hard problem.

An important characteristic of TWNPP is that its computational complexity does not depend (only) on the cardinality of set S, but on the type of input numbers, i.e. the number precision [6, 13]. As it is stated in [8], this is not the case with the multidimensional version of TWNPP; even in the case when $k = 2$, the problem is hard to solve with a moderate number of digits in the input. In the case of $m = 1$ of the MDMWNPP, the problem is considered in [9, 14, 10].

The first generalization which introduces vectors instead of numbers was given by Kojic (2010) in [8]. Kojic [8] presented a mixed-integer linear programming (MILP) formulation of this specific problem. This MILP has n binary variables and $2m$ constraints. The problem is mainly solved by various heuristic approaches such as a genetic algorithm (GA) by Pop and Matei (2013a) in [16], a variable and neighborhood search (VNS) and an electromagnetism-like (EM) by Kratica et al. (2014) in [11], a greedy randomized adaptive search (GRASP) with exterior path-relinking and restricted local search by Rodrigez et al. (2017) in [19], four approaches GA, Simulated annealing (SA), migrating bird optimization algorithm (MBO) and clonal selection algorithms by Hacibeyoglu et al. (2018) in [4], and an Algebraic Differential Evolution and VNS by Santucci et al. (2019) in [20].

Bi-dimensional two-way number partitioning problem has been solved by greedy and genetic algorithms by Hacibeyoglu et al. (2014) in [5]. The same bi-dimensional problem is also solved by Fuksz et al. (2013) in [3] by using three heuristics, greedy, a novel GA and a hybrid of GA and VNS method. The generalized MDMWNPP with arbitrary m and k is introduced by Pop and Matei (2013b) in [17]. In a recent work, Faria et al. (2020) in [2] introduced the first known exact algorithm, a mixed integer programming model, to solve the generalized MDMWNPP. The model has been tested for a smaller number of partitions ($k \in \{2, 3, 4\}$). For $k = 2$ the model has shown as competitive to the model of Kojic [8]. More details about these two models will be discussed in Sections 2.1 and 2.2.

TWNPP and its generalizations faces with many practical applications in areas like public key encryption and task scheduling. Practical applications of the NPP include multiprocessor scheduling [1],public key cryptography [18],and the assignment of tasks in low-power application-specific integrated circuits [12].

Regarding the generalized problems, MDTWNPP and the MDMWNPP are of a practical usage when it is necessary to find a balance between two or more characteristics of elements from a given data set. These applications include public-key cryptography [16], and assigning persons to fair teams [4].

Some concrete examples of the MDTWNPP [8] may concern the following:

- Two travel agents need to have as closely similar revenues from organized tours as possible; the parameters under consideration are the price of the tour, the mileage (due to petrol consumption), special offers (such as restaurants, shopping tours, visits to museums, and so on). This is a three-dimensional, two-way number partitioning problem.

- Two police patrols need to have similar mileage, degree of risk (depending on criminal activities) and time for touring their beat.

Beside further generalizaions of the abovementioned examples, some concrete examples of for the MDMWNPP may be as follows:

- Cities in a country should be organized in regions, in order to achieve an equilibrium between the obtained regions in terms of the observed characteristics, like population, economic power and area.

- A group of scientists wants to form more than one commission, so that, in terms of competencies in the proposed areas, the formed commissions are of uniform quality.

- A professor wants to arrange tests for k students. He disposes with n questions and each question is described by m measurable parameters, such as estimated difficulty and estimated time needed to write the answer. Professor wants to deliver all questions to the students, where each question is assigned to exactly one student and with the requirement that both the overall difficulty and total needed time for each student is as balanced as possible.

The main contributions of our paper are as follows: (i) we present two new mixed integer linear programming models to solve the MDMWNPP with arbitrary number of allowed partitions k; (ii) we give an extensive computational studies also with larger k and compare our models to the model of Faria [2] and (iii) our models have been competitive with this model for smaller k, and significantly outperform it if the number of partitions k grows.

2. Problem Definition and the MILP models from the Literature

The *Multidimensional Multi-Way Number Partitioning Problem* (MDMWNPP) is stated as follows.

Given a set of n vectors $S = \{v_i = (v_{i1}, \dots, v_{im}) \mid i = 1, \dots, n, m \geq 2\}$, the problem asks for partitioning the set into k partitions, that is, $S = S_1 \cup \ldots \cup S_k$, $S_i \cap S_j = \emptyset$, such that sums per every coordinate of elements in each subsets $S_i, i = 1, \dots, k$, are equal or almost equal.

More precisely, the objective of the MDMWNPP is to minimize the greatest difference between maximum and minimum subsets sum per each coordinate. By r, let us denote the greatest difference between these two values as

$$r = \max_{l \in \{1,\dots,m\}} \left(\left| \max_{j \in \{1,\dots,k\}} \left(\sum_{v_i \in S_j} v_{il} \right) - \min_{j \in \{1,\dots,k\}} \left(\sum_{v_i \in S_j} v_{il} \right) \right| \right) \tag{1}$$

The objective of the problem is to minimize r. Recall that for $k = 2$ we get the MDTWNPP.

2.1. The Model for the MDTWNPP

In this section we present the MILP model proposed by Kojic (2010) in [8]. It is about to minimize r under the following constraints

$$-0,5 \cdot r + \sum_{i=1}^{n} v_{il} \cdot x_i \leqslant 0,5 \cdot s_l, \qquad l \in \{1, 2, \dots, m\} \tag{2}$$

$$0,5 \cdot r + \sum_{i=1}^{n} v_{il} \cdot x_i \geqslant 0,5 \cdot s_l, \qquad l \in \{1, 2, \dots, m\} \tag{3}$$

$$x_i \in \{0, 1\}, \qquad i \in \{1, 2, \dots, n\} \tag{4}$$

where $s_l = \sum_{i=1}^{n} v_{il}, l = 1, 2, \dots, m$, and $x_i = 1$, if $v_i \in S_1$, or 0 otherwise. This model has $2 \cdot m$ constraints and n variables. As one could notice, this model allows empty partitions. For example, the optimal solution of the MDTWNPP for $S = \{(-4,-4),(1,1),(1,1),(1,1),(1,1)\}$ can be obtained from partitions

$$S_1 = \{(-4,-4),(1,1),(1,1),(1,1),(1,1)\},$$
$$S_2 = \emptyset.$$

Faria et al. (2020) in [2] include the constraint that partitions must be non-empty. However, original MDMWNPP and Faria et al. (2020) in [2] definitions are the same under the assumption that all vector coordinates are positive real numbers. For $k = 2$, this follows from the simple fact that inequality $|x - y| < |x + y|$ holds for every positive real numbers x, y. Using this inequality, one can conclude that for any $\{S_1, S_2\}$ partition of S, and every $l \in \{1, \dots, m\}$ holds

$$\left|\sum_{v_i \in S_1} v_{il} - \sum_{v_i \in S_2} v_{il}\right| < \left|\sum_{v_i \in S_1} v_{il} + \sum_{v_i \in S_2} v_{il}\right| = \sum_{i=1}^{n} v_{il}.$$

The same can be argued for an any number of partitions k.

2.2. The Model for the MDMWNPP

This model has been proposed by Faria et al. (2020) in [2], and until now, it is the only method to solve the MDMWNPP. As we mentioned in previous Section 2.1, apart of the original MDMWNPP problem, Faria et al. (2020) considered a constraint that each partition has to be non-empty. The model has $(mk(k-1)+k+n)$ constraints and $(nk+1)$ variables.

Variables are (except of r) given by $x_{ij} = 1$, if vector v_i is included in partition S_j, $i \in \{1, \dots, n\}, j \in \{1, \dots, k\}$, or 0 otherwise.

$$\min r \tag{5}$$

s.t.

$$\sum_{j=1}^{k} x_{ij} = 1, \qquad i \in \{1, \dots, n\} \tag{6}$$

$$\sum_{i=1}^{n} x_{ij} \geq 1, \qquad j \in \{1, \dots, k\} \tag{7}$$

$$\sum_{i=1}^{n} v_{il}(x_{ij_1} - x_{ij_2}) \leq r, \qquad 1 \leq j_1 < j_2 \leq k, l \in \{1, \dots, m\} \tag{8}$$

$$\sum_{i=1}^{n} v_{il}(x_{ij_2} - x_{ij_1}) \leq r, \qquad 1 \leq j_1 < j_2 \leq k, l \in \{1, \dots, m\} \tag{9}$$

$$r \in [0, +\infty) \tag{10}$$

$$x_{ij} \in \{0, 1\}, \qquad i \in \{1, \dots, n\}, j \in \{1, \dots, k\} \tag{11}$$

Constraint (6) ensures that each element belongs to exactly one partition, and constraint (7) ensures that each partition is non-empty. Constraints (8) and (9) ensures that for each pair of different partitions, the absolute difference between these two subset sum per each coordinate is at most r.

3. New Models to Solve the MDMWNPP

In this section we give two new MILP models for solving MDMWNPP, denoted by New-1 and New-2. In both these models, n, m and k denote the number of vectors, the dimension of vectors and the number of partitions in the partitioning, respectively. Parameter i always denotes the index of vectors, $i \in \{1, 2, \ldots, n\}$, j denotes the index of a partition, $j \in \{1, 2, \ldots, k\}$ and l denotes the current vector coordinate $l \in \{1, 2, \ldots, m\}$.

Variables introduced in both models are as follows:

- Binary variables x_{ij}: $x_{ij} = 1$, if the i-th vector is included in the j-th partition, otherwise 0;
- real variables y_l, z_l: maximum and minimum sum of l-th coordinate among all partitions;
- a positive real variable r: corresponds to the objective function.

The New-1 model uses additional variables:

- a real variable t_{lj}: the sum of l-th coordinate of the elements in j-th partition.

3.1. *The* New-1 *Model*

This model is stated as follows:

$$
\begin{aligned}
&\min \quad r && && (12)\\
&\text{s.t.} && && \\
&\sum_{j=1}^{k} x_{ij} = 1, && i \in \{1, \ldots, n\} && (13)\\
&\sum_{i=1}^{n} v_{il} \cdot x_{ij} = t_{lj}, && j \in \{1, \ldots, k\}, l \in \{1, \ldots, m\} && (14)\\
&t_{lj} \le y_l, && j \in \{1, \ldots, k\}, l \in \{1, \ldots, m\} && (15)\\
&t_{lj} \ge z_l, && j \in \{1, \ldots, k\}, l \in \{1, \ldots, m\} && (16)\\
&y_l - z_l \le r, && l \in \{1, \ldots, m\} && (17)\\
&x_{ij} \in \{0, 1\}, && i \in \{1, \ldots, n\}, j \in \{1, \ldots, k\} && (18)\\
&t_{lj}, y_l, z_l \in \mathbb{R}, r \in [0, +\infty) && && (19)
\end{aligned}
$$

If we solve Faria et al. [2] version of MDTWNPP, the condition that each of the partitions have to be a non-empty set is expressed by (7) and included in the model (12)–(19).

The task is to minimize $r \in [0, +\infty)$ that is used in the constraint (17) to minimize a difference between y_l and z_l for any coordinate l.

Constraint (13) ensures that each vector is assigned to exactly one partition. Constraint (14) gives the relation between variables x_{ij}, t_{lj} and the vectors coordinates.

From the constraint (15), variables y_l, $l = 1, \ldots, m$, are bounded from below by values t_{lj} and in optimal MILP solution, because of (12), (14) and (17),

$y_l = \max_{j \in \{1,\ldots k\}} t_{lj}$. Similarly, from the constraint (16), variables $z_l,\ l = 1,\ldots,m$, are bounded from above by values t_{lj} and because of (12), (14) and (17), in optimal MILP solution it holds that $z_l = \min_{j \in \{1,\ldots k\}} t_{lj}$.

From (17), it is evident that r is bounded from below by maximum difference between y_l and z_l, which implies that the definition of the variable r is in line with the formula (1). This model has $n \cdot k + m \cdot k + 2 \cdot m + 1$ variables (from which $n \cdot k$ are binary variables) and $n + k(4m + n + 1) + m + 1$ constraints.

3.2. The New-2 *Model*

The New-2 model is derived from the New-1 by omitting the variables $t_{lj}, l \in \{1,\ldots,m\}, j \in \{1,\ldots,k\}$. More precisely, the New-2 model uses variables y_l and z_l, $l \in \{1,\ldots,m\}$, to directly bound the sums of l-th coordinates of vectors in a partition set.

Instead of constraints (14)–(16), the following constraints are introduced for New2 model:

$$\sum_{i=1}^{n} v_{il} \cdot x_{ij} \leq y_l, \qquad j \in \{1,\ldots,k\}, l \in \{1,\ldots,m\} \tag{20}$$

$$\sum_{i=1}^{n} v_{il} \cdot x_{ij} \geq z_l, \qquad j \in \{1,\ldots,k\}, l \in \{1,\ldots,m\} \tag{21}$$

All other constraints remain the same, with the exception of constraint 19, which is replaced by

$$y_l, z_l \in \mathbb{R}, r \in [0, +\infty) \tag{22}$$

since t_{lj} variables do not exist here. Thus, New-2 is stated as:

$$\min \quad r$$

with the constraints (13), (17),(18), (20)–(22). Constraint (7) is added if we not allow non-empty partitions as in Faria et al. (2020). This model has $n{\cdot}k{+}2{\cdot}m{+}1$ variables (from which $n{\cdot}k$ are binary variables) and $n{+}3m{+}k{\cdot}(n{+}2{\cdot}m{+}1){+}1$ constraints.

Remark 1. Please, note that another MILP model can be derived from New-1 model, by omitting y_l and z_l, $l = 1,\ldots,m$, variables and replacing constraints (15)–(17) by

$$t_{lj_1} - t_{lj_2} \leq r, \qquad 1 \leq j_1 < j_2 \leq k, l \in \{1,\ldots,m\} \tag{23}$$

$$t_{lj_2} - t_{lj_1} \leq r, \qquad 1 \leq j_1 < j_2 \leq k, l \in \{1,\ldots,m\} \tag{24}$$

However, preliminary experimental evaluation has indicated that this variant delivers worse results than New-1 and New-2, and results similar as Faria model, so due to clear presentation and comparison it is not further considered in the experimental section.

Remark 2. Based on the MILP model presented in [8] and [2] and the MILP models presented in Section 3, corresponding constraint programming (CP) models can be constructed. All four CP models are listed in Appendix B. Again, preliminary evaluation of these CP models has indicated that they deliver significantly worse results than NEW-1 and NEW-2 MILP models. For that reason, they are not further considered in the experimental section, due to clear presentation and comparison.

4. Experimental results

All proposed algorithms were implemented in C++ using GCC 5.4.2. All experiments were performed on a cluster of machines with Intel Xeon E5649 CPUs with 2.53 GHz and a memory limit of 8 GB in single-threaded mode. The maximum computation time allowed for each run was limited to 20 minutes, i.e., 1200 seconds.

The both mathematical models we proposed in this paper were implemented using CPLEX solver version 12.5.1, with the default configuration, in C++ and GCC compiler version 5.4.2. The both mathematical models include the constraint which ensures that partitions are not empty in order to be on a fair side with the model proposed by Faria et al. (2020) in [2]. Faria's et al. model and Kojic's model have been re-implemented in the form as they presented in the original papers. We recall that the model of Kojic would need additional constraint of non-emptiness of partitions, but it is not required due to the problem instances used in the experiments (all values of all vectors include positive real numbers). The following models are compared:

- the model of Kojic [8], labelled by KOJIC; the model is limited to the instances with $k = 2$;
- the model by Faria et al. [2], labelled by FDSDS;
- NEW-1 model;
- NEW-2 model.

The benchmark set used in our experiments was originally generated by Kojic [8]. All the published papers on this topic used the same starting set of instances to generate the final benchmark sets apart of the paper by Hacibeyoglu et al. (2014) in [5]. In set of instances proposed by Kojic (2010) in [8], there are 5 types of instances, denoted as $type \in \{a, b, c, d, e\}$, that is 5 files in which a matrix of dimension 500×200 has been generated. Each element of the matrices is a float number with 4 decimal places that are all randomly generated. The names of the starting instances are `mdtwnpp_500_20a`, `mdtwnpp_500_20b`, `mdtwnpp_500_20c`, `mdtwnpp_500_20d`, `mdtwnpp_500_20e`, respectively. For instance, in order to generate a specific instance with $n = 30, m = 10, k = 5$, and $type = a$, we take a starting instance `mdtwnpp_500_20a` and store the upper-left matrix of dimension 30×10 which exactly gives us 30 vectors (rows) each of dimension 10. In the same manner, for each of the following configurations:

- the number of vectors $n \in \{30, 40, 50, 100, 200, 300, 400, 500\}$,
- the vector's dimension $m \in \{2, 3, 5, 10, 15, 20\}$,
- the number of allowed partitions $k \in \{2, 3, 4, 5, 10, 20, 30\}$,

we generate one problem instance, which gives us, in overall, 1680 problem instances. Note that, until now, there have not been performed any experiments with $k \geq 5$ in the literature. Thus, this paper further expands the experimental evaluation by considering for the first time the problem instances with larger k, $k \in \{5, 10, 20, 30\}$. Also, the instances with $n \in \{30, 40\}$ are also considered for the first time in order to get a deeper insight into exact solving and its limitation which was not so clear in the literature. Our executable file of our implementations, the benchmark sets and complete results can be found at https://github.com/markodjukanovic90/mtwnpp-2021.

In order to check the statistical significance of differences of our algorithms, we generated Figures 6a–12b. This test is performed in the following way. Friedman's test was performed simultaneously on the groups of results of all models (four in case of $k = 2$, or three otherwise). The results are grouped w.r.t. different number of allowed partitions k. Given that in all cases the test rejected the hypothesis that the same group of results for each model are (statistically) equal, pairwise comparisons are performed using the Nemenyi post-hoc test [15]. The outcome is shown by means of so-called critical difference plots, three (comparing objective values, run-times and lower bound) for each group of results. In short, each model is positioned in the horizontal segment according to its average ranking concerning the considered set of instances. Then, the critical difference (CD) is computed for a significance level of 0.05 and the performance of those algorithms that have a difference lower than CD are considered as equal—that is, no difference of statistical significance can be detected. This is indicated in the graphics by horizontal bars joining the respective models.

In the descriptions of the obtained results, we make a small convention. By writing Kojic, FdSdS, and New-1 or New-2 model, we always refer to the Cplex solver which is applied on specific model.

Tables (1)–(4) presents summaries over the results which compare number of best achieved solutions, average best solutions, number of optimal solutions found, and average lower bounds, all grouped by different k. In Table (1), the number of achieved best results for each of the compared algorithms is presented. These results are given also by Figure 1. The following observations can be made:

- For $k = 2$, Kojic model finds the best solutions in a slightly more cases than the models FdSdS and New-1. It seems that the performance of the model New-2 is a bit worse than the other competitors.
- For $k = 3$, FdSdS model achieves significantly more best results than the other two competitors.
- Starting with $k \geq 4$, New-1 and New-2 models almost always achieve significantly more instances a best solution, in comparison to FdSdS. It is especially noticed for larger k, where New-2 model shows its dominance.

In Table (2), average objective values for each model grouped by k are presented. These results are also presented by Figure 2. We observe the main conclusions:

- For $k = 2$, New-1 model delivers the best solutions. The second best is kojic model which is slightly better than FdSdS model. However, these differences are not significant, see Figure 6a. New-2 model delivers significantly worse solutions quality in this case than the competitors.

- For $k \in \{3, 4\}$, FdSdS model delivers slightly better solutions than the other three algorithms, see Figures 7a and 8a, these differences are not significant on comparison to our models in case when $k = 4$, whereas for $k = 3$ no significant difference between model New-1 and FdSdS.
- For $k \geq 5$, New-2 model delivers significantly better results than the other models. It is especially noticed for $k \geq 10$, see Figures 9a, 10a, 11a and 12a.

In Table (3), the number of instances where optimality could be proven within the given time limit for each model and each k are presented. These results are also shown by Figure 3. The following observations can be made:

- For $k = 2$, Kojic model proves 45 instances to optimality, whereas FdSdS and New-1 model were successful in 44 cases, and New-2 model proves optimality in 43 cases.
- For $k \in \{3, 4, 5\}$, the situation is similar as for $k = 2$. All three models proves optimality for a similar number of instances but note that the numbers are significantly smaller than in the case when $k = 2$. This number of instances where optimality could be proven decreases as k increases (except in the case when $k = 30$)
- For $k \geq 10$, New-2 model is able to prove optimiality in a significantly more instances than the other two models. The difference is especially large for $k = 2$.
- In order to compare the run-times on those instances where optimality could be proven, we provide a summary in Table 5 and ploted in Figure 5. For a fair comparison, we compare only on those instances where all algorithms have been able to prove optimality (see column #common optima which gives the number of common instances, grouped by different k, for which the average run-times are calculated). For $k = 2$, Kojic model proves optimality in less time than FdSdS, but FdSdS model needs significantly less time than New-1 and New-2 model for the common instances where optimality is proven by the all algorithms (41 case). For $2 \leq k \leq 10$, FdSdS model still delivers smaller run-times for the common optima or at least equally good as the other two models. For higher $k \in \{20, 30\}$, New-1 and New-2 models need significantly less time to prove optimality than Kojic model. It is especially noticed for $k = 30$ where on the 26 common cases, avg. run-time of New-2 was $3.10s$ in comparison to FdSdS model where it was $127.40s$.

In Table (4), average lower bound for each model and each k is presented. These results are also given by Figure 4. The following observations can be made:

- For $k = 2$, the best lower bounds are achieved by Kojic model, slightly worse lower bound is delivered by FdSdS, New-1 model, and New-2 model. These differences are not significant, see Figure 6b.

- For $k \in \{3, 4, 5\}$, FdSdS model delivers better lower bounds than New-1 and New-2 model, but again, there is no significant difference, see Figures 7b and 8b.

- For $k \geq 10$, it is the other way around, that is, New-2 model delivers better lower bound that New-1 model (but not significant, which can be seen in Figures 10b, 11b, and 12b), while they obtain significantly better lower bounds than FdSdS.

Table 1: Number of achieved best results.

k	Kojic	FdSdS	New-1	New-2
2	115	109	107	82
3	n/a	114	86	99
4	n/a	92	97	97
5	n/a	94	80	99
10	n/a	63	83	113
20	n/a	43	96	143
30	n/a	46	113	177

Table 2: Average value of the objective function.

k	Kojic	FdSdS	New-1	New-2
2	19614.52941	19717.17353	19430.85745	20602.41718
3		33883.36726	35086.09534	34714.65171
4		44353.14992	44732.9819	44882.12398
5		52237.99449	52826.20617	51422.91392
10		77569.10346	75623.24447	74297.48153
20		110263.4594	101539.2615	97814.84431
30		158659.1417	123724.2726	108460.7812

Table 3: Number of proved optimal solutions

k	Kojic	FdSdS	New-1	New-2
2	45	44	44	43
3	n/a	29	29	31
4	n/a	18	17	18
5	n/a	15	13	14
10	n/a	8	3	11
20	n/a	7	5	13
30	n/a	30	35	35

Table 4: Average lower bound

k	Kojic	FdSdS	New-1	New-2
2	7134.645622	7084.572575	6822.179415	6667.217115
3	n/a	7308.104003	7181.417467	7072.641941
4	n/a	7385.246268	7196.975629	7233.539356
5	n/a	7824.52937	7435.768993	7640.947335
10	n/a	9051.497475	9731.603429	10452.52839
20	n/a	16788.6601	19116.00774	20145.72036
30	n/a	23754.11973	28625.13209	29596.43516

Table 5: Average time to reach optima (common)

k	Kojic	FdSdS	New-1	New-2	#common optima
2	127.24	140.30	217.28	200.37	41
3	n/a	144.23	179.87	210.81	28
4	n/a	249.41	200.17	233.76	15
5	n/a	221.30	336.78	373.02	12
10	n/a	213.93	202.92	217.19	3
20	n/a	248.13	100.99	49.36	4
30	n/a	127.40	15.91	3.10	26

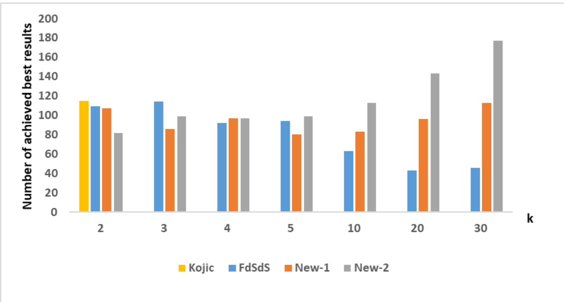


Figure 1: Number of achieved best results

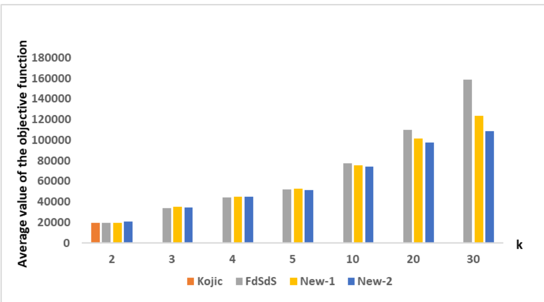


Figure 2: Average value of the objective function

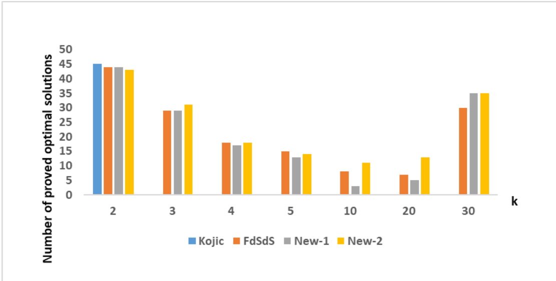


Figure 3: Number of proved optimal solutions

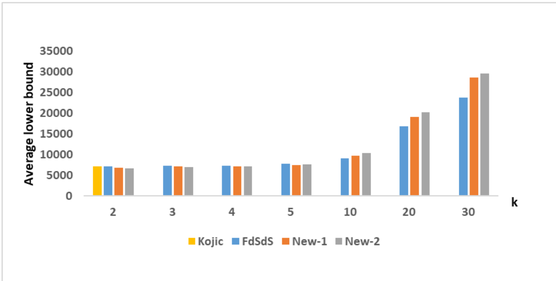


Figure 4: Average lower bound

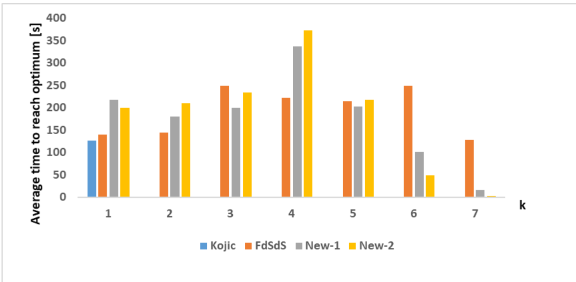


Figure 5: Average time to reach common optima

(a) Objective values comparisons.

(b) Lower bounds comparisons.

Figure 6: The plots showing statistical differences: $k = 2$.

(a) Objective values comparisons.

(b) Lower bounds comparisons.

Figure 7: The plots showing statistical differences: $k = 3$.

(a) Objective values comparisons.

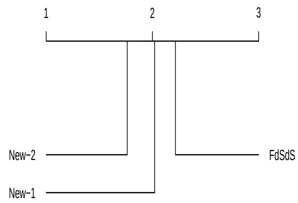
(b) Lower bounds comparisons.

Figure 8: The plots showing statistical differences: $k = 4$.

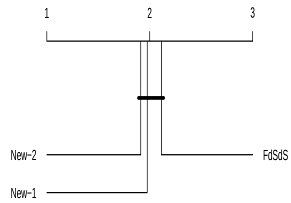
(a) Objective values comparisons.

(b) Lower bounds comparisons.

Figure 9: The plots showing statistical differences: $k = 5$.

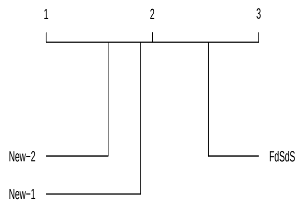


(a) Objective values comparisons.

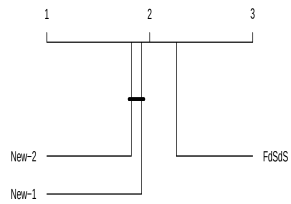


(b) Lower bounds comparisons.

Figure 10: The plots showing statistical differences: $k = 10$.

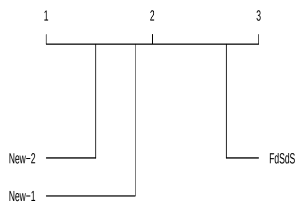


(a) Objective values comparisons.

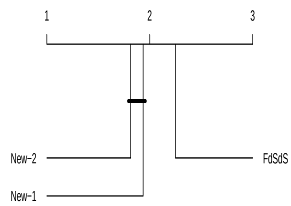


(b) Lower bounds comparisons.

Figure 11: The plots showing statistical differences: $k = 20$.



(a) Objective values comparisons.



(b) Lower bounds comparisons.

Figure 12: The plots showing statistical differences: $k = 30$.

5. Conclusions

In this paper we proposed two new MILP models to solve the MDMWNPP. The experimental evaluation from the literature has been further expanded to consider a new set of instances, which includes the number of vectors $n \in \{30, 40\}$ and a higher number of allowed partitions $k \in \{5, 10, 20, 30\}$. Both of proposed models have been shown as competitive with the model from literature for smaller k, and outperform it w.r.t. the number of best achieved solutions as well as average solutions quality for larger k. This difference was statistically significant, according to our statistical evaluation performed.

In future work, we can consider the mechanisms for breaking symmetries in our models to further boost the quality of solutions. Also, developing heuristic methods that are hybridized with MILP could be a promising research direction to consider.

Acknowledgments

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Appendices

A The complete results

In this section we provide complete results obtained by testing following MILP models:

- the model Kojic; the model is limited to the instances with $k = 2$;
- the model FdSdS;
- New-1 model;
- New-2 model.

Results are organized in Tables 1-21 as follows:

- $k = 2$
  - Table 1: results for $n = 30,\ 40,\ 50$;
  - Table 2: results for $n = 100,\ 300,\ 500$;
  - Table 3: results for $n = 200, 400$.
- $k = 3$
  - Table 4: results for $n = 30,\ 40,\ 50$;
  - Table 5: results for $n = 100,\ 300,\ 500$;
  - Table 6: results for $n = 200, 400$.
- $k = 4$
  - Table 7: results for $n = 30,\ 40,\ 50$;
  - Table 8: results for $n = 100,\ 300,\ 500$;
  - Table 9: results for $n = 200, 400$.
- $k = 5$
  - Table 10: results for $n = 30,\ 40,\ 50$;
  - Table 11: results for $n = 100,\ 300,\ 500$;
  - Table 12: results for $n = 200, 400$.
- $k = 10$
  - Table 13: results for $n = 30,\ 40,\ 50$;
  - Table 14: results for $n = 100,\ 300,\ 500$;
  - Table 15: results for $n = 200, 400$.
- $k = 20$
  - Table 16: results for $n = 30,\ 40,\ 50$;
  - Table 17: results for $n = 100,\ 300,\ 500$;
  - Table 18: results for $n = 200, 400$.
- $k = 30$
  - Table 19: results for $n = 30,\ 40,\ 50$;
  - Table 20: results for $n = 100,\ 300,\ 500$;
  - Table 21: results for $n = 200, 400$.

Each of the tables (except Tables 1–3) is organized as follows. The first three columns represent the instance characteristics which is followed by three (or four in cases of Tables 1–3) blocks. Each block reports the results of the compared models: (Kojic model in case of Tables 1–3), Faria model, New-1 model and New-2 model, respectively. Each of these blocks consists of 3 columns which report objective value (r), run-time ($t[s]$) in seconds and achieved lower bound (LB). The minimum objective value obtained for the observed three algorithms (or four algorithms in case of Tables 1–3) is highlighted in bold font, and, if optimality can be proved for this value, then it is additionally noted by underlining.

Table 1: Results for $n = 30,\ 40,\ 50$ and $k = 2$

n	m	$type$	Kojic			FdSdS			New-1			New-2		
			r	$t[s]$	LB	r	$t[s]$	LB	r	$t[s]$	LB	r	$t[s]$	LB
30	2	a	**10.719**	323.48	10.719	**10.719**	614.92	10.719	**10.719**	938.52	10.719	**10.719**	586.03	10.719
30	2	b	**17.686**	369.76	17.686	**17.686**	436.05	17.686	**17.686**	944.33	17.686	**17.686**	548.83	17.686
30	2	c	**11.857**	440.07	11.857	**11.857**	521.74	11.857	**11.857**	1173.77	11.857	**11.857**	1200	0
30	2	d	**33.359**	228.95	33.359	**33.359**	150.88	33.359	**33.359**	447.41	33.359	**33.359**	278.41	33.3581
30	2	e	**12.411**	151.39	12.411	**12.411**	161.77	12.411	**12.411**	470.32	12.411	**12.411**	444.89	12.411
30	3	a	**458.689**	185.61	458.689	**458.689**	188.48	458.689	**458.689**	339.8	458.689	**458.689**	334.05	458.689
30	3	b	**374.662**	120.25	374.662	**374.662**	157.66	374.662	**374.662**	212.12	374.662	**374.662**	277.11	374.662
30	3	c	**397.202**	118.19	397.202	**397.202**	131.4	397.202	**397.202**	252.54	397.202	**397.202**	235.24	397.202
30	3	d	**431.835**	200.51	431.835	**431.835**	227.97	431.835	**431.835**	511.4	431.835	**431.835**	438.82	431.835
30	3	e	**529.719**	107.92	529.719	**529.719**	143.79	529.719	**529.719**	285.33	529.719	**529.719**	196.8	529.719
30	5	a	**2027.599**	31.86	2027.599	**2027.599**	28.08	2027.599	**2027.599**	38.46	2027.599	**2027.599**	40.75	2027.599
30	5	b	**4137.914**	53.92	4137.914	**4137.914**	42.09	4137.914	**4137.914**	44.22	4137.914	**4137.914**	82.72	4137.914
30	5	c	**5036.236**	34.1	5036.236	**5036.236**	33.06	5036.236	**5036.236**	36.85	5036.236	**5036.236**	55.58	5036.236
30	5	d	**5067.523**	35.27	5067.523	**5067.523**	34.92	5067.523	**5067.523**	48.95	5067.523	**5067.523**	40.76	5067.523
30	5	e	**4854.22**	30.27	4854.22	**4854.22**	31.73	4854.22	**4854.22**	49.71	4854.22	**4854.22**	47.69	4854.22
30	10	a	**35838.597**	8.68	35838.597	**35838.597**	9.61	35838.597	**35838.597**	21	35838.597	**35838.597**	29.71	35838.597
30	10	b	**32215.215**	9.35	32215.215	**32215.215**	6.23	32215.215	**32215.215**	13.27	32215.215	**32215.215**	9.86	32215.215
30	10	c	**27339.421**	6.49	27339.421	**27339.421**	4.85	27339.421	**27339.421**	7.93	27339.421	**27339.421**	8.62	27339.421
30	10	d	**27913.194**	5.34	27913.194	**27913.194**	4.05	27913.194	**27913.194**	13.47	27913.194	**27913.194**	7.49	27913.194
30	10	e	**31773.234**	7.33	31773.234	**31773.234**	6.15	31773.234	**31773.234**	14.67	31773.234	**31773.234**	13.13	31773.234
30	15	a	**62635.735**	9	62635.735	**62635.735**	7.68	62635.735	**62635.735**	15.91	62635.735	**62635.735**	32.07	62635.735
30	15	b	**58270.95**	6.92	58270.95	**58270.95**	6.04	58270.95	**58270.95**	15.01	58270.95	**58270.95**	9.55	58270.95
30	15	c	**57945.688**	5	57945.688	**57945.688**	4.22	57945.688	**57945.688**	11.31	57945.688	**57945.688**	15.96	57945.688
30	15	d	**59496.708**	6.63	59496.708	**59496.708**	7.37	59496.708	**59496.708**	11.08	59496.708	**59496.708**	20.34	59496.708
30	15	e	**63144.72**	8.22	63144.72	**63144.72**	5.86	63144.72	**63144.72**	18.8	63144.72	**63144.72**	15.49	63144.72
30	20	a	**78879.971**	6.43	78879.971	**78879.971**	4.95	78879.971	**78879.971**	10.34	78879.971	**78879.971**	20.68	78879.971
30	20	b	**83283.709**	7.65	83283.709	**83283.709**	11.59	83283.709	**83283.709**	17.92	83283.709	**83283.709**	27.01	83283.709
30	20	c	**93979.726**	16.58	93979.726	**93979.726**	12.1	93979.726	**93979.726**	35.93	93979.726	**93979.726**	33.69	93979.726
30	20	d	**86948.525**	10.68	86948.525	**86948.525**	9.54	86948.525	**86948.525**	25.63	86948.525	**86948.525**	14.12	86948.525
30	20	e	**83060.654**	7.19	83060.654	**83060.654**	6.81	83060.654	**83060.654**	16.72	83060.654	**83060.654**	10.12	83060.654
40	2	a	**21.425**	1200	0	27.22	1200	0	25.62	1200	0	28.674	1200	0
40	2	b	1.525	1200	0	**0.8812**	1200	0	7.629	1200	0	1.527	1200	0
40	2	c	1.525	1200	0	**0.7003**	1200	0	1.527	1200	0	1.527	1200	0
40	2	d	5.739	1200	0	**5.423**	1200	0	11.527	1200	0	8.141	1200	0
40	2	e	1.5283	1200	0	**1.5214**	1200	0	1.526	1200	0	1.524	1200	0
40	3	a	140.937	1200	0	**137.885**	1200	0	**137.885**	1200	0	**137.885**	1200	0
40	3	b	**1.523**	1200	0	3.05	1200	0	3.05	1200	0	4.579	1200	0
40	3	c	1.529	1200	0	1.524	1200	0	**0.8971**	1200	0	3.048	1200	0
40	3	d	280.939	1200	0	**239.539**	1200	0	280.937	1200	0	283.989	1200	0
40	3	e	**1.5254**	1200	0	4.576	1200	0	289.512	1200	0	4.58	1200	0
40	5	a	3598.944	1200	0	**3595.894**	1200	0	3598.944	1200	0	**3595.894**	1200	0
40	5	b	3.052	1200	0	4.575	1200	0	**1.5238**	1200	0	4.576	1200	0
40	5	c	**3769.863**	1200	0	**3769.863**	1200	0	**3769.863**	1200	0	3775.969	1200	0
40	5	d	**3204.371**	1200	0	4072.443	1200	0	**3204.371**	1200	0	4072.445	1200	0
40	5	e	9.154	1200	0	**3.05**	1200	0	7.63	1200	0	7.628	1200	0
40	10	a	**16516.276**	345.83	16516.276	**16516.276**	284.08	16516.276	**16516.276**	342.49	16516.276	**16516.276**	285.48	16516.276
40	10	b	**20877.577**	501.51	20877.577	**20877.577**	561.06	20877.577	**20877.577**	488.34	20877.577	**20877.577**	584.42	20877.577
40	10	c	**21841.318**	489.97	21841.318	**21841.318**	510.28	21841.318	**21841.318**	507.48	21841.318	**21841.318**	584.74	21841.318
40	10	d	**17717.383**	256.48	17717.383	**17717.383**	334.84	17717.383	**17717.383**	299.55	17717.383	**17717.383**	758.23	17717.383
40	10	e	**16493.277**	358.11	16493.277	**16493.277**	323.12	16493.277	**16493.277**	488.44	16493.277	**16493.277**	397.6	16493.277
40	15	a	**42266.364**	279.8	42266.364	**42266.364**	240.99	42266.364	**42266.364**	536.16	42266.364	**42266.364**	295.24	42266.364
40	15	b	**43167.289**	143.29	43167.289	**43167.289**	170.24	43167.289	**43167.289**	156.82	43167.289	**43167.289**	206.4	43167.289
40	15	c	**44707.051**	200.82	44707.051	**44707.051**	193.87	44707.051	**44707.051**	222.24	44707.051	**44707.051**	204.97	44707.051
40	15	d	**36352.069**	67.7	36352.069	**36352.069**	57.07	36352.069	**36352.069**	113.52	36352.069	**36352.069**	274.57	36352.069
40	15	e	**44955.036**	262.27	44955.036	**44955.036**	328.06	44955.036	**44955.036**	479.13	44955.036	**44955.036**	256.71	44955.036
40	20	a	**63771.744**	137.21	63771.744	**63771.744**	199	63771.7432	**63771.744**	657.26	63771.744	**63771.744**	663.22	63771.744
40	20	b	**65467.198**	89.86	65467.198	**65467.198**	104.63	65467.198	**65467.198**	224.48	65467.1976	**65467.198**	219.5	65467.198
40	20	c	**65383.92**	153.39	65383.92	**65383.92**	130.41	65383.92	**65383.92**	292.98	65383.92	**65383.92**	264.35	65383.92
40	20	d	**66769.009**	131.02	66769.009	**66769.009**	125.19	66769.009	**66769.009**	257.52	66769.009	**66769.009**	248.19	66769.009
40	20	e	**63179.416**	142.63	63179.416	**63179.416**	154.14	63179.416	**63179.416**	302.26	63179.416	**63179.416**	257.27	63179.416
50	2	a	0.7918	1200	0	0.969	1200	0	**0.1587**	1200	0	0.973	1200	0
50	2	b	5.752	1200	0	3.309	1200	0	**3.087**	1200	0	8.918	1200	0
50	2	c	9.189	1200	0	**7.277**	1200	0	9.191	1200	0	9.195	1200	0
50	2	d	1.9507	1200	0	**1.703**	1200	0	5.134	1200	0	7.668	1200	0
50	2	e	**9.044**	1200	0	9.342	1200	0	12.092	1200	0	11.792	1200	0
50	3	a	**291.369**	1200	0	297.477	1200	0	300.529	1200	0	454.112	1200	0
50	3	b	**314.84**	1200	0	350.834	1200	0	359.984	1200	0	468.118	1200	0
50	3	c	**142.937**	1200	0	145.985	1200	0	152.093	1200	0	145.985	1200	0
50	3	d	105.566	1200	0	105.572	1200	0	**99.464**	1200	0	192.39	1200	0
50	3	e	220.957	1200	0	**217.901**	1200	0	220.953	1200	0	230.107	1200	0
50	5	a	926.168	1200	0	**920.066**	1200	0	923.118	1200	0	926.164	1200	0
50	5	b	3205.933	1200	0	**3151.534**	1200	0	3205.927	1200	0	3205.927	1200	0
50	5	c	2702.809	1200	0	**2327.471**	1200	0	2705.861	1200	0	2708.911	1200	0
50	5	d	**2275.79**	1200	0	**2275.79**	1200	0	2275.792	1200	0	**2275.79**	1200	0
50	5	e	**4453.267**	1200	0	4456.321	1200	0	4830.037	1200	0	4485.518	1200	0
50	10	a	16176.582	1200	0	**16173.526**	1200	0	16173.53	1200	0	16173.53	1200	0
50	10	b	19560.314	1200	0	**19554.21**	1200	0	19560.316	1200	0	20793.183	1200	0
50	10	c	14134.693	1200	0	17750.997	1200	0	**14125.537**	1200	0	17750.997	1200	0
50	10	d	14921.977	1200	0	14925.025	1200	0	14921.973	1200	0	**14915.871**	1200	0
50	10	e	19303.638	1200	0	19309.742	1200	0	15365.959	1200	0	**15365.957**	1200	0
50	15	a	**33208.019**	1200	0	**33208.019**	1200	0	33211.071	1200	0	**33208.019**	1200	0
50	15	b	**33240.094**	1200	12535.332	**33240.094**	1200	6984.182	**33240.094**	1200	4201.5198	**33240.094**	1200	2159.4063
50	15	c	**29456.854**	1200	6112.3355	**29456.854**	1200	4227.6527	**29456.854**	1200	0	29923.977	1200	0
50	15	d	**21652.841**	1200	5438.0953	**21652.841**	1200	5940.3798	**21652.841**	1200	0	**21652.841**	1200	1998.2114
50	15	e	**31800.69**	1200	0	**31800.69**	1200	1559.1991	**31800.69**	1200	0	**31800.69**	1200	0
50	20	a	**52826.34**	1200	30251.9683	**52826.34**	1200	25618.8992	**52826.34**	1200	17759.4165	**52826.34**	1200	7216.5883
50	20	b	**51917.902**	1200	32580.2509	**51917.902**	1200	32174.3343	56089.632	1200	19413.9407	**51917.902**	1200	15117.5657
50	20	c	**50560.864**	1200	31217.3734	**50560.864**	1200	31970.0527	**50560.864**	1200	23964.8391	**50560.864**	1200	12409.0941
50	20	d	**53955.965**	1200	30577.5703	**53955.965**	1200	32981.8731	**53955.965**	1200	21434.7778	**53955.965**	1200	12137.7008
50	20	e	**48281.499**	1200	28009.4185	**48281.499**	1200	23248.241	**48281.499**	1200	14955.9612	**48281.499**	1200	13512.794

Table 2: Results for $n = 100,\ 300,\ 500$ and $k = 2$

n	m	$type$	Kojic			FdSdS			New-1			New-2		
			r	$t[s]$	LB	r	$t[s]$	LB	r	$t[s]$	LB	r	$t[s]$	LB
100	2	a	17.303	1200	0	**14.1401**	1200	0	25.615	1200	0	18.405	1200	0
100	2	b	**3.646**	1200	0	3.913	1200	0	6.886	1200	0	5.539	1200	0
100	2	c	5.231	1200	0	4.53	1200	0	**2.783**	1200	0	5.2129	1200	0
100	2	d	**3.0495**	1200	0	4.431	1200	0	6.437	1200	0	6.433	1200	0
100	2	e	3.189	1200	0	1.071	1200	0	1.377	1200	0	**1.067**	1200	0
100	3	a	133.309	1200	0	**124.157**	1200	0	136.361	1200	0	298.65	1200	0
100	3	b	107.087	1200	0	**97.937**	1200	0	104.041	1200	0	**97.937**	1200	0
100	3	c	266.251	1200	0	231.514	1200	0	234.566	1200	0	**231.512**	1200	0
100	3	d	250.087	1200	0	**247.045**	1200	0	256.201	1200	0	256.203	1200	0
100	3	e	150.565	1200	0	147.511	1200	0	**141.407**	1200	0	147.513	1200	0
100	5	a	3592.841	1200	0	2453.685	1200	0	**2432.329**	1200	0	2447.591	1200	0
100	5	b	2825.846	1200	0	2077.632	1200	0	**2068.476**	1200	0	2086.786	1200	0
100	5	c	2839.322	1200	0	**2788.966**	1200	0	2839.326	1200	0	2845.426	1200	0
100	5	d	**2978.991**	1200	0	2982.037	1200	0	2982.041	1200	0	**2978.991**	1200	0
100	5	e	3784.8	1200	0	3778.694	1200	0	3778.694	1200	0	**3778.692**	1200	0
100	10	a	16008.039	1200	0	16011.091	1200	0	16004.985	1200	0	**16001.935**	1200	0
100	10	b	**13600.363**	1200	0	18996.495	1200	0	18996.489	1200	0	18999.545	1200	0
100	10	c	16575.456	1200	0	15389.616	1200	0	**15383.52**	1200	0	15386.57	1200	0
100	10	d	**15556.06**	1200	0	18282.346	1200	0	15562.16	1200	0	15559.11	1200	0
100	10	e	**15822.361**	1200	0	16513.219	1200	0	15834.561	1200	0	16507.127	1200	0
100	15	a	**32149.343**	1200	0	**32149.343**	1200	0	32155.451	1200	0	34161.617	1200	0
100	15	b	**28717.693**	1200	0	28723.797	1200	0	28720.747	1200	0	30794.75	1200	0
100	15	c	32668.12	1200	0	32674.23	1200	0	**31115.39**	1200	0	33363.204	1200	0
100	15	d	30693.964	1200	0	30697.008	1200	0	**30690.906**	1200	0	30697.012	1200	0
100	15	e	30268.637	1200	0	**30256.433**	1200	0	30265.587	1200	0	30274.747	1200	0
100	20	a	51814.656	1200	0	**48152.897**	1200	0	49210.389	1200	0	55236.884	1200	0
100	20	b	52010.092	1200	0	46688.441	1200	0	46688.443	1200	0	**46685.387**	1200	0
100	20	c	**46912.193**	1200	0	47647.322	1200	0	51767.883	1200	0	51880.438	1200	0
100	20	d	**43786.661**	1200	0	45923.664	1200	0	50361.173	1200	0	50848.357	1200	0
100	20	e	**46124.493**	1200	0	48073.751	1200	0	50840.841	1200	0	56561.872	1200	0
300	2	a	2.909	1200	0	2.66	1200	0	2.561	1200	0	**1.245**	1200	0
300	2	b	3.643	1200	0	3.814	1200	0	4.224	1200	0	**3.502**	1200	0
300	2	c	**0.783**	1200	0	2.003	1200	0	4.481	1200	0	4.473	1200	0
300	2	d	**2.377**	1200	0	2.577	1200	0	3.447	1200	0	6.531	1200	0
300	2	e	**0.7692**	1200	0	6.234	1200	0	4.832	1200	0	10.118	1200	0
300	3	a	190.808	1200	0	158.821	1200	0	221.258	1200	0	**151.388**	1200	0
300	3	b	**113.201**	1200	0	147.881	1200	0	175.343	1200	0	153.985	1200	0
300	3	c	248.295	1200	0	**62.615**	1200	0	75.735	1200	0	212.889	1200	0
300	3	d	132.863	1200	0	134.541	1200	0	71.471	1200	0	**53.153**	1200	0
300	3	e	203.831	1200	0	208.981	1200	0	**185.527**	1200	0	195.258	1200	0
300	5	a	**9.154**	1200	0	1850.81	1200	0	1859.96	1200	0	1869.11	1200	0
300	5	b	4342.959	1200	0	2150.515	1200	0	2742.529	1200	0	**2147.463**	1200	0
300	5	c	**921.98**	1200	0	2667.158	1200	0	2654.944	1200	0	3187.061	1200	0
300	5	d	**2124.567**	1200	0	2155.091	1200	0	2372.511	1200	0	2730.907	1200	0
300	5	e	1562.015	1200	0	1574.215	1200	0	**1555.903**	1200	0	3794.385	1200	0
300	10	a	**14416.036**	1201.11	0	21452.305	1200.1	0	14428.242	1200	0	17861.274	1200	0
300	10	b	18428.266	1203.13	0	18152.52	1200.26	0	18170.834	1200	0	**16255.54**	1200	0
300	10	c	20508.846	1200.52	0	16816.5	1200.15	0	**15987.045**	1200	0	22092.947	1200	0
300	10	d	19229.684	1200.59	0	**17188.357**	1200.18	0	17869.437	1200	0	18149.577	1200	0
300	10	e	21478.476	1201.55	0	18111.889	1200.06	0	**15303.985**	1200	0	16908.62	1200	0
300	15	a	**27488.05**	1200	0	33959.196	1200	0	37683.185	1200	0	31629.89	1200	0
300	15	b	40518.826	1200	0	36869.59	1200.83	0	**29204.895**	1200	0	42190.439	1200	0
300	15	c	41365.389	1200	0	38065.407	1200	0	**34250.673**	1200	0	36783.579	1200	0
300	15	d	39604.987	1200	0	**32730.409**	1200	0	34216.595	1200	0	37571.36	1200	0
300	15	e	**34148.776**	1200	0	37371.088	1200	0	36045.472	1200	0	36829.396	1200	0
300	20	a	**49011.001**	1200	0	57772.474	1200	0	51608.636	1200	0	60016.225	1200	0
300	20	b	53183.241	1200	0	47929.938	1200	0	**46988.897**	1200	0	56765.672	1200	0
300	20	c	53965.75	1200	0	61778.549	1200	0	**51153.359**	1200	0	57422.802	1200	0
300	20	d	48272.879	1200	0	**47342.561**	1200	0	52131.141	1200	0	61173.493	1200	0
300	20	e	**43641.886**	1200	0	47273.654	1200	0	54139.14	1200	0	60025.185	1200	0
500	2	a	**5.1326**	1200	0	12.49	1200	0	14.008	1200	0	16.3	1200	0
500	2	b	1.969	1200	0	1.677	1200	0	**1.385**	1200	0	1.957	1200	0
500	2	c	3.097	1200	0	4.898	1200	0	**1.129**	1200.23	0	4.904	1200	0
500	2	d	**1.673**	1200	0	3.187	1200	0	7.276	1200	0	16.045	1200	0
500	2	e	**3.318**	1200	0	3.6968	1200	0	6.007	1200	0	6.08	1200	0
500	3	a	**10.675**	1200	0	138.998	1200	0	138.984	1200.17	0	281.282	1200	0
500	3	b	129.954	1200	0	267.37	1200	0	**88.147**	1200	0	296.543	1200	0
500	3	c	160.334	1200	0	94.262	1200	0	85.106	1200	0	**79**	1200	0
500	3	d	185.238	1200	0	66.696	1201.12	0	172.559	1200	0	**57.55**	1200	0
500	3	e	**4.5769**	1200	0	91.379	1200	0	97.477	1200	0	178.662	1200	0
500	5	a	1849.275	1200	0	**1742.128**	1200	0	1852.325	1200	0	3683.8	1200	0
500	5	b	2728.162	1200	0	2727.795	1200	0	**2712.549**	1200	0	2894.805	1200	0
500	5	c	2744.85	1200	0	**1900.642**	1200.44	0	2524.797	1200	0	1912.864	1200	0
500	5	d	**13.732**	1200	0	2424.008	1200	0	1964.553	1200	0	1982.865	1200	0
500	5	e	2833.654	1204.03	0	**2667.309**	1200	0	3812.182	1200	0	3196.62	1200.09	0
500	10	a	20076.202	1202.07	0	**17695.694**	1202.7	0	18370.441	1200	0	20070.104	1200	0
500	10	b	18781.808	1200.01	0	21930.449	1202.76	0	**12176.614**	1200	0	20431.433	1200	0
500	10	c	20434.322	1200.92	0	22464.14	1200	0	**19350.369**	1200	0	21892.12	1200	0
500	10	d	16197.362	1200.41	0	**12894.202**	1200	0	16064.226	1200.02	0	21291.872	1200	0
500	10	e	19379.451	1200.42	0	19894.967	1200.11	0	**18858.793**	1200	0	22961.576	1200	0
500	15	a	35225.494	1200	0	34987.747	1200	0	**29225.196**	1200	0	38486.452	1200	0
500	15	b	38371.227	1200	0	**31840.388**	1200	0	36149.015	1200	0	34864.475	1200	0
500	15	c	41517.228	1200	0	**35025.472**	1200	0	36928.512	1200	0	44855.907	1200	0
500	15	d	**37491.829**	1200	0	39253.958	1200	0	40289.508	1200	0	37979.168	1200	0
500	15	e	**33823.836**	1200	0	43558.088	1200	0	41859.683	1200	0	42331.336	1200	0
500	20	a	**52880.099**	1200	0	52917.094	1200	0	59685.126	1200	0	60012.96	1200	0
500	20	b	**55750.197**	1200	0	58205.477	1200	0	56222.519	1200	0	57775.366	1200	0
500	20	c	53450.357	1200	0	54605.922	1200	0	**47597.318**	1200	0	63364.387	1200	0
500	20	d	56538.543	1200	0	58465.438	1200	0	**54536.544**	1200	0	62097.2	1200	0
500	20	e	**48558.694**	1200	0	54778.504	1200	0	58151.481	1200	0	57691.895	1200	0

Table 3: Results for $n = 200,\ 400$ and $k = 2$

n	m	$type$	Kojic			FdSdS			New-1			New-2		
			r	$t[s]$	LB	r	$t[s]$	LB	r	$t[s]$	LB	r	$t[s]$	LB
200	2	a	16.836	1200	0	19.2545	1200	0	**4.331**	1200	0	16.531	1200	0
200	2	b	2.485	1200	0	**0.3408**	1200	0	8.103	1200	0	3.343	1200	0
200	2	c	3.74	1200	0	0.85	1200	0	**0.5878**	1200	0	0.85	1200	0
200	2	d	**1.207**	1200	0	3.423	1200	0	1.2555	1200.31	0	6.499	1200	0
200	2	e	2.907	1200	0	9.833	1200	0	**1.5825**	1200	0	4.717	1200	0
200	3	a	**181.368**	1200	0	294.516	1200	0	337.78	1200	0	327.024	1200	0
200	3	b	**137.584**	1200	0	331.04	1200	0	328.524	1200	0	154.969	1200	0
200	3	c	**13.734**	1200	0	236.269	1200	0	239.445	1200	0	245.425	1200	0
200	3	d	132.859	1200	0	139.691	1200	0	**124.433**	1200	0	140.643	1200	0
200	3	e	203.517	1200.09	0	117.488	1200.03	0	**108.34**	1200	0	306.625	1200	0
200	5	a	1918.864	1200	0	1918.874	1200	0	**1912.76**	1200	0	1937.18	1200	0
200	5	b	2709.857	1200	0	1882.624	1200	0	**1876.52**	1200	0	1885.68	1200	0
200	5	c	**10.684**	1200	0	18.317	1200	0	1840.896	1200	0	1855.385	1200	0
200	5	d	2384.733	1200	0	2363.363	1200	0	**2124.573**	1200	0	2127.629	1200	0
200	5	e	3798.609	1200	0	3786.407	1200	0	3615.962	1200	0	**3591.542**	1200	0
200	10	a	17479.552	1213.33	0	**16515.071**	1200	0	17491.929	1200	0	16551.689	1200	0
200	10	b	17806.694	1210.43	0	20734.057	1201.4	0	**17797.536**	1200	0	17797.538	1200	0
200	10	c	16170.862	1200	0	**12788.826**	1200	0	12797.99	1200	0	20442.573	1200	0
200	10	d	18095.191	1200	0	**18092.139**	1202.57	0	18101.299	1200	0	22763.495	1200	0
200	10	e	18589.563	1200.21	0	18595.665	1200	0	**17806.154**	1200	0	17818.554	1200	0
200	15	a	**29403.65**	1200	0	32633.798	1200	0	29409.752	1200	0	35354.676	1200	0
200	15	b	32106.644	1200.38	0	35203.966	1200	0	**30495.007**	1200	0	38227.303	1200	0
200	15	c	**30485.324**	1200	0	30500.582	1200	0	30503.644	1200	0	37235.134	1200	0
200	15	d	26480.488	1200	0	33113.727	1200	0	**22850.196**	1200	0	37907.791	1200	0
200	15	e	28630.001	1200	0	33319.974	1200	0	28617.803	1200	0	**28611.703**	1200	0
200	20	a	53385.485	1200	0	**43788.36**	1200	0	53376.335	1200	0	57762.378	1200	0
200	20	b	51663.038	1200	0	**49722.198**	1200	0	49914.86	1200	0	63713.47	1200	0
200	20	c	46379.626	1200	0	**45552.728**	1200	0	51478.592	1200	0	51760.034	1200	0
200	20	d	48629.33	1200	0	53277.609	1200	0	**44105.198**	1200	0	58910.412	1200	0
200	20	e	50942.559	1200	0	**39420.737**	1200	0	45808.517	1200	0	50431.937	1200	0
400	2	a	19.158	1200	0	**4.886**	1200	0	19.308	1200	0	6.84	1200	0
400	2	b	4.144	1200	0	**0.62**	1200	0	5.502	1200	0	6.734	1200	0
400	2	c	**1.73**	1200	0	6.35	1200	0	8.56	1200	0	4.904	1200	0
400	2	d	**0.842**	1200	0	3.846	1200	0	9.952	1200	0	4.495	1200	0
400	2	e	1.548	1200	0	**0.891**	1200	0	9.552	1200	0	8.101	1200	0
400	3	a	198.164	1200	0	108.548	1200.16	0	98.832	1200	0	**90.244**	1200	0
400	3	b	211.739	1200	0	**149.585**	1200	0	172.752	1200	0	175.794	1200	0
400	3	c	**97.1**	1200	0	206.864	1200	0	203.735	1200	0	206.876	1200	0
400	3	d	**251.739**	1200	0	269.846	1200	0	274.533	1200	0	292.841	1200	0
400	3	e	185.529	1200	0	179.417	1200	0	208.047	1200	0	**149.797**	1200	0
400	5	a	**2472.552**	1200	0	2658.672	1200	0	2658.672	1200	0	2646.458	1200	0
400	5	b	**1491.263**	1200	0	2026.252	1200	0	2001.84	1200	0	3078.764	1200	0
400	5	c	**920.063**	1200	0	3162.647	1200	0	3087.802	1200	0	3100.008	1200	0
400	5	d	2511.647	1200	0	2690.316	1200	0	2690.318	1200	0	**2502.495**	1200	0
400	5	e	1631.646	1200	0	**1622.496**	1200	0	2851.873	1200	0	2839.667	1200	0
400	10	a	18436.079	1200.8	0	19013.111	1200.09	0	21581.081	1200	0	**16725.126**	1200	0
400	10	b	21190.523	1200.17	0	18702.998	1200	0	18712.146	1200	0	**17893.723**	1200	0
400	10	c	20709.911	1203.85	0	22009.308	1200.15	0	**18198.234**	1200	0	20391.976	1200	0
400	10	d	13735.6	1200	0	**13726.44**	1200.11	0	13747.816	1200	0	18123.474	1200	0
400	10	e	22024.199	1201.54	0	18896.512	1200	0	**16165.645**	1200	0	19541.989	1200	0
400	15	a	37709.021	1200	0	**30462.143**	1200	0	40511.241	1200	0	38765.615	1200	0
400	15	b	38534.025	1200.26	0	**34731.633**	1200.09	0	35414.982	1200	0	37984.444	1200	0
400	15	c	38658.121	1200	0	41839.477	1200	0	38846.33	1200	0	**36866.684**	1200	0
400	15	d	34646.147	1200	0	39582.518	1200	0	**34493.042**	1200	0	47523.21	1200	0
400	15	e	33711.591	1200.42	0	33624.756	1200	0	**31785.462**	1200	0	38910.69	1200	0
400	20	a	62625.795	1200	0	62211.657	1200	0	**51322.434**	1200	0	57268.898	1200	0
400	20	b	56002.291	1200	0	**54093.022**	1200	0	60459.223	1200	0	55606.06	1200	0
400	20	c	61141.342	1200	0	59878.639	1200	0	**45800.096**	1200	0	58489.442	1200	0
400	20	d	**49621.586**	1200	0	54203.142	1200	0	54839.093	1200	0	55671.767	1200	0
400	20	e	54729.968	1200	0	59583.114	1200	0	**52210.992**	1200	0	61708.882	1200	0

Table 4: Results for $n = 30,\ 40,\ 50$ and $k = 3$

n	m	$type$	FdSdS r	FdSdS $t[s]$	FdSdS LB	New-1 r	New-1 $t[s]$	New-1 LB	New-2 r	New-2 $t[s]$	New-2 LB
30	2	a	**80.259**	292.55	80.259	**80.259**	261.55	80.259	**80.259**	950.19	80.259
30	2	b	215.741	1200	0	441.903	1200	0	**78**	281.02	78
30	2	c	**144.433**	285.15	144.433	**144.433**	639.97	144.433	**144.433**	386.84	144.433
30	2	d	**135.582**	223.79	135.582	**135.582**	160.93	135.582	**135.582**	198.95	135.582
30	2	e	**129.451**	724.8	129.451	**129.451**	329.79	129.451	**129.451**	151.53	129.451
30	3	a	**773.446**	513.78	773.4451	**773.446**	134.69	773.446	**773.446**	56.7	773.446
30	3	b	**1378.271**	81.68	1378.271	**1378.271**	95.64	1378.271	**1378.271**	90.57	1378.271
30	3	c	**1012.334**	96.13	1012.334	**1012.334**	89.89	1012.334	**1012.334**	107.21	1012.334
30	3	d	**1192.211**	469.36	1192.211	**1192.211**	99.28	1192.211	**1192.211**	462.71	1192.211
30	3	e	**1202.613**	88.08	1202.613	**1202.613**	312.77	1202.613	**1202.613**	104.51	1202.613
30	5	a	**11072.565**	49.09	11072.565	**11072.565**	48.44	11072.565	**11072.565**	60.42	11072.565
30	5	b	**9259.137**	27.76	9259.137	**9259.137**	65.25	9259.137	**9259.137**	92.56	9259.137
30	5	c	**8484.901**	21.2	8484.901	**8484.901**	38.94	8484.901	**8484.901**	46.64	8484.901
30	5	d	**10773.195**	257.77	10773.195	**10773.195**	70.1	10773.195	**10773.195**	81.51	10773.195
30	5	e	**10826.653**	49.29	10826.653	**10826.653**	138.22	10826.653	**10826.653**	92.58	10826.653
30	10	a	**42823.408**	28.6	42823.408	**42823.408**	207.43	42823.408	**42823.408**	137.18	42823.408
30	10	b	**42828.972**	24.03	42828.972	**42828.972**	38.42	42828.972	**42828.972**	38.28	42828.972
30	10	c	**45214.814**	32.62	45214.814	**45214.814**	30.66	45214.814	**45214.814**	26.54	45214.814
30	10	d	**41095.153**	36.98	41095.153	**41095.153**	50.88	41095.153	**41095.153**	54.95	41095.153
30	10	e	**47539.404**	68.16	47539.404	**47539.404**	63.85	47539.404	**47539.404**	61.12	47539.404
30	15	a	**67164.828**	76.17	67164.828	**67164.828**	90.94	67164.828	**67164.828**	266.96	67164.828
30	15	b	**68745.794**	85.16	68745.794	**68745.794**	71.53	68745.794	**68745.794**	97.53	68745.794
30	15	c	**73912.823**	48.67	73912.823	**73912.823**	96.41	73912.823	**73912.823**	172.4	73912.823
30	15	d	**73713.771**	57.31	73713.771	**73713.771**	1130.98	73713.771	**73713.771**	116.77	73713.771
30	15	e	**76456.772**	65.39	76456.772	**76456.772**	141.57	76456.772	**76456.772**	87.5	76456.772
30	20	a	**90909.077**	111.83	90909.077	**90909.077**	134.38	90909.077	**90909.077**	671.22	90909.077
30	20	b	**93941.054**	206.05	93941.054	**93941.054**	204.95	93941.054	**93941.054**	561.78	93941.054
30	20	c	**89914.579**	83.59	89914.579	**89914.579**	112.42	89914.579	**89914.579**	140.22	89914.579
30	20	d	**92540.911**	137.8	92540.911	**92540.911**	167.22	92540.911	**92540.911**	111.6	92540.911
30	20	e	**95571.301**	309.51	95571.301	**95571.301**	143.89	95571.301	**95571.301**	532.54	95571.301
40	2	a	320.608	1200	0	**220.465**	1200	0	507.208	1200	0
40	2	b	**323.36**	1200	0	327.095	1200	0	374.141	1200	0
40	2	c	**122.389**	1200	0	305.685	1200	0	322.694	1200	0
40	2	d	244.968	1200	0	**156.027**	1200	0	169.011	1200	0
40	2	e	318.291	1200	0	129.517	1200	0	**123.328**	1200	0
40	3	a	2145.481	1200	0	**570.024**	1200	0	2135.036	1200	0
40	3	b	**1515.038**	1200	0	2041.601	1200	0	2854.636	1200	0
40	3	c	1469.431	1200	0	1372.011	1200	0	**1108.191**	1200	0
40	3	d	2157.667	1200	0	**1860.23**	1200	0	3218.618	1200	0
40	3	e	2157.026	1200	0	**1955.783**	1200	0	2313.412	1200	0
40	5	a	**8787.489**	1200	0	9762.916	1200	0	9318.52	1200	0
40	5	b	**10144.831**	1200	0	10302.597	1200	0	11014.181	1200	0
40	5	c	8864.727	1200	0	**7304**	1200	0	9265.389	1200	0
40	5	d	9575.019	1200	0	11001.387	1200	0	**9388.777**	1200	0
40	5	e	**10905.765**	1200	0	14405.095	1200	0	14737.84	1200	0
40	10	a	**27751.902**	1200	23871.0066	34228.892	1200	17642.1798	36844.128	1200	14691.011
40	10	b	40558.087	1200	12227.3042	**33831.339**	1200	21936.6283	40293.232	1200	4127.8009
40	10	c	40615.414	1200	13274.2334	31963.509	1200	23375.0918	**30202.157**	1165.8	30202.157
40	10	d	**32276.056**	1200	26157.6046	34652.582	1200	17706.245	35549.589	1200	12818.2165
40	10	e	42077.713	1200	5059.9956	38361.494	1200	16423.6278	**37283.926**	1200	23039.5066
40	15	a	**59645.354**	1200	40390.477	62649.602	1200	41953.4399	67271.245	1200	30485.574
40	15	b	65941.57	1200	40033.5941	66106.024	1200	31363.645	**60322.254**	1200	44435.5893
40	15	c	**55700.432**	1013.33	55700.432	70386.087	1200	40272.4282	**55700.432**	1200	50797.3659
40	15	d	60545.075	1200	53421.8363	**58287.145**	1200	41699.3327	63928.402	1200	45564.2238
40	15	e	65523.233	1200	41455.003	**58473.2**	1200	46135.1299	62618.34	1200	39324.6893
40	20	a	91489.348	1200	51161.6224	93043.1	1200	54409.0482	**91311.519**	1200	41554.8479
40	20	b	**89628.016**	1200	56861.4346	95050.823	1200	49953.2838	98972.779	1200	42560.6136
40	20	c	89721.493	1200	57697.798	93660.204	1200	43632.6679	**88495.385**	1200	53118.2211
40	20	d	91368.765	1200	56478.1937	99113.298	1200	57617.8785	**83662.159**	1200	61730.033
40	20	e	**91021.352**	1200	55972.5547	95531.054	1200	48023.0509	96655.744	1200	54319.4004
50	2	a	317.893	1200	0	**269.245**	1200	0	309.154	1200	0
50	2	b	203.182	1200	0	261.361	1200	0	**129.132**	1200	0
50	2	c	**219.844**	1200	0	222.594	1200	0	262.014	1200	0
50	2	d	276.328	1200	0	**214.582**	1200	0	264.321	1200	0
50	2	e	**131.002**	1200	0	285.743	1200	0	244.691	1200	0
50	3	a	1405.353	1200	0	1809.443	1200	0	**1244.386**	1200	0
50	3	b	1993.619	1200	0	**1633.551**	1200	0	1697.217	1200	0
50	3	c	1522.863	1200	0	1381.3	1200	0	**617.054**	1200	0
50	3	d	1801.01	1200	0	2168.423	1200	0	**1456.272**	1200	0
50	3	e	2340.947	1200	0	1732.583	1200	0	**1573.148**	1200	0
50	5	a	10308.708	1200	0	**9881.023**	1200	0	11238.054	1200	0
50	5	b	**7840.887**	1200	0	8929.619	1200	0	12079.316	1200	0
50	5	c	10348.392	1200	0	**7468.694**	1200	0	11772.818	1200	0
50	5	d	**9094.253**	1200	0	11632.594	1200	0	11139.726	1200	0
50	5	e	10219.303	1200	0	12541.514	1200	0	**9845.438**	1200	0
50	10	a	28724.766	1200	0	**28494.061**	1200	0	40640.306	1200	0
50	10	b	**41959.849**	1200	0	42468.854	1200	0	44059.321	1200	0
50	10	c	38073.507	1200	0	**37954.257**	1200	0	44430.482	1200	0
50	10	d	**31457.444**	1200	0	38504.288	1200	0	42588.436	1200	0
50	10	e	**39290.617**	1200	0	39989.334	1200	0	42942.348	1200	0
50	15	a	**63736.587**	1200	0	65229.815	1200	0	69707.444	1200	0
50	15	b	**67792.368**	1200.19	0	71689.739	1200	2190.5335	68758.332	1200	3443.2027
50	15	c	67964.141	1200	0	**66378.695**	1200	4693.0729	71341.735	1200	0
50	15	d	69620	1200	0	65905.913	1200	0	**59654.074**	1200	0
50	15	e	**59269.802**	1200	0	65731.089	1200	0	68855.733	1200	0
50	20	a	96373.413	1200	9540.3793	**90493.9**	1200	15584.9752	91361.477	1200	7348.0489
50	20	b	91380.297	1200	11181.5726	98150.072	1200	10866.7143	**89459.974**	1200	1739.7672
50	20	c	94780.729	1200	13746.2538	**90795.538**	1200	18139.102	97502.087	1200	13049.1647
50	20	d	**83460.991**	1200	18177.2677	97082.284	1200	8114.9238	87870.423	1200	12117.2923
50	20	e	**85985.393**	1200	12698.6859	91666.526	1200	12969.4806	90422.946	1200	12051.6278

Table 5: Results for $n = 100,\ 300,\ 500$ and $k = 3$

n	m	$type$	FdSdS			New-1			New-2		
			r	$t[s]$	LB	r	$t[s]$	LB	r	$t[s]$	LB
100	2	a	193.192	1200.1	0	279.332	1200	0	**92.688**	1200	0
100	2	b	**161.118**	1200	0	235.509	1200	0	167.797	1200	0
100	2	c	**148.432**	1200	0	207.94	1200.65	0	157.28	1200	0
100	2	d	233.734	1200	0	164.166	1200	0	**160.461**	1200	0
100	2	e	255.704	1200	0	**129.942**	1200.32	0	375.311	1200	0
100	3	a	1249.459	1200	0	**678.115**	1200	0	1208.584	1200	0
100	3	b	1697.015	1200	0	2253.52	1200	0	**1158.115**	1200	0
100	3	c	1411.766	1200	0	**1410.407**	1200	0	1550.38	1200	0
100	3	d	1529.636	1200	0	**588.487**	1200	0	1403.444	1200	0
100	3	e	1668.658	1200	0	**1157.319**	1200	0	1428.696	1200	0
100	5	a	**10418.016**	1200	0	10521.278	1200	0	11028.8	1200	0
100	5	b	11399.312	1200	0	**7125.513**	1200	0	11433.442	1200.3	0
100	5	c	**9443.727**	1200	0	10347.103	1200	0	11893.044	1200	0
100	5	d	12551.612	1200	0	10514.574	1200	0	**8808.212**	1200	0
100	5	e	10884.699	1200	0	**8596.977**	1200	0	10456.419	1200	0
100	10	a	39730.923	1200	0	**38202.361**	1200	0	44090.095	1200	0
100	10	b	33281.848	1200	0	**29725.95**	1200	0	38945.758	1200	0
100	10	c	40129.175	1200	0	**37759.036**	1200	0	39453.714	1200	0
100	10	d	38729.231	1200	0	**32093.057**	1200	0	41833.15	1200	0
100	10	e	39419.147	1200	0	41833.845	1200	0	**38264.1**	1200	0
100	15	a	**59056.436**	1200	0	72219.436	1200	0	60969.112	1200	0
100	15	b	67856.899	1200	0	62345.794	1200	0	**62057.077**	1200	0
100	15	c	59762.607	1200	0	65954.578	1200	0	**55986.222**	1200	0
100	15	d	68501.063	1200	0	67206.269	1200	0	**58434.533**	1200	0
100	15	e	74391.616	1200	0	63602.138	1200	0	**59853.418**	1200	0
100	20	a	95286.274	1200	0	91820.012	1200	0	**85561.934**	1200	0
100	20	b	**83957.155**	1200	0	86217.197	1200	0	92309.958	1200	0
100	20	c	**86583.714**	1200	0	100212.651	1200	0	89632.159	1200	0
100	20	d	86749.649	1200	0	**86168.17**	1200	0	92177.919	1200	0
100	20	e	88077.106	1200	0	93027.195	1200	0	**86400.076**	1200	0
300	2	a	202.223	1200.94	0	**158.878**	1205.58	0	239.22	1205.04	0
300	2	b	287.741	1200	0	207.592	1208.42	0	**169.5**	1204.07	0
300	2	c	66.088	1201.25	0	**55.372**	1214.65	0	127.907	1200	0
300	2	d	305.74	1200.37	0	257.236	1214.35	0	**170.905**	1204.66	0
300	2	e	**67.659**	1200	0	119.083	1202.86	0	174.28	1200	0
300	3	a	1625.162	1200.12	0	1807.164	1209.32	0	**1256.199**	1200	0
300	3	b	1772.102	1200	0	**1397.856**	1200.61	0	2030.697	1200	0
300	3	c	**842.076**	1200	0	1237.107	1200	0	1565.426	1200	0
300	3	d	1980.135	1200	0	**1601.351**	1200.37	0	2317.517	1200	0
300	3	e	**1647.167**	1200.23	0	2796.311	1200.24	0	2534.023	1200	0
300	5	a	**9535.332**	1200	0	11682.709	1200	0	11754.545	1200	0
300	5	b	10410.224	1200	0	10948.572	1200	0	**9616.228**	1200	0
300	5	c	8799.906	1200	0	**8744.229**	1200	0	9642.188	1200.26	0
300	5	d	10916.582	1200	0	**7034.639**	1200	0	14517.019	1200.07	0
300	5	e	**9601.762**	1200	0	11904.136	1200	0	9810.493	1200	0
300	10	a	36747.868	1200	0	**33363.813**	1200	0	35056.145	1200	0
300	10	b	39436.93	1200	0	34226.044	1200	0	**33392.556**	1200	0
300	10	c	41958.988	1200	0	45942.419	1200	0	**29139.115**	1200	0
300	10	d	38330.055	1200	0	**32442.707**	1200	0	42759.786	1200	0
300	10	e	40191.523	1200	0	41741.095	1200	0	**36208.151**	1200	0
300	15	a	66082.967	1200	0	65357.609	1200	0	**57421.682**	1200	0
300	15	b	**50990.525**	1200	0	57117.968	1200	0	61602.229	1200	0
300	15	c	**66680.359**	1200	0	71082.798	1200	0	68861.33	1200	0
300	15	d	68634.577	1200	0	**65134.845**	1200	0	71338.187	1200	0
300	15	e	**63722.366**	1200	0	71550.673	1200	0	67399.53	1200	0
300	20	a	90556.088	1200	0	**88499.986**	1200	0	90005.626	1200	0
300	20	b	93738.707	1200	0	**86253.546**	1200	0	101511.242	1200	0
300	20	c	89663.736	1200	0	**83562.993**	1200	0	88623.772	1200	0
300	20	d	89832.556	1200	0	92267.461	1200	0	**88468.376**	1200	0
300	20	e	**88725.587**	1200	0	97870.072	1200	0	94730.287	1200	0
500	2	a	**123.884**	1208.21	0	240.708	1200	0	162.664	1200	0
500	2	b	279.341	1208.87	0	**228.501**	1202.49	0	262.908	1200	0
500	2	c	**160.498**	1206.03	0	279.711	1210.38	0	258.503	1200	0
500	2	d	225.848	1205.93	0	206.771	1204	0	**134.418**	1209.13	0
500	2	e	103.044	1206.33	0	141.975	1208.76	0	**39.711**	1200	0
500	3	a	**1380.356**	1200.64	0	2604.61	1203.84	0	1534.254	1200	0
500	3	b	1723.319	1204.74	0	2019.824	1203.8	0	**1643.997**	1200	0
500	3	c	1430.253	1206.72	0	1918.567	1201.48	0	**885.304**	1200	0
500	3	d	2025.618	1202.91	0	2166.65	1218.89	0	**1531.129**	1200	0
500	3	e	**1415.841**	1202.85	0	1619.036	1205.05	0	1604.97	1200	0
500	5	a	10880.144	1200	0	9372.421	1201.56	0	**9196.418**	1200	0
500	5	b	12377.821	1200	0	12097.825	1200	0	**10478.277**	1200	0
500	5	c	**7058.296**	1200	0	10294.711	1201.66	0	10195.353	1200.13	0
500	5	d	9421.06	1200	0	**6092.68**	1200	0	8587.859	1200	0
500	5	e	**9092.992**	1200	0	10363.611	1200	0	10688.123	1200.05	0
500	10	a	**36935.645**	1200	0	43234.285	1200	0	43258.487	1200	0
500	10	b	**34545.025**	1200	0	43035.159	1200	0	43644.319	1200	0
500	10	c	**36715.003**	1200	0	42043.809	1200	0	39497.799	1200	0
500	10	d	**31334.361**	1200	0	34411.693	1200	0	44922.956	1200	0
500	10	e	**33245.182**	1200	0	37570.549	1200	0	40463.256	1200	0
500	15	a	**62192.663**	1200	0	75763.045	1200	0	73221.412	1200	0
500	15	b	**62743.504**	1200	0	69163.348	1200	0	70867.434	1200	0
500	15	c	63852.549	1200	0	**63255.691**	1200	0	67274.186	1200	0
500	15	d	**60902.016**	1200	0	70480.502	1200	0	67187.431	1200	0
500	15	e	**64902.84**	1200	0	76205.427	1200	0	65833.195	1200	0
500	20	a	**85958.535**	1200.01	0	92799.387	1200	0	91286.375	1200	0
500	20	b	85716.808	1200	0	91424.717	1200	0	**82184.912**	1200	0
500	20	c	**71964.643**	1200	0	91342.311	1200	0	90785.725	1200	0
500	20	d	97886.603	1200	0	96731.31	1200	0	**91457.664**	1200	0
500	20	e	91827.787	1200	0	101980.413	1200	0	**91263.67**	1200	0

Table 6: Results for $n = 200,\ 400$ and $k = 3$

n	m	$type$	FdSdS			New-1			New-2		
			r	$t[s]$	LB	r	$t[s]$	LB	r	$t[s]$	LB
200	2	a	159.974	1200	0	151.867	1207.53	0	**79.569**	1200	0
200	2	b	**136.248**	1200	0	162.397	1203.07	0	261.653	1200.43	0
200	2	c	**91.293**	1200	0	117.124	1200	0	228.456	1200	0
200	2	d	148.054	1200.24	0	**111.209**	1205.19	0	261.916	1200	0
200	2	e	180.024	1200.27	0	173.711	1201.78	0	**131.543**	1200	0
200	3	a	1196.877	1200	0	1513.98	1205.27	0	**631.15**	1200	0
200	3	b	**1125.584**	1200	0	2085.407	1207.96	0	2231.055	1200	0
200	3	c	**1369.679**	1200	0	1711.714	1209.99	0	2466.571	1200	0
200	3	d	1687.252	1200	0	**1425.548**	1201.71	0	2146.71	1200	0
200	3	e	**1506.17**	1200	0	2025.263	1200	0	1545.151	1200	0
200	5	a	9764.38	1200	0	11424.929	1200	0	**9127.63**	1200	0
200	5	b	**9418.789**	1200	0	10471.75	1200	0	12411.434	1200.97	0
200	5	c	**5839.467**	1200	0	7496.691	1200	0	10195.665	1200	0
200	5	d	9873.172	1200	0	**8959.76**	1200	0	11196.528	1200.76	0
200	5	e	**10606.923**	1200	0	12772.025	1200	0	13243.998	1201.17	0
200	10	a	**36350.164**	1200	0	37635.086	1200	0	40544.755	1200	0
200	10	b	42913.81	1200	0	42540.114	1200	0	**39322.039**	1200	0
200	10	c	**34076.224**	1200	0	43665.643	1200	0	36116.264	1200	0
200	10	d	37926.069	1200	0	**28073.452**	1200	0	42110.976	1200	0
200	10	e	45851.853	1200	0	**36214.653**	1200	0	42212.323	1200	0
200	15	a	**54012.679**	1200	0	65223.41	1200	0	69195.547	1200	0
200	15	b	62910.295	1200	0	65620.155	1200	0	**62388.548**	1200	0
200	15	c	**58550.943**	1200	0	69511.473	1200	0	65953.103	1200	0
200	15	d	**65169.349**	1200	0	67273.819	1200	0	71979.057	1200	0
200	15	e	**63557.545**	1200	0	64281.355	1200	0	70975.367	1200	0
200	20	a	85440.475	1200	0	87960.287	1200	0	**80396.46**	1200	0
200	20	b	89580.693	1200	0	91315.981	1200	0	**82141.502**	1200	0
200	20	c	**74128.064**	1200	0	101381.699	1200	0	85432.046	1200	0
200	20	d	**84400.188**	1200	0	92583.608	1200	0	86626.178	1200	0
200	20	e	**88915.728**	1200	0	100022.199	1200	0	92624.3	1200	0
400	2	a	227.071	1203.63	0	194.826	1211.09	0	**89.081**	1200	0
400	2	b	192.535	1203.95	0	284.6047	1203.48	0	**176.891**	1200	0
400	2	c	273.407	1201.1	0	269.772	1223.39	0	**81.132**	1208.28	0
400	2	d	360.078	1202.35	0	324.873	1219.63	0	**263.478**	1200	0
400	2	e	**124.102**	1202.55	0	153.638	1213.1	0	194.826	1200.13	0
400	3	a	**1072.531**	1200	0	2506.665	1201.11	0	1128.616	1200	0
400	3	b	1840.54	1201.92	0	**1561.935**	1209.07	0	2462.509	1200	0
400	3	c	**1190.525**	1203.14	0	2123.557	1217.06	0	2436.516	1200	0
400	3	d	**794.487**	1202.08	0	1836.051	1208.8	0	2131.033	1200	0
400	3	e	2208.49	1200.22	0	**2164.061**	1209.99	0	2240.918	1200	0
400	5	a	10065.049	1200	0	**7174.857**	1200	0	9628.135	1200.11	0
400	5	b	**8620.8**	1200	0	10884.778	1200	0	10819.646	1200.28	0
400	5	c	9930.805	1200	0	9321.404	1200	0	**7145.684**	1200.15	0
400	5	d	**6414.779**	1200	0	10299.745	1201.06	0	12751.181	1200	0
400	5	e	**10638.446**	1200	0	11241.726	1200.78	0	12737.588	1200.17	0
400	10	a	33062.416	1200	0	40858.272	1200	0	**29030.453**	1200	0
400	10	b	36772.451	1200	0	**36699.434**	1200	0	44567.119	1200	0
400	10	c	42124.733	1200	0	39172.757	1200	0	**37127.974**	1200	0
400	10	d	**35066.124**	1200	0	39039.355	1200	0	46377.238	1200	0
400	10	e	**36408.221**	1200	0	44140.625	1200	0	37784.038	1200	0
400	15	a	64132.673	1200	0	**62033.431**	1200	0	64629.296	1200	0
400	15	b	66505.745	1200	0	69386.038	1200	0	**62238.581**	1200	0
400	15	c	75579.191	1200	0	67197.069	1200	0	**58805.149**	1200	0
400	15	d	67175.98	1200	0	68416.76	1200	0	**60406.197**	1200	0
400	15	e	66247.82	1200	0	68863.21	1200	0	**60822.148**	1200	0
400	20	a	81817.325	1200	0	93352.965	1200	0	**79522.755**	1200	0
400	20	b	**85452.88**	1200	0	88155.773	1200	0	85940.935	1200	0
400	20	c	90070.181	1200	0	94315.197	1200	0	**87392.614**	1200	0
400	20	d	95652.502	1200	0	**95036.676**	1200	0	96400.061	1200	0
400	20	e	**78702.317**	1200	0	84394.101	1200	0	93572.512	1200	0

Table 7: Results for $n = 30,\ 40,\ 50$ and $k = 4$

n	m	$type$	FdSdS			New-1			New-2		
			r	$t[s]$	LB	r	$t[s]$	LB	r	$t[s]$	LB
30	2	a	**476.461**	1200	381.7173	1470.831	1200	0	**476.461**	704.22	476.461
30	2	b	**434.266**	275.54	434.266	**434.266**	214.74	434.266	**434.266**	144.36	434.266
30	2	c	690.456	1200	214.4163	**493.275**	410.06	493.275	**493.275**	1200	438.447
30	2	d	**404.159**	276.14	404.159	**404.159**	89.71	404.159	**404.159**	190.53	404.159
30	2	e	**486.858**	253.55	486.858	**486.858**	411.7	486.858	**486.858**	360.75	486.858
30	3	a	**2691.514**	1028.84	2691.514	**2691.514**	59.21	2691.514	**2691.514**	82.41	2691.514
30	3	b	**3859.603**	307.41	3859.603	**3859.603**	444.8	3859.603	**3859.603**	352.29	3859.603
30	3	c	**3312.903**	116.57	3312.903	**3312.903**	76.49	3312.903	**3312.903**	333.56	3312.903
30	3	d	**2996.785**	134.05	2996.785	**2996.785**	88.35	2996.785	**2996.785**	71.03	2996.785
30	3	e	**3164.05**	163.51	3164.05	**3164.05**	48.63	3164.05	**3164.05**	161.84	3164.05
30	5	a	**15662.239**	87.55	15662.239	**15662.239**	59.44	15662.239	**15662.239**	472.48	15662.239
30	5	b	**16329.085**	130.58	16329.085	**16329.085**	74.76	16329.085	**16329.085**	106.47	16329.085
30	5	c	**14719.782**	81.17	14719.782	**14719.782**	91.51	14719.782	**14719.782**	346.55	14719.782
30	5	d	**18241.625**	221.3	18241.625	**18241.625**	100.18	18241.625	**18241.625**	148.38	18241.625
30	5	e	**16472.702**	148.37	16472.702	**16472.702**	321.01	16472.702	**16472.702**	141.69	16472.702
30	10	a	**47407.536**	103.62	47407.536	**47407.536**	845.06	47407.536	**47407.536**	410.12	47407.536
30	10	b	**60485.478**	786.66	60485.478	**60485.478**	1200	58897.1223	**60485.478**	574.22	60485.478
30	10	c	**59732.302**	719.41	59732.302	**59732.302**	1176.48	59732.302	**59732.302**	1200	59164.7272
30	10	d	**42615.872**	412.98	42615.872	**42615.872**	77	42615.872	**42615.872**	183.89	42615.872
30	10	e	**56243.651**	404.07	56243.651	**56243.651**	1200	51229.9233	**56243.651**	917.19	56243.651
30	15	a	86159.422	1200	63695.4803	91179.388	1200	55360.7661	**86015.468**	1200	64446.6463
30	15	b	89239.516	1200	72923.299	**89088.511**	1200	71097.692	92378.586	1200	74788.0937
30	15	c	**88457.13**	1200	79756.5843	91028.378	1200	74452.8843	**88457.13**	1200	74777.486
30	15	d	**84269.4**	1200	65954.8538	**84269.4**	1200	80634.7171	**84269.4**	1200	71393.8213
30	15	e	87270.621	1200	77814.3283	87270.621	1200	80083.4063	**86602.419**	1200	77294.9317
30	20	a	**106025.717**	1200	88029.6953	114011.061	1200	85605.4551	107436.246	1200	81656.4923
30	20	b	**107509.547**	1200	87025.5497	115330.048	1200	88562.716	111681.539	1200	88054.4327
30	20	c	108128.536	1200	94684.4633	**107783.722**	1200	86213.9653	**107783.722**	1200	95923.0087
30	20	d	111428.33	1200	91612.321	**108979.717**	1200	83722.7702	110715.98	1200	85141.7088
30	20	e	**102003.679**	1200	91387.1624	**102003.679**	1200	84319.1067	**102003.679**	1200	84640.2681
40	2	a	1293.599	1200	0	**1180.738**	1200	0	1274.943	1200	0
40	2	b	897.264	1200	0	**760.018**	1200	0	1130.51	1200	0
40	2	c	**888.049**	1200	0	1211.397	1200	0	1103.024	1200	0
40	2	d	1052.271	1200	0	**792.688**	1200	0	1079.849	1200	0
40	2	e	915.122	1200	0	**796.85**	1200	0	962.333	1200	0
40	3	a	5105.505	1200	0	5149.997	1200	0	**3483.441**	1200	0
40	3	b	6004.476	1200	0	**5245.663**	1200	0	7708.947	1200	0
40	3	c	4915.436	1200	0	**3990.067**	1200	0	4914.408	1200	0
40	3	d	5394.131	1200	0	**5362.181**	1200	0	6295.255	1200	0
40	3	e	**4364.643**	1200	0	5673.365	1200	0	6119.401	1200	0
40	5	a	16744.835	1200	0	14426.917	1200	3400.7966	**13762.232**	1200	0
40	5	b	20117.902	1200	0	**7869.66**	1200	5301.6407	18911.636	1200	0
40	5	c	18139.451	1200	0	**11775.621**	1200	4702.427	18363.485	1200	0
40	5	d	23293.091	1200	0	**22999.061**	1200	0	24867.165	1200	0
40	5	e	22559.158	1200	3322.2897	**10273.927**	1200	4548.0893	22149.327	1200	0
40	10	a	49129.67	1200	23165.883	60621.128	1200	14810.926	**48258.578**	1200	18737.0289
40	10	b	50748.499	1200	24309.4807	43081.913	1200	24382.6794	**40172.57**	1200	25666.5327
40	10	c	63658.595	1200	19633.9605	57935.929	1200	16326.223	**55516.159**	1200	15704.9368
40	10	d	53190.053	1200	14954.3794	60421.576	1200	18665.1454	**51738.302**	1200	22756.482
40	10	e	**53060.095**	1200	24687.9005	68331.264	1200	9258.2233	61895.388	1200	18892.259
40	15	a	**79138.683**	1200	39194.4547	**79138.683**	1200	23274.2223	84909.701	1200	33933.6584
40	15	b	**77920.308**	1200	37952.2043	**77920.308**	1200	39012.1783	87096.929	1200	31323.7889
40	15	c	**78489.705**	1200	40067.8451	84350.918	1200	40617.3171	91077.877	1200	30967.9922
40	15	d	87568.196	1200	33599.2379	**80726.573**	1200	39099.9272	97105.447	1200	40014.9527
40	15	e	**86758.055**	1200	31712.8129	90742.026	1200	27081.3247	87387.643	1200	36756.916
40	20	a	**103991.524**	1200	40552.5475	106300.774	1200	36389.256	109272.442	1200	38878.7293
40	20	b	111111.261	1200	55677.5169	**105824.367**	1200	50609.5511	114917.443	1200	45420.6349
40	20	c	**105854.558**	1200	39491.3997	105900.783	1200	37845.3904	118919.746	1200	44030.8456
40	20	d	**103465.009**	1200	44064.186	104341.565	1200	37999.1385	108646.541	1200	35971.6247
40	20	e	109577.983	1200	39935.9243	108986.675	1200	36943.6877	**106184.045**	1200	35723.7755
50	2	a	1270.716	1200	0	**658.036**	1200	0	1190.588	1200	0
50	2	b	1010.463	1200	0	703.344	1200	0	**568.648**	1200	0
50	2	c	1045.69	1200	0	1096.866	1200	0	**586.357**	1200	0
50	2	d	954.772	1200	0	**818.101**	1200	0	935.151	1200	0
50	2	e	927.791	1200	0	**678.724**	1200	0	845.418	1200	0
50	3	a	4970.049	1200	0	**4257.352**	1200	0	5623.479	1200	0
50	3	b	5365.222	1200	0	**3414.396**	1200	0	5417.589	1200	0
50	3	c	3773.888	1200	0	**3711.143**	1200	0	4417.341	1200	0
50	3	d	4043.057	1200	0	**3393.003**	1200	0	4755.618	1200	0
50	3	e	6537.878	1200	0	4153.511	1200	0	**3950.427**	1200	0
50	5	a	20847.197	1200	0	20618.834	1200	0	**15534.998**	1200	0
50	5	b	20360.081	1200	0	18382.098	1200	0	**15395.31**	1200	0
50	5	c	20833.769	1200	0	**16743.309**	1200	0	19874.039	1200	0
50	5	d	**18438.365**	1200	0	20243.75	1200	0	24473.527	1200	0
50	5	e	25379.872	1200	0	22131.43	1200	0	**21132.106**	1200	0
50	10	a	51730.47	1200	0	57348.325	1200	0	**50849.672**	1200	0
50	10	b	**56732.249**	1200	0	63828.682	1200	0	57866.583	1200	0
50	10	c	**56862.357**	1200	0	59485.911	1200	0	58387.013	1200	0
50	10	d	55202.779	1200	0	**47752.942**	1200	0	60693.211	1200	0
50	10	e	56228.286	1200	0	64572.974	1200	0	**55308.656**	1200	0
50	15	a	85889.285	1200	730.2231	85815.126	1200	0	**79702.344**	1200	8550.6077
50	15	b	87960.652	1200	3698.5228	89138.113	1200	863.2263	**85350.282**	1200	9278.318
50	15	c	92169.504	1200	0	92272.519	1200	8359.4062	**85913.35**	1200	5284.692
50	15	d	95542.932	1200	551.3026	**75582.218**	1200	7296.5382	85979.168	1200	4212.3993
50	15	e	86218.14	1200	4160.976	**85410.662**	1200	3999.6805	88546.938	1200	0
50	20	a	**101370.594**	1200	20323.0975	106213.503	1200	22597.3526	109674.131	1200	10018.0514
50	20	b	116428.597	1200	16171.2156	111387.618	1200	19515.0228	**102743.811**	1200	15967.182
50	20	c	114529.921	1200	14384.8757	**110235.008**	1200	20240.7686	113852.427	1200	17871.1492
50	20	d	113643.808	1200	13960.6758	**108284.302**	1200	10660.413	109217.665	1200	17147.7899
50	20	e	**107132.182**	1200	7415.911	107559.568	1200	14268.5181	107829.074	1200	9214.4655

Table 8: Results for $n = 100,\ 300,\ 500$ and $k = 4$

n	m	$type$	FdSdS			New-1			New-2		
			r	$t[s]$	LB	r	$t[s]$	LB	r	$t[s]$	LB
100	2	a	595.724	1200	0	**564.717**	1200	0	956.496	1200	0
100	2	b	714.742	1200	0	**474.591**	1200	0	871.927	1200	0
100	2	c	**363.377**	1200	0	578.064	1200	0	566.449	1200	0
100	2	d	563.518	1200	0	**394.623**	1200	0	676.398	1200	0
100	2	e	**496.016**	1200	0	795.998	1200	0	616.444	1200	0
100	3	a	4939.118	1200	0	3548.249	1200	0	**2658.49**	1200	0
100	3	b	**2479.666**	1200	0	4199.276	1200	0	3255.663	1200	0
100	3	c	4143.471	1200	0	**2921.075**	1200	0	3272.253	1200	0
100	3	d	4629.729	1200	0	4217.105	1200	0	**3296.405**	1200	0
100	3	e	3963.063	1200	0	4722.962	1200	0	**3473.167**	1200	0
100	5	a	16172.561	1200	0	20635.316	1200	0	**15846.7**	1200	0
100	5	b	18907.036	1200	0	**17550.956**	1200	0	20534.701	1200	0
100	5	c	16945.923	1200	0	**15900**	1200	0	17692.02	1200	0
100	5	d	20029.396	1200.01	0	**17180.421**	1200	0	21602.706	1200	0
100	5	e	**19208.476**	1200	0	20393.569	1200	0	19349.279	1200	0
100	10	a	54141.023	1200	0	**49627.888**	1200	0	61419.317	1200	0
100	10	b	55077.644	1200	0	**48877.159**	1200	0	53654.723	1200	0
100	10	c	**54477.969**	1200	0	56755.418	1200	0	62963.804	1200	0
100	10	d	52962.124	1200	0	**51101.161**	1200	0	57517.593	1200	0
100	10	e	55841.734	1200	0	58825.479	1200	0	**53968.704**	1200	0
100	15	a	91919.613	1200	0	87210.491	1200	0	**70988.514**	1200	0
100	15	b	84651.933	1200	0	96522.09	1200	0	**83342.016**	1200	0
100	15	c	79786.998	1200	0	85665.539	1200	0	**75650.515**	1200	0
100	15	d	90022.191	1200	0	84153.708	1200	0	**82535.41**	1200	0
100	15	e	91503.321	1200	0	89640.28	1200	0	**84928.803**	1200	0
100	20	a	**108824.236**	1200	0	111969.14	1200	0	111404.082	1200	0
100	20	b	104658.659	1200	0	107642.012	1200	0	**101590.043**	1200	0
100	20	c	**107854.678**	1200	0	109289.859	1200	0	117774.724	1200	0
100	20	d	**85110.668**	1200	0	106646.615	1200	0	102187.647	1200	0
100	20	e	107521.91	1200	0	109184.216	1200	0	**103551.823**	1200	0
300	2	a	**356.348**	1200	0	632.8	1201.27	0	653.277	1200.36	0
300	2	b	**332.779**	1200	0	991.454	1200.15	0	770.923	1200	0
300	2	c	**341.217**	1200	0	441.734	1210.98	0	538.076	1200	0
300	2	d	**368.973**	1200	0	527.609	1213.56	0	536.085	1200	0
300	2	e	**265.534**	1200	0	481.058	1208.97	0	926.708	1200	0
300	3	a	3061.626	1200	0	**2600.075**	1200	0	3878.235	1200	0
300	3	b	4656.793	1200	0	**4052.803**	1200	0	4967.458	1200	0
300	3	c	2987.569	1200	0	**2225.575**	1200	0	3230.507	1200	0
300	3	d	3320.671	1200	0	**1762.421**	1200	0	3687.999	1200	0
300	3	e	4867.315	1200	0	3798.905	1200.16	0	**2503.874**	1200	0
300	5	a	**15494.229**	1200	0	16097.526	1200	0	18396.643	1200	0
300	5	b	19683.825	1200	0	**19581.381**	1200	0	20255.88	1200.06	0
300	5	c	**9908.01**	1200	0	19848.218	1200	0	19873.337	1200.23	0
300	5	d	**15948.913**	1200	0	17119.261	1200	0	19808.573	1200	0
300	5	e	16095.236	1200	0	**16074.562**	1200	0	19256.628	1200	0
300	10	a	61483.568	1200	0	54421.124	1200	0	**51246.59**	1200	0
300	10	b	59882.383	1200	0	54907.192	1200	0	**45416.699**	1200	0
300	10	c	55372.209	1200	0	51844.786	1200	0	**50915.561**	1200	0
300	10	d	**47100.92**	1200	0	50685.176	1200	0	52007.031	1200	0
300	10	e	**40091.803**	1200	0	50282.988	1200	0	59154.964	1200	0
300	15	a	80227.971	1200	0	80561.979	1200	0	**77845.709**	1200	0
300	15	b	81761.657	1200.01	0	**79824.221**	1200	0	88537.173	1200	0
300	15	c	81371.639	1200	0	**77201.074**	1200	0	86517.069	1200	0
300	15	d	82988.183	1200	0	**69516.505**	1200	0	84788.623	1200	0
300	15	e	86560.061	1200	0	**69981.033**	1200	0	84629.765	1200	0
300	20	a	113361.298	1200	0	**104482.011**	1200	0	116904.643	1200	0
300	20	b	104006.668	1200	0	103934.312	1200	0	**101363.135**	1200	0
300	20	c	113599.65	1200	0	**106238.443**	1200	0	112436.843	1200	0
300	20	d	**106537.921**	1200	0	107726.7	1200	0	113009.361	1200	0
300	20	e	**101565.382**	1200	0	113476.031	1200	0	107075.776	1200	0
500	2	a	488.441	1200	0	517.807	1200.85	0	**187.483**	1200	0
500	2	b	**358.612**	1200	0	976.611	1202.73	0	487.461	1200	0
500	2	c	521.578	1200	0	465.065	1211.63	0	**313.964**	1200	0
500	2	d	622.315	1200	0	**483.472**	1219.34	0	518.012	1200	0
500	2	e	**239.685**	1200.62	0	614.093	1203.52	0	624.592	1200	0
500	3	a	3019.177	1200	0	**2041.091**	1200	0	2776.812	1200	0
500	3	b	3977.862	1200	0	3499.541	1200	0	**3426.192**	1200	0
500	3	c	**2176.776**	1200.01	0	3197.691	1200	0	3466.192	1200	0
500	3	d	2874.47	1200	0	**1241.874**	1211.56	0	5003.594	1200	0
500	3	e	**3252.676**	1200	0	4020.268	1200	0	5714.109	1200.03	0
500	5	a	17083.923	1200	0	18895.099	1200	0	**14662.666**	1200	0
500	5	b	**13628.326**	1200.68	0	19563.879	1200	0	13821.586	1200	0
500	5	c	15589.318	1200	0	**12689.993**	1200	0	17023.73	1200	0
500	5	d	**13501.305**	1201.99	0	16961.48	1201.37	0	18681.569	1200	0
500	5	e	**18926.81**	1202.64	0	19880.285	1200	0	20008.499	1200	0
500	10	a	**49737.444**	1200.01	0	61375.949	1200	0	52908.221	1200	0
500	10	b	50432.167	1200	0	**49818.927**	1200	0	56325.795	1200	0
500	10	c	49814.366	1200	0	**49207.936**	1200	0	49229.729	1200	0
500	10	d	**44864.73**	1200	0	56974.674	1200	0	54889.61	1200	0
500	10	e	**42803.001**	1200	0	50485.183	1200	0	45898.261	1200	0
500	15	a	90576.717	1200	0	85460.432	1200	0	**73107.323**	1200	0
500	15	b	79804.023	1200.01	0	95477.733	1200	0	**76924.702**	1200	0
500	15	c	80109.987	1200	0	**77973.747**	1200	0	90077.324	1200	0
500	15	d	83008.823	1200	0	88325.776	1200	0	**80939.675**	1200	0
500	15	e	**74257.658**	1200	0	85703.948	1200	0	83493.111	1200.01	0
500	20	a	106488.337	1200	0	107435.665	1200	0	**105849.943**	1200	0
500	20	b	118156.812	1200	0	110976.254	1200	0	**110275.4**	1200	0
500	20	c	112240.9	1200	0	**107858.145**	1200	0	109250.435	1200	0
500	20	d	107254.527	1200	0	107662.072	1200	0	**103271.843**	1200	0
500	20	e	**105773.54**	1200	0	108087.712	1200	0	107698.151	1200	0

Table 9: Results for $n = 200,\ 400$ and $k = 4$

n	m	$type$	FdSdS			New-1			New-2		
			r	$t[s]$	LB	r	$t[s]$	LB	r	$t[s]$	LB
200	2	a	702.307	1200	0	**534.375**	1200	0	564.955	1200	0
200	2	b	**333.273**	1200	0	775.376	1200	0	667.393	1200	0
200	2	c	442.229	1200	0	**426.607**	1200	0	479.365	1200	0
200	2	d	633.389	1200	0	581.435	1200	0	**449.81**	1200	0
200	2	e	720.175	1200	0	489.207	1200	0	**422.577**	1200	0
200	3	a	**2929.723**	1200	0	4835.643	1200	0	4218.361	1200	0
200	3	b	4047.762	1200	0	5233.246	1200	0	**3521.923**	1200	0
200	3	c	3949.6	1200	0	3830.06	1200	0	**3139.607**	1200	0
200	3	d	**3455.488**	1200	0	4104.711	1200	0	4075.068	1200	0
200	3	e	4722.186	1200	0	**2992.715**	1200	0	4527.365	1200	0
200	5	a	20537.627	1200	0	20077.544	1200	0	**15639.584**	1200	0
200	5	b	19806.458	1200	0	**16487.626**	1200	0	21358.7	1200	0
200	5	c	17962.712	1200	0	16528.556	1200	0	**13351.277**	1200	0
200	5	d	16966.555	1200	0	19128.772	1200	0	**16301.54**	1201.4	0
200	5	e	17809.808	1200	0	19070.364	1200	0	**16861.759**	1200	0
200	10	a	**48206.927**	1200	0	57008.139	1200	0	54829.421	1200	0
200	10	b	**55185.276**	1200	0	59902.796	1200	0	55867.189	1200	0
200	10	c	53082.197	1200	0	55678.868	1200	0	**47886.633**	1200	0
200	10	d	55158.827	1200	0	55973.567	1200	0	**54342.489**	1200	0
200	10	e	**50136.101**	1200	0	58071.788	1200	0	57582.023	1200	0
200	15	a	**80443.27**	1200	0	86468.766	1200	0	85806.647	1200	0
200	15	b	85674.378	1200	0	87197.077	1200	0	**84465.181**	1200	0
200	15	c	84609.145	1200	0	93650.983	1200	0	**82218.399**	1200	0
200	15	d	85528.325	1200	0	**84302.265**	1200	0	85189.152	1200	0
200	15	e	87637.541	1200	0	90167.891	1200	0	**87183.752**	1200	0
200	20	a	**103684.152**	1200	0	108229.191	1200	0	107296.909	1200	0
200	20	b	120558.859	1200	0	**85921.888**	1200	0	111812.132	1200	0
200	20	c	**90341.889**	1200	0	100861.591	1200	0	119475.228	1200	0
200	20	d	**92801.027**	1200.01	0	106768.818	1200	0	113174.703	1200	0
200	20	e	**105673.291**	1200	0	106363.065	1200	0	106143.533	1200	0
400	2	a	362.782	1200	0	991.244	1202.38	0	**331.58**	1200	0
400	2	b	**420.701**	1200	0	661.407	1201.31	0	542.84	1200	0
400	2	c	605.155	1200	0	664.554	1202.02	0	**230.752**	1200	0
400	2	d	362.039	1200.54	0	714.593	1202.59	0	**333.371**	1200	0
400	2	e	397.252	1200	0	**392.864**	1205.22	0	445.575	1200	0
400	3	a	3559.881	1200	0	**2842.855**	1200	0	2910.901	1200	0
400	3	b	3819.704	1200	0	5370.136	1200	0	**3649.796**	1200	0
400	3	c	**4417.381**	1200	0	4873.568	1200	0	4467.348	1200	0
400	3	d	**3079.263**	1200	0	5366.367	1200	0	3272.192	1200	0
400	3	e	3879.743	1200	0	3350.74	1200.56	0	**2171.671**	1200	0
400	5	a	17654.703	1200	0	**17125.601**	1200	0	18880.054	1200	0
400	5	b	18101.46	1200	0	**16535.578**	1200.17	0	19145.46	1200.01	0
400	5	c	**16014.152**	1200	0	18930.623	1200	0	16051.007	1200	0
400	5	d	16064.241	1200	0	16978.594	1200	0	**15973.311**	1200	0
400	5	e	**15683**	1200	0	16732.491	1200	0	17458.567	1200.03	0
400	10	a	51921.18	1200	0	58975.085	1200	0	**50692.653**	1200	0
400	10	b	55393.75	1200	0	56400.433	1200	0	**52357.794**	1200	0
400	10	c	**46399.087**	1200	0	50257.977	1200	0	53444.267	1200	0
400	10	d	55930.094	1200	0	50245.683	1200	0	**47044.923**	1200	0
400	10	e	50937.271	1200	0	55867.986	1200	0	**46829.706**	1200	0
400	15	a	77242.294	1200	0	**72447.19**	1200	0	84042.066	1200	0
400	15	b	79220.65	1200	0	95112.189	1200	0	**73856.481**	1200	0
400	15	c	**75928.349**	1200	0	79419.922	1200	0	79743.783	1200	0
400	15	d	**80717.721**	1200	0	88920.999	1200	0	85529.174	1200	0
400	15	e	85113.266	1200.01	0	**76694.943**	1200	0	87233.423	1200	0
400	20	a	108518.022	1200	0	**99624.829**	1200	0	107393.94	1200	0
400	20	b	111306.079	1200	0	**94467.118**	1200	0	109474.008	1200	0
400	20	c	**102739.801**	1200	0	107730.912	1200	0	103282.855	1200	0
400	20	d	111623.102	1200.01	0	**101855.768**	1200	0	107568.668	1200	0
400	20	e	**94245.268**	1200	0	112128.101	1200	0	106948.966	1200	0

Table 10: Results for $n = 30,\ 40,\ 50$ and $k = 5$

n	m	$type$	FdSdS			New-1			New-2		
			r	$t[s]$	LB	r	$t[s]$	LB	r	$t[s]$	LB
30	2	a	**891.409**	144.74	891.409	**891.409**	71.22	891.409	1356.036	1200	709.6505
30	2	b	**1351.374**	186.63	1351.374	**1351.374**	277.82	1351.374	**1351.374**	346.56	1351.374
30	2	c	**1082.819**	277.46	1082.819	**1082.819**	527.07	1082.819	**1082.819**	280.85	1082.819
30	2	d	**1289.324**	636.95	1289.324	**1289.324**	1200	1266.9987	**1289.324**	238.51	1289.324
30	2	e	**1289.501**	162.94	1289.501	**1289.501**	143.12	1289.501	**1289.501**	779.5	1289.501
30	3	a	**5736.417**	201.4	5736.417	**5736.417**	189.05	5736.417	**5736.417**	117.58	5736.417
30	3	b	**5767.678**	225.76	5767.678	**5767.678**	78.07	5767.678	**5767.678**	116.17	5767.678
30	3	c	**5471.117**	259.5	5471.117	**5471.117**	544.04	5471.117	**5471.117**	257.99	5471.117
30	3	d	**6839.582**	228.54	6839.582	**6839.582**	94.77	6839.582	**6839.582**	347.14	6839.582
30	3	e	**7991.13**	374.68	7991.13	**7991.13**	1200	7420.8502	**7991.13**	336.15	7991.13
30	5	a	**21117.612**	160.76	21117.612	**21117.612**	135.43	21117.612	**21117.612**	204.85	21117.612
30	5	b	**22230.203**	234.69	22230.203	**22230.203**	158.38	22230.203	**22230.203**	253.05	22230.203
30	5	c	**21958.479**	244.42	21958.479	**21958.479**	514.67	21958.479	**21958.479**	242.08	21958.479
30	5	d	**25263.452**	344.77	25263.452	**25263.452**	459.15	25263.452	**25263.452**	699.71	25263.452
30	5	e	**23714.885**	128.76	23714.885	**23714.885**	919.74	23714.885	**23714.885**	830.77	23714.885
30	10	a	**62020.545**	1200	52981.2699	63265.912	1200	54398.444	**62020.545**	1200	53063.2013
30	10	b	**58107.539**	1200	54733.6968	**58107.539**	1200	57440.6482	60879.343	1200	53141.5527
30	10	c	69300.651	1200	60229.287	69300.651	1200	65270.1335	**67482.481**	1200	66022.3467
30	10	d	**64486.754**	1200	55484.3191	65629.967	1200	46051.2198	65241.808	1200	54259.7457
30	10	e	67712.734	1200	61943.8577	**67155.645**	1200	65884.261	67273.266	1200	64983.8755
30	15	a	88663.941	1200	68702.7311	88663.941	1200	70760.821	**88587.947**	1200	62723.2853
30	15	b	87893.29	1200	71534.2284	**82769.916**	1200	73962.1354	**82769.916**	1200	73017.403
30	15	c	107114.488	1200	73542.4861	101989.062	1200	64603.1732	**97324.35**	1200	70399.933
30	15	d	**91891.539**	1200	66679.4439	99291.123	1200	58130.1057	**91891.539**	1200	68107.4246
30	15	e	93658.303	1200	75880.897	**92547.019**	1200	73386.263	96734.729	1200	72455.0235
30	20	a	**110541.287**	1200	77123.754	112147.446	1200	75125.8162	114675.691	1200	75377.5038
30	20	b	110986.236	1200	82000.831	111288.438	1200	80248.2888	**107852.462**	1200	78085.732
30	20	c	**114367.656**	1200	86783.5278	116507.643	1200	75189.0755	115843.711	1200	85817.054
30	20	d	115617.832	1200	79876.294	**109869.523**	1200	73365.0783	120574.844	1200	75202.2852
30	20	e	**107697.217**	1200	85948.2144	118378.721	1200	79528.9471	112999.481	1200	83966.8107
40	2	a	**1955.671**	1200	0	2501.26	1200	0	2356.018	1200	0
40	2	b	2287.848	1200	0	2218.563	1200	0	**1485.355**	1200	0
40	2	c	**1689.939**	1200	0	2272.693	1200	0	1795.291	1200	0
40	2	d	1880.818	1200	0	**1726.553**	1200	0	1737.118	1200	0
40	2	e	2027.019	1200	0	1951.428	1200	0	**1739.354**	1200	0
40	3	a	**7200.643**	1200	505.9455	7561.931	1200	0	9729.708	1200	0
40	3	b	9522.165	1200	0	**4785.665**	1200	677.9622	6523.842	1200	628.2685
40	3	c	7748.729	1200	0	**5784.625**	1200	363.5423	7475.508	1200	0
40	3	d	**8914.064**	1200	0	12264.235	1200	0	9776.177	1200	0
40	3	e	**8867.984**	1200.01	0	10091.742	1200	0	9130.659	1200	0
40	5	a	13912.173	1200	6303.6713	**12639.186**	1200	8190.2517	22708.596	1200	4694.9314
40	5	b	30523.208	1200	3948.9077	**23620.481**	1200	5141.478	25625.785	1200	5016.8055
40	5	c	**17449.709**	1200	5204.0435	28227.151	1200	0	29095.026	1200	0
40	5	d	**22216.136**	1200	0	26259.634	1200	4371.226	31805.037	1200	0
40	5	e	32661.279	1200	0	**17509.157**	1200	6845.1205	27668.84	1200	5222.4035
40	10	a	57380.345	1200	25468.0226	65799.617	1200	20051.305	**55902.036**	1200	26024.182
40	10	b	**62313.319**	1200	29455.2395	65733.923	1200	19130.8278	66108.425	1200	25566.3596
40	10	c	**65519.813**	1200	28522.981	76153.809	1200	20926.3935	66482.996	1200	28908.095
40	10	d	73449.313	1200	26494.2778	69956.572	1200	20558.2	**68095.21**	1200	21469.9841
40	10	e	**73325.253**	1200	27790.8405	85653.492	1200	16391.8267	75734.973	1200	5554.3751
40	15	a	**91445.311**	1200	28062.9228	92247.377	1200	35513.5239	96820.641	1200	28455.9132
40	15	b	**91194.427**	1200	32333.9617	97493.699	1200	43999.7331	99858.651	1200	38266.7752
40	15	c	**96095.185**	1200	26082.1019	99980.967	1200	40666.8322	100382.48	1200	34805.4563
40	15	d	102848.416	1200	38891.0564	97188.143	1200	43239.1953	**96056.1**	1200	42437.6962
40	15	e	**95522.274**	1200	28343.4529	103539.904	1200	30841.8637	100151.558	1200	37107.1998
40	20	a	115871.303	1200	44861.6269	**104927.708**	1200	40008.4569	116379.918	1200	40021.4459
40	20	b	**115691.688**	1200	51517.6843	117657.495	1200	33429.9988	117125.746	1200	47847.5167
40	20	c	125991.023	1200	40293.5617	**115201.205**	1200	37265.8148	115537.077	1200	37160.4929
40	20	d	126007.885	1200	50191.3277	**120657.587**	1200	37402.0966	121761.116	1200	44705.1709
40	20	e	122951.888	1200	46602.5711	123423.637	1200	35021.2831	**117509.04**	1200	35417.7641
50	2	a	**1741.22**	1200	0	1747.33	1200	0	2972.38	1200	0
50	2	b	1867.984	1200	0	**1362.671**	1200	0	1892.319	1200	0
50	2	c	1807.117	1200	0	**1621.845**	1200	0	2054.746	1200	0
50	2	d	**1714.102**	1200	0	1757.21	1200	0	1841.962	1200	0
50	2	e	2234.212	1200	0	**1494.724**	1200	0	1519.651	1200	0
50	3	a	9668.464	1200	0	8613.419	1200	0	**8455.544**	1200	0
50	3	b	8191.982	1200	0	**7210.421**	1200	0	8124.133	1200	0
50	3	c	6689.257	1200	0	**4486.808**	1200	0	6871.106	1200	0
50	3	d	8530.814	1200	0	**6052.152**	1200	0	9728.342	1200	0
50	3	e	9654.501	1200	0	8810.451	1200	0	**7612.808**	1200	0
50	5	a	**24355.467**	1200	0	29209.868	1200	0	26344.391	1200	0
50	5	b	28035.908	1200	0	**26727.179**	1200	0	27627.385	1200	0
50	5	c	28558.454	1200	0	**27175.992**	1200	0	27500.286	1200	0
50	5	d	**27595.36**	1200	0	29434.197	1200	0	28193.333	1200.01	0
50	5	e	36340.338	1200	0	**34545.832**	1200	0	34790.68	1200	0
50	10	a	71374.498	1200	0	70740.946	1200	0	**64304.349**	1200	0
50	10	b	60213.316	1200	0	73989.378	1200	2864.6293	**59746.907**	1200	0
50	10	c	**69231.261**	1200	0	76880.958	1200	0	78636.852	1200	0
50	10	d	**70599.268**	1200	0	73455.301	1200	0	73925.863	1200	0
50	10	e	76372.484	1200	0	75957.159	1200	5000.5808	**67416.293**	1200	0
50	15	a	94815.653	1200	6423.1616	**93119.943**	1200	11125.0451	102185.759	1200	7903.9
50	15	b	97639.353	1200	13699.0234	98059.611	1200	14754.5015	**88449.312**	1200	6142.5114
50	15	c	**86025.805**	1200	3253.8467	100425.906	1200	2051.3222	97079.536	1200	8988.468
50	15	d	101579.097	1200	4991.5886	99575.381	1200	3940.2882	**99335.356**	1200	8251.1848
50	15	e	**93925.875**	1200	5259.3224	100601.109	1200	5446.5505	98033.635	1200	6398.3928
50	20	a	112340.52	1200	18923.9742	117733.579	1200	7510.3968	**109890.706**	1200	14814.5283
50	20	b	**114887.287**	1200	24246.3898	126801.658	1200	16727.5034	123875.647	1200	21874.7231
50	20	c	125318.261	1200	17929.7699	120628.775	1200	25446.4792	**115056.474**	1200	19656.4877
50	20	d	114225.541	1200	17185.0033	129445.204	1200	10578.0007	**106141.884**	1200	24274.9711
50	20	e	123446.106	1200	19660.9519	132353.573	1200	14355.5408	**108747.409**	1200	17774.9569

Table 11: Results for $n = 100,\ 300,\ 500$ and $k = 5$

n	m	$type$	FdSdS			New-1			New-2		
			r	$t[s]$	LB	r	$t[s]$	LB	r	$t[s]$	LB
100	2	a	1753.057	1200	0	**1610.323**	1200	0	1666.825	1200	0
100	2	b	1456.644	1200	0	1522.957	1200	0	**991.751**	1200	0
100	2	c	1952.464	1200	0	**1166.659**	1200	0	1497.297	1200	0
100	2	d	1399.64	1200	0	**674.466**	1200	0	1417.34	1200	0
100	2	e	1912.683	1200	0	**1420.324**	1200	0	1441.08	1200	0
100	3	a	8182.275	1200	0	7670.809	1200	0	**5078.36**	1200	0
100	3	b	**6590.428**	1200	0	8082.319	1200	0	7444.946	1200	0
100	3	c	7883.189	1200	0	**5501.299**	1200	0	6232.255	1200	0
100	3	d	7289.946	1200	0	**6417.61**	1200	0	8045.481	1200	0
100	3	e	8939.621	1200	0	8671.309	1200	0	**7486.493**	1200	0
100	5	a	27285.2	1200	0	20442.187	1200	0	**19363.929**	1200	0
100	5	b	26200.544	1200	0	**25105.62**	1200	0	26583.683	1200	0
100	5	c	24411.48	1200	0	**19533.008**	1200	0	19533.373	1200	0
100	5	d	29640.458	1200.1	0	27996.896	1200	0	**19394.201**	1200	0
100	5	e	**24048.758**	1200	0	26497.084	1200	0	29406.79	1200	0
100	10	a	66570.337	1200	0	70269.351	1200	0	**61770.82**	1200	0
100	10	b	**68104.66**	1200	0	73614.683	1200	0	71703.817	1200	0
100	10	c	66521.635	1200	0	67196.557	1200	0	**56456.727**	1200	0
100	10	d	70875.06	1200	0	70437.117	1200	0	**52141.445**	1200	0
100	10	e	70296.576	1200	0	**68218.867**	1200	0	71588.324	1200	0
100	15	a	**88954.685**	1200	0	98176.624	1200	0	93744.832	1200	0
100	15	b	99048.668	1200	0	**95061.493**	1200	0	98043.567	1200	0
100	15	c	**95427.824**	1200	0	100237.23	1200	0	106176.679	1200	0
100	15	d	99974.989	1200	0	104002.892	1200	0	**87354.745**	1200	0
100	15	e	**89184.477**	1200	0	98653.814	1200	0	94679.467	1200	0
100	20	a	120874.749	1200	0	**112440.79**	1200	0	126688.638	1200	0
100	20	b	119589.385	1200	0	107034.993	1200	0	**106870.861**	1200	0
100	20	c	**120812.752**	1200	0	132675.766	1200	0	127640.055	1200	0
100	20	d	119458.706	1200	0	124449.226	1200	0	**108736.931**	1200	0
100	20	e	**112999.044**	1200	0	121428.408	1200	0	124749.813	1200	0
300	2	a	1269.853	1200	0	**1108.349**	1200	0	1592.685	1200	0
300	2	b	920.436	1200	0	1131.175	1200	0	**799.833**	1200	0
300	2	c	1260.761	1200	0	**386.077**	1200	0	801.478	1200	0
300	2	d	972.73	1200	0	1298.142	1200	0	**723.738**	1200	0
300	2	e	**668.026**	1200	0	850.776	1200	0	1147.186	1200	0
300	3	a	**5993.818**	1200	0	7410.486	1200	0	6558.442	1200.35	0
300	3	b	**6549.059**	1200	0	6698.177	1200	0	6972.714	1200	0
300	3	c	6994.853	1200	0	5949.919	1200	0	**4023.872**	1200	0
300	3	d	8386.31	1200	0	**5482.5**	1200	0	6642.394	1200	0
300	3	e	**3979.316**	1200	0	8288.479	1200	0	8370.502	1200	0
300	5	a	**21954.511**	1200	0	26128.132	1200	0	24755.728	1200	0
300	5	b	**20780.985**	1200	0	29572.809	1200	0	24808.945	1201.32	0
300	5	c	25941.601	1200	0	**20775.553**	1200	0	21942.158	1200.15	0
300	5	d	22967.776	1200	0	24080.364	1200	0	**22908.327**	1200.47	0
300	5	e	24998.661	1200	0	**20155.004**	1200	0	27958.306	1200	0
300	10	a	66057.981	1200	0	**59813.579**	1200	0	66502.949	1200	0
300	10	b	66810.825	1200	0	69495.4	1200	0	**63371.024**	1200	0
300	10	c	70523.065	1200	0	75845.995	1200	0	**57221.876**	1200.87	0
300	10	d	60075.134	1200	0	70141.9	1200	0	**59930.957**	1200	0
300	10	e	75343.148	1200	0	**57260.235**	1200	0	69771.176	1200	0
300	15	a	99645.928	1200.01	0	96126.429	1200	0	**92866.517**	1200	0
300	15	b	97758.181	1200	0	98371.647	1200	0	**97113.204**	1200	0
300	15	c	107359.567	1200	0	102139.161	1200	0	**101402.98**	1200	0
300	15	d	104598.806	1200	0	**91633.285**	1200	0	97254	1200	0
300	15	e	105197.678	1200.01	0	101276.094	1200	0	**97879.52**	1200	0
300	20	a	127522.964	1200.01	0	122857.354	1200	0	**117801.041**	1200	0
300	20	b	120493.758	1200	0	127395.501	1200	0	**109206.42**	1200.01	0
300	20	c	119401.101	1200	0	**105016.948**	1200	0	107088.288	1200	0
300	20	d	126165.176	1200	0	**112638.084**	1200	0	121665.434	1200	0
300	20	e	119455.629	1200.01	0	122436.981	1200	0	**109607.603**	1200	0
500	2	a	996.511	1200	0	1090.422	1200.01	0	**701.835**	1200	0
500	2	b	**1038.216**	1200	0	2050.7	1202.32	0	1160.437	1202.84	0
500	2	c	865.956	1200	0	**557.239**	1200	0	1560.493	1200	0
500	2	d	1079.131	1200	0	1179.261	1200	0	**712.291**	1200	0
500	2	e	1002.797	1200	0	1020.683	1200.59	0	**950.081**	1200	0
500	3	a	7827.759	1200	0	5145.864	1200	0	**5070.884**	1200	0
500	3	b	**5287.602**	1200	0	7359.502	1200	0	7620.695	1200.06	0
500	3	c	4509.017	1200.01	0	6689.57	1200	0	**4479.932**	1200	0
500	3	d	**5680.724**	1200	0	7710.599	1200.49	0	6747.823	1200.01	0
500	3	e	6140.74	1201.21	0	**6087.025**	1200	0	6180.715	1200	0
500	5	a	**18698.696**	1202.38	0	22005.836	1200	0	23440.767	1200	0
500	5	b	23342.768	1200	0	29854.711	1200	0	**21536.634**	1200	0
500	5	c	**22615.969**	1200.01	0	24186.616	1202.91	0	23747.863	1200	0
500	5	d	21314.06	1200	0	26144.702	1203.06	0	**17991.923**	1200	0
500	5	e	20581.337	1200.01	0	27200.076	1200	0	**18378.399**	1200.18	0
500	10	a	**59106.015**	1200	0	65733.892	1200	0	70732.885	1200	0
500	10	b	**56279.095**	1200	0	68616.841	1200	0	65423	1200	0
500	10	c	**57457.478**	1200.01	0	61785.181	1200	0	66655.859	1200	0
500	10	d	60438.692	1200	0	62857.248	1200	0	**59499.086**	1200	0
500	10	e	61604.972	1200.01	0	**58444.262**	1200	0	61905.447	1200	0
500	15	a	100211.007	1200	0	**95907.281**	1200	0	102815.529	1200	0
500	15	b	101335.124	1200	0	93280.575	1200	0	**86516.564**	1200.01	0
500	15	c	102432.458	1200.01	0	96611.383	1200	0	**92899.735**	1200	0
500	15	d	96792.18	1200.01	0	**86326.166**	1200	0	93887.974	1200	0
500	15	e	**95801.196**	1200.01	0	96646.155	1200	0	98505.624	1200	0
500	20	a	122524.284	1200.01	0	**112480.07**	1200	0	126588.182	1200	0
500	20	b	130697.276	1200.01	0	134423.773	1200	0	**117255.441**	1200	0
500	20	c	**120057.386**	1200.01	0	122187.89	1200	0	126522.997	1200	0
500	20	d	**121454.931**	1200	0	128924.905	1200	0	131500.091	1200	0
500	20	e	**118897.724**	1200.01	0	130340.607	1200	0	122430.038	1200	0

Table 12: Results for $n = 200,\ 400$ and $k = 5$

n	m	$type$	FdSdS			New-1			New-2		
			r	$t[s]$	LB	r	$t[s]$	LB	r	$t[s]$	LB
200	2	a	1412.202	1200	0	1911.611	1200	0	**1278.489**	1200	0
200	2	b	1353.401	1200	0	**764.204**	1200	0	1565.967	1200	0
200	2	c	871.755	1200	0	807.778	1200	0	**509.206**	1200	0
200	2	d	1604.183	1200	0	**900.817**	1200	0	1398.813	1200	0
200	2	e	**1242.338**	1200	0	1511.446	1200	0	1623.048	1200	0
200	3	a	9597.509	1200	0	**5684.372**	1200	0	8569.376	1200	0
200	3	b	6904.299	1200	0	6491.401	1200	0	**6363.146**	1200	0
200	3	c	7908.2	1200	0	6992.444	1200	0	**4513.367**	1200.23	0
200	3	d	5336.164	1200	0	**3864.897**	1200	0	8411.871	1200	0
200	3	e	**7085.501**	1200	0	7538.209	1200	0	9632.596	1200	0
200	5	a	26493.978	1200	0	26159.424	1200	0	**24875.209**	1200	0
200	5	b	27720.335	1200	0	28111.297	1200	0	**25953.985**	1200.45	0
200	5	c	**26493.145**	1200	0	28103.214	1200	0	26501.558	1200.68	0
200	5	d	26888.581	1200	0	**21111.495**	1200	0	26353.255	1201.44	0
200	5	e	28371.453	1200	0	**26290.283**	1200	0	27361.9	1200	0
200	10	a	66950.371	1200	0	63451.947	1200	0	**59866.964**	1200	0
200	10	b	65603.989	1200	0	**53783.551**	1200	0	63882.577	1200	0
200	10	c	64763.98	1200	0	68452.707	1200	0	**55570.486**	1200	0
200	10	d	55088.744	1200	0	56816.671	1200	0	**50821.812**	1200	0
200	10	e	**59897.967**	1200	0	78973.936	1200	0	69190.046	1200	0
200	15	a	**86786.125**	1200	0	96465.499	1200	0	95219.075	1200	0
200	15	b	**88354.845**	1200	0	93378.98	1200	0	94344.372	1200	0
200	15	c	96107.31	1200	0	**89354.846**	1200	0	89458.164	1200	0
200	15	d	99556.499	1200	0	**90741.129**	1200	0	97094.749	1200	0
200	15	e	**91868.938**	1200	0	94810.619	1200	0	94591.504	1200	0
200	20	a	115625.214	1200	0	113983.302	1200	0	**113681.354**	1200	0
200	20	b	**99848.117**	1200	0	120621.71	1200	0	126635.134	1200	0
200	20	c	122541.257	1200.01	0	**119309.357**	1200	0	121661.115	1200	0
200	20	d	113443.96	1200	0	125970.87	1200	0	**110748.65**	1200	0
200	20	e	120164.343	1200.01	0	130752.594	1200	0	**113706.458**	1200	0
400	2	a	**606.266**	1200	0	926.107	1200	0	1271.204	1200	0
400	2	b	1689.228	1200	0	1376.983	1200	0	**763.6**	1200	0
400	2	c	**597.324**	1200	0	644.3	1200	0	1060.598	1201.47	0
400	2	d	**661.702**	1200.01	0	1230.106	1205.3	0	742.708	1200	0
400	2	e	**973.819**	1200	0	1849.184	1203.75	0	1233.92	1200.47	0
400	3	a	**5902.945**	1200.03	0	6498.348	1200	0	6807.487	1200	0
400	3	b	**6657.956**	1200	0	6989.359	1207.99	0	8797.939	1200	0
400	3	c	**5461.741**	1200	0	6831.316	1200	0	6218.346	1200	0
400	3	d	**4985.92**	1200	0	8017.735	1200	0	6634.411	1200	0
400	3	e	6335.469	1200	0	6344.066	1200.18	0	**5497.894**	1200	0
400	5	a	**22495.75**	1200	0	23666.11	1200	0	22658.679	1200	0
400	5	b	26735.781	1200	0	**22984.095**	1200	0	23345.183	1200.03	0
400	5	c	23598.277	1200	0	20824.243	1202.82	0	**20608.989**	1200	0
400	5	d	**19562.2**	1200	0	25029.819	1200	0	25062.085	1200	0
400	5	e	23849.271	1200	0	25147.961	1201.28	0	**20779.797**	1200	0
400	10	a	**66061.217**	1200	0	68787.77	1200	0	68620.685	1200	0
400	10	b	66761.656	1200	0	73528.822	1200	0	**63933.479**	1200	0
400	10	c	67048.645	1200	0	**56127.294**	1200	0	64609.35	1200	0
400	10	d	61890.201	1200	0	65481.532	1200	0	**57656.105**	1200	0
400	10	e	60455.085	1200.01	0	67837.077	1200	0	**60084.905**	1200	0
400	15	a	97094.861	1200	0	97611.946	1200	0	**90109.304**	1200	0
400	15	b	**86938.826**	1200	0	98862.944	1200	0	89774.097	1200	0
400	15	c	**90524.857**	1200.01	0	96609.004	1200	0	92884.902	1200	0
400	15	d	88785.928	1200.01	0	96364.794	1200	0	**87472.092**	1200	0
400	15	e	94193.778	1200	0	**90618.527**	1200	0	95603.89	1200	0
400	20	a	129515.411	1200.02	0	125832.225	1200	0	**96557.069**	1200.01	0
400	20	b	**116896.827**	1200	0	131853.297	1200	0	116984.352	1200	0
400	20	c	123595.013	1200	0	114602.381	1200	0	**113780.698**	1200	0
400	20	d	128648.269	1200	0	118659.243	1200	0	**116658.25**	1200	0
400	20	e	133451.752	1200	0	**109262.484**	1200	0	112338.979	1200	0

Table 13: Results for $n = 30,\ 40,\ 50$ and $k = 10$

n	m	$type$	FdSdS			New-1			New-2		
			r	$t[s]$	LB	r	$t[s]$	LB	r	$t[s]$	LB
30	2	a	16743.296	1200	8009.657	18077.54	1200	3517.3726	**15805.822**	549.61	15805.822
30	2	b	**14338.294**	121.34	14338.294	**14338.294**	76.71	14338.294	**14338.294**	72.1	14338.294
30	2	c	**10114.876**	217.73	10114.876	**10114.876**	390.98	10114.876	**10114.876**	103.7	10114.876
30	2	d	**15460.745**	357.69	15460.745	15471.083	1200	4554.298	**15460.745**	382.68	15460.745
30	2	e	**25472.952**	1200	15812.7384	**25472.952**	1200	9222.7819	**25472.952**	1200	20477.4021
30	3	a	**24862.404**	895.03	24862.404	**24862.404**	1200	21175.3459	**24862.404**	380.78	24862.404
30	3	b	**27031.183**	313.15	27031.183	**27031.183**	1200	17328.3301	**27031.183**	317.76	27031.183
30	3	c	**28024.872**	1200	23662.1754	28328.679	1200	12700.0981	**28024.872**	634.57	28024.872
30	3	d	**28713.894**	302.73	28713.894	**28713.894**	141.08	28713.894	**28713.894**	475.77	28713.894
30	3	e	35722.587	1200	25799.6708	**34320.271**	1200	18011.2938	**34320.271**	1200	32699.374
30	5	a	52549.355	1200	36067.7907	**50024.911**	1200	36612.365	**50024.911**	1200	43720.957
30	5	b	53662.784	1200	40654.6242	52686.584	1200	23255.2037	**51908.359**	829.52	51908.359
30	5	c	**47240.179**	967.97	47240.179	47708.141	1200	33851.5651	47708.141	1200	36139.1661
30	5	d	**52348.676**	901.23	52348.676	**52348.676**	1200	33162.9667	**52348.676**	774.26	52348.676
30	5	e	63550.074	1200	41897.6053	62079.814	1200	35739.4652	**61496.275**	1200	38449.5057
30	10	a	**79838.68**	1200	59188.7643	85865.607	1200	57169.2918	82943.184	1200	60565.4428
30	10	b	89847.954	1200	56073.5906	**78854.718**	1200	51789.3951	**78854.718**	1164.2	78854.718
30	10	c	105431.087	1200	62350.5332	**101535.501**	1200	56554.2015	105081.051	1200	56600.2839
30	10	d	**92142.036**	1200	51361.6976	93935.54	1200	54813.7743	96606.441	1200	50945.5829
30	10	e	100755.765	1200	56436.5126	97249.64	1200	53218.99	**90903.216**	1200	57134.3367
30	15	a	113115.879	1200	63871.9971	116765.555	1200	65732.9381	**109074.393**	1200	69466.6572
30	15	b	114660.351	1200	62007.852	108549.274	1200	72847.9909	**104998.947**	1200	72967.2617
30	15	c	121774.618	1200	62832.9864	126831.889	1200	70059.2032	**121202.645**	1200	64921.0754
30	15	d	117118.916	1200	63544.0438	**112270.108**	1200	62926.8125	113690.304	1200	65340.8659
30	15	e	**112080.893**	1200	72384.6616	116393.885	1200	68630.3801	112998.546	1200	69903.5466
30	20	a	127085.015	1200	57748.7159	**124031.987**	1200	76728.7685	131281.205	1200	71068.9409
30	20	b	134270.101	1200.01	64326.7853	127927.928	1200	72837.4879	**124363.794**	1200	72291.9948
30	20	c	146075.447	1200	60195.4972	**135095.637**	1200	71022.4839	139585.583	1200	63786.5985
30	20	d	134737.779	1200	64889.0342	**122567.331**	1200	71054.2665	124575.507	1200	73143.6478
30	20	e	131065.169	1200.03	71248.8252	129430.554	1200	81036.1933	**123481.164**	1200	76527.927
40	2	a	**7476.126**	1200.02	903.5897	9894.227	1200	571.1224	7650.608	1200	370.2398
40	2	b	9589.554	1200	976.401	5674.364	1200	2315.2134	**4030.423**	1200	2227.006
40	2	c	8959.665	1200.11	443.6914	**2577.04**	1200	2489.0659	4510.013	1200	1386.224
40	2	d	7661.001	1200	744.8182	**4713.077**	1200	0	7760.991	1200	0
40	2	e	**7719.627**	1200	34.3983	10420.704	1200	266.1118	10796.679	1200	57.9289
40	3	a	24322.689	1200	2432.065	**22709.777**	1200	5117.423	24609.248	1200	3873.9721
40	3	b	19976.764	1200	5835.6451	17089.67	1200	6531.9163	**12355.128**	1200	8519.676
40	3	c	**15493.807**	1200	4603.274	23797.645	1200	1289.1379	21658.817	1200	0
40	3	d	**20544.016**	1200	4428.8921	21936.823	1200	2956.6179	23742.051	1200	2933.317
40	3	e	25924.232	1200	0	22476.587	1200	2655.2806	**18158.855**	1200	5183.5709
40	5	a	58935.906	1200.06	12268.8218	62009.528	1200	12920.1796	**53098.727**	1200	12236.3663
40	5	b	**51491.482**	1200	15514.4251	51604.47	1200	14256.066	51657.648	1200	14880.4011
40	5	c	54856.708	1200	13943.89	**46513.356**	1200	18769.1397	47867.277	1200	2575.8033
40	5	d	58798.244	1200	8597.6264	55296.201	1200	14538.5923	**45451.383**	1200	11593.2226
40	5	e	61857.777	1200	6118.2802	60682.581	1200	11286.7123	**47697.53**	1200	8395.5307
40	10	a	101779.746	1200	20515.7538	89912.882	1200	24220.2648	**85086.796**	1200	36485.906
40	10	b	**84962.05**	1200	34101.6898	96782.462	1200	37274.7188	86479.276	1200	41749.9974
40	10	c	101107.733	1200	45523.2973	101384.555	1200	26914.2367	**96829.735**	1200	45638.5505
40	10	d	95703.676	1200	23896.1787	104897.802	1200	27821.9357	**90327.978**	1200	34235.3444
40	10	e	**92638.59**	1200	35885.0325	103000.638	1200	24422.0849	100924.456	1200	26446.3186
40	15	a	124284.666	1200	38727.3793	129522	1200	36990.4781	**112175.426**	1200	47019.7874
40	15	b	127509.473	1200	36629.0669	**118349.835**	1200	49363.9099	124270.311	1200	47492.5137
40	15	c	137420.673	1200	47893.5418	124248.453	1200	47984.3166	**122053.188**	1200	47275.483
40	15	d	125284.992	1200	29780.2086	**111486.96**	1200	35823.2078	113256.68	1200	41592.7602
40	15	e	**115833.882**	1200	37339.4732	122114.754	1200	51409.0482	119490.895	1200	50133.7485
40	20	a	144376.101	1200	35206.732	**137142.056**	1200	54806.9489	140111.38	1200	48192.648
40	20	b	137863.526	1200.01	47452.5038	144015.549	1200	56178.5795	**134880.367**	1200	59080.4256
40	20	c	143909.104	1200	36989.1516	155439.223	1200	54804.1994	**140516.345**	1200	46726.7029
40	20	d	153065.349	1200	37368.6684	140064.684	1200	57481.0284	**138410.79**	1200	44170.0835
40	20	e	**137668.811**	1200	34869.1815	146782.531	1200.01	46592.765	146217.841	1200	47126.7457
50	2	a	7825.187	1200	0	**7735.505**	1200	0	9213.474	1200	0
50	2	b	**7094.15**	1200.14	0	8411.793	1200	0	8904.598	1200	0
50	2	c	8376.015	1200	0	9182.242	1200	0	**7024.686**	1200	0
50	2	d	**7462.355**	1200	0	10039.313	1200	0	7613.919	1200	0
50	2	e	8215.921	1200	0	**5590.218**	1200	0	5622.727	1200	0
50	3	a	**19308.283**	1200	0	20361.463	1200	1265.7321	21825.874	1200	0
50	3	b	26340.221	1200.04	0	**17070.858**	1200	2113.4536	25583.338	1200	0
50	3	c	20410.117	1200	0	25426.531	1200	0	**19734.347**	1200	0
50	3	d	22486.779	1200	0	22334.452	1200	0	**18537.054**	1200	643.1786
50	3	e	24970.673	1200	0	24738.63	1200	0	**21150.799**	1200	0
50	5	a	54108.859	1200	0	57869.862	1200	1597.0327	**51325.464**	1200	0
50	5	b	53863.404	1200	0	**50301.861**	1200	0	54875.399	1200	0
50	5	c	58865.321	1200.05	0	43269.366	1200	0	**42483.444**	1200	4118.3953
50	5	d	54780.905	1200	0	**54535.933**	1200	0	55634.6	1200	3662.5137
50	5	e	59986.841	1200	7163.1907	58468.653	1200	6738.7754	**56807.046**	1200	0
50	10	a	**92018.735**	1200	7849.1128	99068.519	1200	5943.4485	101400.02	1200	3943.6014
50	10	b	99868.286	1200	14721.4268	**92610.275**	1200	8488.5708	110606.209	1200	14193.0378
50	10	c	104348.355	1200	2965.2275	109740.16	1200	4718.3331	**101447.605**	1200	5491.5898
50	10	d	**99954.748**	1200	316.135	103306.216	1200	6737.8351	102915.169	1200	3411.2022
50	10	e	**104689.79**	1200	0	107645.87	1200	9477.6834	107174.111	1200	8674.0114
50	15	a	**120465.811**	1200	12792.0485	123559.569	1200	18967.0653	123756.312	1200	18485.5736
50	15	b	**124076.118**	1200	20793.0244	132033.53	1200	20793.8161	126429.281	1200	24565.1666
50	15	c	136527.715	1200	10053.5942	130140.899	1200	22207.1078	**119494.77**	1200	31162.3267
50	15	d	125795.835	1200	18681.8556	**119768.239**	1200	6279.2769	130240.951	1200	17708.1491
50	15	e	131022.359	1200	5297.0884	131034.401	1200	11898.8729	**118708.617**	1200	27337.5399
50	20	a	142767.696	1200.01	24010.3658	**138374.841**	1200	38581.9783	144054.429	1200	30417.1866
50	20	b	**143870.22**	1200.01	23014.5446	149746.288	1200	44664.7346	154426.38	1200	38588.4554
50	20	c	162304.788	1200.01	23101.0829	153346.847	1200	33608.1096	**151491.628**	1200	25059.2081
50	20	d	150571.166	1200.01	24658.2572	151358.344	1200	34021.0563	**149409.19**	1200	29955.7514
50	20	e	154697.18	1200.01	17442.7523	**136581.384**	1200	40711.3109	142353.148	1200	39975.2415

Table 14: Results for $n = 100,\ 300,\ 500$ and $k = 10$

n	m	$type$	FdSdS			New-1			New-2		
			r	$t[s]$	LB	r	$t[s]$	LB	r	$t[s]$	LB
100	2	a	10217.795	1200	0	**6125.913**	1200	0	6483.596	1200	0
100	2	b	8456.231	1200	0	**5911.812**	1200	0	6095.066	1200	0
100	2	c	7716.136	1200	0	**7389.585**	1200	0	8450.859	1200	0
100	2	d	6308.793	1200.04	0	**4693.922**	1200	0	4982.22	1200	0
100	2	e	7043.447	1200.04	0	5947.918	1200	0	**5551.36**	1200	0
100	3	a	**19020.57**	1200	0	22863.198	1200	0	23575.178	1200	0
100	3	b	22905.656	1200	0	24705.26	1200	0	**21585.425**	1200	0
100	3	c	22996.983	1200	0	**20209.978**	1200	0	27354.103	1200	0
100	3	d	23225.952	1200	0	21477.192	1200	0	**21204.957**	1200	0
100	3	e	22516.75	1200.1	0	**19547.033**	1200	0	20652.286	1200	0
100	5	a	**46686.552**	1200	0	49686.535	1200	0	49439.904	1200	0
100	5	b	57461.065	1200	0	51629.385	1200	0	**45667.151**	1200	0
100	5	c	**48919.067**	1200	0	48957.612	1200	0	54558.075	1200	0
100	5	d	**44361.398**	1200	0	57475.003	1200	0	54321.64	1200	0
100	5	e	58217.601	1200	0	**54222.128**	1200	0	58613.784	1200	0
100	10	a	98002.595	1200.01	0	**92506.584**	1200	0	94279.675	1200	0
100	10	b	99689.039	1200	0	**90430.851**	1200	0	98038.033	1200	0
100	10	c	102448.603	1200	0	**100901.864**	1200	0	102966.706	1200	0
100	10	d	97589.549	1200	0	92482.534	1200	0	**90686.963**	1200	0
100	10	e	106363.288	1200	0	109315.746	1200	0	**94539.15**	1200	0
100	15	a	136182.168	1200.01	0	127212.894	1200	0	**122351.041**	1200	0
100	15	b	**118799.555**	1200.01	0	132587.917	1200	0	128913.719	1200	0
100	15	c	139184.896	1200.02	0	**126665.839**	1200	0	132615.718	1200	0
100	15	d	129047.329	1200.01	0	**126816.572**	1200	0	127458.892	1200	0
100	15	e	139011.901	1200.01	0	128616.964	1200	0	**126846.425**	1200	0
100	20	a	**145449.749**	1200.01	0	147991.574	1200	0	147587.071	1200	0
100	20	b	163083.845	1200.01	0	155306.879	1200	0	**144902.832**	1200	0
100	20	c	**153705.967**	1200	0	157224.833	1200	0	155083.79	1200	0
100	20	d	**156217.662**	1200	0	156381.53	1200	0	160926.581	1200	0
100	20	e	158437.855	1200.01	0	165589.402	1200	0	**148801.022**	1200	0
300	2	a	**4961.886**	1200.16	0	7045.071	1200.03	0	5080.848	1200.01	0
300	2	b	6720.246	1200.12	0	7643.984	1200	0	**5920.86**	1200.33	0
300	2	c	5580.573	1200.19	0	**5046.808**	1200	0	6746.552	1200.26	0
300	2	d	6266.951	1200	0	**5404.059**	1200.01	0	5892.042	1200.01	0
300	2	e	6583.48	1200	0	5630.779	1200	0	**5128.287**	1200.11	0
300	3	a	19819.72	1200	0	**19770.11**	1200	0	20364.328	1200.08	0
300	3	b	**17901.199**	1200	0	19507.819	1200.34	0	23140.629	1200	0
300	3	c	**15566.036**	1200	0	16948.493	1200	0	18568.673	1200	0
300	3	d	20014.683	1200	0	19511.571	1200	0	**17615.407**	1200	0
300	3	e	17229.803	1200	0	21960.354	1200.34	0	**17145.253**	1200.01	0
300	5	a	50746.148	1200	0	**41587.657**	1200	0	45931.483	1200.01	0
300	5	b	51272.991	1200.01	0	52576.543	1200	0	**44077.404**	1200	0
300	5	c	51812.119	1200	0	**44070.427**	1200	0	44251.874	1200	0
300	5	d	47816.913	1200	0	**40786.658**	1200	0	44539.812	1200	0
300	5	e	51244.875	1200	0	53099.945	1200	0	**40937.038**	1200	0
300	10	a	**92753.12**	1200	0	95175.908	1200	0	103841.455	1200	0
300	10	b	101321.285	1200	0	112526.612	1200	0	**100564.042**	1200	0
300	10	c	**90432.596**	1200.01	0	99830.399	1200	0	98877.948	1200.01	0
300	10	d	**101693.301**	1200	0	102166.158	1200	0	105908.481	1200	0
300	10	e	108755.769	1200.01	0	101730.871	1200	0	**96517.769**	1200	0
300	15	a	136577.262	1200.02	0	134715.495	1200	0	**134521.208**	1200	0
300	15	b	140153.794	1200.04	0	**119630.658**	1200	0	143218.446	1200.01	0
300	15	c	141029.966	1200.03	0	**122783.591**	1200	0	130198.209	1200	0
300	15	d	137507.656	1200.02	0	**124467.37**	1200	0	130680.152	1200.01	0
300	15	e	133494.025	1200.02	0	132849.048	1200	0	**130663.166**	1200.01	0
300	20	a	170145.411	1200.02	0	166724.492	1200.01	0	**144255.041**	1200.01	0
300	20	b	154868.717	1200.02	0	**148608.504**	1200	0	149490.952	1200.01	0
300	20	c	181708.414	1200.04	0	166221.345	1201.28	0	**150740.768**	1200.01	0
300	20	d	165575.474	1200.03	0	157935.05	1200	0	**153206.214**	1200	0
300	20	e	179410.19	1200.03	0	**156419.602**	1200	0	158598.026	1200.01	0
500	2	a	5700.803	1202.37	0	7454.449	1200	0	**5636.472**	1200	0
500	2	b	**4817.564**	1200.08	0	5015.021	1200	0	5912.672	1200.01	0
500	2	c	**3556.699**	1200.74	0	6973.92	1200.01	0	4297.899	1200	0
500	2	d	3365.703	1200.01	0	5136.59	1200	0	**2773.682**	1200.16	0
500	2	e	**3364.124**	1200.19	0	5131.719	1200	0	5409.395	1200.01	0
500	3	a	**13118.375**	1200.01	0	18577.636	1200.71	0	15930.6	1200	0
500	3	b	13553.494	1200.01	0	17580.499	1200.27	0	**13360.084**	1200	0
500	3	c	17508.299	1200	0	18642.636	1200.8	0	**16890.376**	1200	0
500	3	d	16894.249	1200.01	0	16654.955	1200.01	0	**15933.689**	1200	0
500	3	e	19021.005	1200.01	0	21054.564	1200.15	0	**13070.063**	1200.01	0
500	5	a	45053.551	1200.02	0	48469.194	1200	0	**42017.179**	1200.01	0
500	5	b	43269.234	1200	0	56301.778	1201.27	0	**40218.557**	1200	0
500	5	c	46626.338	1200.01	0	48506.804	1201.45	0	**45673.377**	1200	0
500	5	d	43697.288	1200.01	0	**40238.477**	1200	0	50252.031	1200	0
500	5	e	**43658.067**	1200.01	0	55367.861	1200	0	45923.22	1200	0
500	10	a	**96092.4**	1200	0	99433.709	1200	0	98007.211	1200	0
500	10	b	104203.902	1200.01	0	111658.307	1200.01	0	**99843.97**	1200	0
500	10	c	113905.74	1200.02	0	**95518.692**	1200	0	100314.285	1200	0
500	10	d	103066.875	1200.01	0	97032.298	1200	0	**96176.947**	1200	0
500	10	e	106866.732	1200	0	100328.37	1200	0	**99517.884**	1200	0
500	15	a	**112134.423**	1200.03	0	135077.625	1200	0	121912.28	1200	0
500	15	b	141722.814	1200.01	0	**125059.216**	1200	0	126145.441	1200	0
500	15	c	132680.413	1200.02	0	131435.852	1200	0	**112433.881**	1200.01	0
500	15	d	**128963.681**	1200.01	0	137551.809	1200	0	135161.335	1200.01	0
500	15	e	146526.217	1200.01	0	127598.749	1200.01	0	**127221.26**	1200.01	0
500	20	a	179791.117	1200.07	0	146299.895	1200	0	**139766**	1200.01	0
500	20	b	173529.376	1200.05	0	164340.008	1200	0	**150091.091**	1200.01	0
500	20	c	159180.16	1200.02	0	163329.733	1200.01	0	**151908.843**	1200.01	0
500	20	d	168615.743	1200.02	0	**151826.536**	1200.01	0	158222.647	1200.01	0
500	20	e	161578.316	1200.01	0	158428.051	1200.01	0	**142127.447**	1200.01	0

Table 15: Results for $n = 200,\ 400$ and $k = 10$

n	m	$type$	FdSdS			New-1			New-2		
			r	$t[s]$	LB	r	$t[s]$	LB	r	$t[s]$	LB
200	2	a	8342.642	1200	0	8436.134	1200	0	6572.731	1200	0
200	2	b	**5471.875**	1200	0	7157.339	1200	0	6557.579	1200.65	0
200	2	c	6337.022	1200	0	6125.065	1200.66	0	**4419.093**	1200	0
200	2	d	6747.101	1200.02	0	**5210.191**	1200	0	6192.754	1203.55	0
200	2	e	7978.879	1200	0	**6717.912**	1200	0	7277.181	1200	0
200	3	a	20195.653	1200	0	21632.644	1200.01	0	**18785.063**	1200	0
200	3	b	20727.1	1200	0	**18577.312**	1206.54	0	21172.585	1200	0
200	3	c	17838.778	1200	0	17965.479	1200	0	**17300.364**	1200	0
200	3	d	25218.702	1200	0	24002.889	1200.65	0	**22439.172**	1200.47	0
200	3	e	19647.917	1200	0	26907.182	1200	0	**18804.464**	1200	0
200	5	a	52051.462	1200	0	**49153.349**	1200	0	53845.513	1200	0
200	5	b	**40382.121**	1200	0	56986.484	1200	0	47049.083	1200	0
200	5	c	49070.363	1200	0	42061.441	1200	0	**39738.35**	1200	0
200	5	d	48247.48	1200.01	0	50710.374	1200	0	**35617.311**	1200	0
200	5	e	**49375.07**	1200	0	53009.261	1200	0	52554.684	1200	0
200	10	a	98850.362	1200	0	104568.066	1200	0	**94064.272**	1200	0
200	10	b	104817.821	1200	0	102068.514	1200	0	**98862.078**	1200	0
200	10	c	111578.125	1200.01	0	**96985.523**	1200	0	105269.399	1200	0
200	10	d	102070.958	1200.01	0	103294.461	1200	0	**95202.969**	1200.1	0
200	10	e	106729.052	1200.01	0	100941.301	1200	0	**97975.182**	1200	0
200	15	a	134902.226	1200.01	0	133870.624	1200	0	**124774.26**	1200	0
200	15	b	139867.38	1200.02	0	129459.15	1200	0	**126795.768**	1200	0
200	15	c	136512.413	1200.03	0	**129883.936**	1200	0	139779.641	1200.01	0
200	15	d	**121978.338**	1200.02	0	128202.107	1200	0	125946.505	1200	0
200	15	e	128092.551	1200.02	0	125852.615	1200	0	**125193.001**	1200	0
200	20	a	**136374.85**	1200.01	0	150193.585	1200	0	165447.16	1200.01	0
200	20	b	164710.982	1200.07	0	**143526.653**	1200.01	0	153157.476	1200	0
200	20	c	168947.604	1200.02	0	**148388.832**	1200	0	151106.981	1200.01	0
200	20	d	170014.179	1200	0	**148402.904**	1200	0	158813.94	1200.01	0
200	20	e	163138.695	1200.03	0	161765.991	1200.01	0	**150987.338**	1200.01	0
400	2	a	4972.049	1200.28	0	5428.264	1204.72	0	**3213.235**	1200.36	0
400	2	b	5764.149	1200	0	5007.432	1200	0	**4850.378**	1201.32	0
400	2	c	**3358.247**	1200.01	0	3629.53	1202.38	0	4890.896	1202.83	0
400	2	d	6387.233	1200	0	5390.807	1200.96	0	**5039.965**	1200.84	0
400	2	e	4954.328	1200	0	5392.409	1200	0	**3524.055**	1200	0
400	3	a	18948.797	1200	0	**15490.285**	1200	0	22149.104	1200	0
400	3	b	14149.297	1200	0	**13691.48**	1200	0	24129.418	1200.17	0
400	3	c	**16405.439**	1200.01	0	21245.335	1200	0	17192.915	1200.01	0
400	3	d	19674.39	1200.01	0	**19150.36**	1200	0	19173.98	1200.24	0
400	3	e	21467.393	1200	0	**20764.483**	1200.31	0	21093.346	1200.66	0
400	5	a	44988.607	1200	0	**43251.101**	1200	0	49252.943	1200	0
400	5	b	**38070.534**	1200	0	40150.744	1200	0	51622.195	1200	0
400	5	c	40891.673	1200	0	**40703.055**	1200	0	41452.624	1200.01	0
400	5	d	47950.876	1200	0	**33752.212**	1200	0	43161.918	1200	0
400	5	e	**40209.675**	1200.01	0	50770.962	1200	0	46062.431	1200	0
400	10	a	106521.979	1200	0	**92386.923**	1200	0	107565.538	1200	0
400	10	b	98511.27	1200	0	**81756.98**	1200	0	87803.613	1200.01	0
400	10	c	102267.964	1200.01	0	97955.326	1200	0	**95241.887**	1200.01	0
400	10	d	93813.294	1200.02	0	**88529.254**	1200	0	93176.117	1200	0
400	10	e	115606.38	1200.01	0	**90820.488**	1200	0	102498.846	1200.01	0
400	15	a	141484.454	1200.03	0	144390.266	1200	0	**117154.978**	1200.01	0
400	15	b	142438.636	1200.02	0	**118576.866**	1200	0	130848.939	1200	0
400	15	c	137807.196	1200.02	0	**120212.218**	1200	0	131572.004	1200	0
400	15	d	141351.358	1200.02	0	**130910.476**	1200	0	136643.873	1200.01	0
400	15	e	135148.223	1200.02	0	127060.469	1200	0	**121458.613**	1200.01	0
400	20	a	165711.384	1200.02	0	**161930.992**	1200	0	164831.954	1200.01	0
400	20	b	165799.641	1200.04	0	**138342.217**	1200.01	0	147932.315	1200.01	0
400	20	c	**146783.606**	1200.07	0	159064.797	1200.01	0	148475.2	1200.01	0
400	20	d	160168.297	1200.11	0	170825.679	1200	0	**156165.317**	1200.01	0
400	20	e	158539.795	1200.02	0	166588.202	1200	0	**148615.861**	1200.01	0

Table 16: Results for $n = 30,\ 40,\ 50$ and $k = 20$

n	m	$type$	FdSdS			New-1			New-2		
			r	$t[s]$	LB	r	$t[s]$	LB	r	$t[s]$	LB
30	2	a	**62110.281**	201.03	62110.281	**62110.281**	1200	43748.9164	**62110.281**	1196.86	62110.281
30	2	b	**63290.646**	179.58	63290.646	**63290.646**	182.87	63290.646	**63290.646**	19.59	63290.646
30	2	c	**54965.246**	1200	53101.959	**54965.246**	287.51	54965.246	**54965.246**	159.52	54965.246
30	2	d	**65185.379**	128.75	65185.379	**65185.379**	86.05	65185.379	**65185.379**	21.25	65185.379
30	2	e	**67072.783**	74.06	67072.783	**67072.783**	8.93	67072.783	**67072.783**	5.14	67072.783
30	3	a	**76464.457**	1200	60446.4724	**76464.457**	1200	66412.1216	**76464.457**	1200	62791.2844
30	3	b	**82012.052**	870.96	82012.052	**82012.052**	1200	72787.6185	**82012.052**	684.74	82012.052
30	3	c	**72673.436**	988.25	72673.436	**72673.436**	1200	45626.8186	**72673.436**	85.11	72673.436
30	3	d	**79765.282**	1200	67571.2229	**79765.282**	1200	41372.3668	**79765.282**	103.8	79765.282
30	3	e	**79943.575**	610.12	79943.575	**79943.575**	126.09	79943.575	**79943.575**	151.46	79943.575
30	5	a	**90730.863**	1200	84536.1549	**90730.863**	1200	79955.0502	**90730.863**	657.3	90730.863
30	5	b	**102068.245**	1200	80883.2496	**102068.245**	1200	82619.6479	**102068.245**	1200	84842.212
30	5	c	99523.402	1200	70426.6584	**95235.01**	1200	80118.3976	**95235.01**	1200	88440.4772
30	5	d	108556.435	1200	72535.0018	**102185.82**	1200	79875.7224	**102185.82**	1200	85948.2279
30	5	e	**108252.316**	1200	78936.8855	**108252.316**	1200	99928.5088	**108252.316**	292.46	108252.316
30	10	a	118037.213	1200.01	70236.3495	**115493.647**	1200	82577.3716	117217.197	1200	74237.732
30	10	b	130884.393	1200.01	86823.0079	**128163.277**	1200	93985.631	129726.102	1200	93427.9017
30	10	c	130407.051	1200.01	75890.1428	127712.193	1200	102079.1222	**125698.173**	1200	92132.63
30	10	d	**127270.932**	1200.01	81872.442	128408.98	1200	93107.516	128875.922	1200	89514.934
30	10	e	**124219.432**	1200	85735.0215	**124219.432**	1200	99624.6305	**124219.432**	1200	91485.0145
30	15	a	133560.303	1200.01	68500.7102	134655.695	1200	76891.0825	**133268.037**	1200	80006.7992
30	15	b	137839.065	1200	81919.8107	141949.002	1200	96960.9158	**134616.04**	1200	96539.7417
30	15	c	138685.673	1200.01	78268.1528	134333.76	1200	82567.439	**132465.705**	1200	84175.2223
30	15	d	138215.808	1200.01	72978.3546	135879.253	1200	79678.12	**133565.717**	1200	92111.3242
30	15	e	140278.83	1200.01	90825.4867	**135950.694**	1200	107427.741	136005.333	1200	97067.0968
30	20	a	145778.996	1200.02	68083.6763	143290.233	1200	77458.3194	**140793.801**	1200	77060.1359
30	20	b	147057.015	1200.01	89381.0109	147689.99	1200	104339.3432	**144450.92**	1200	104005.7428
30	20	c	150623.014	1200.04	77131.6405	142746.854	1200	96390.4736	**141697.364**	1200	96757.4365
30	20	d	145083.796	1200.01	72463.6894	141821.889	1200	91857.7884	**137670.065**	1200	82125.1734
30	20	e	154006.34	1200.01	83705.0855	144835.187	1200	92278.4776	**141934.512**	1200	91823.0705
40	2	a	**35259.236**	1200	15348.3129	**35259.236**	1200	7732.3109	**35259.236**	1200	18232.1719
40	2	b	**27900.963**	1200	24072.3368	**27900.963**	1200	27251.84	**27900.963**	596.02	27900.963
40	2	c	36452.606	1200	17750.5882	**35901.091**	1200	4985.0396	**35901.091**	1200	14377.7715
40	2	d	30019.654	1200	22399.197	30019.654	1200	6118.7245	**29810.145**	1200	25910.4891
40	2	e	25457.357	1200	15714.959	**24902.634**	1200	19625.5116	**24902.634**	519.94	24902.634
40	3	a	**53476.87**	1200	30234.3031	**53476.87**	1200	27468.1877	**53476.87**	1200	29242.6573
40	3	b	52373.906	1200	31021.0492	52373.906	1200	27699.4828	**52009.002**	1200	30978.4565
40	3	c	52333.626	1200.01	29954.416	51366.045	1200	35382.6288	**51322.951**	1200	37581.2979
40	3	d	51166.999	1200	26839.0619	**50370.795**	1200	29580.5534	50757.037	1200	26603.811
40	3	e	**54956.724**	1200.01	29475.5672	**54956.724**	1200	33820.506	**54956.724**	1200	26045.0585
40	5	a	71618.785	1200.01	39177.2568	73049.681	1200	34739.7051	**70895.778**	1200	45810.6773
40	5	b	77126.647	1200.01	36290.3379	74430.809	1200	40541.5152	**72079.643**	1200	55079.7355
40	5	c	82364.14	1200	39897.0494	81791.328	1200	41357.291	**75750.252**	1200	49673.5283
40	5	d	80585.323	1200	45973.438	78961.353	1200	46475.5817	**78405.812**	1200	49914.3276
40	5	e	86765.54	1200.01	36115.8354	**73441.218**	1200	40701.8148	74442.559	1200	44640.0847
40	10	a	110928.686	1200.01	41941.462	107276.673	1200	49363.135	**98692.56**	1200	64431.7372
40	10	b	117338.066	1200.01	60551.9961	106199.098	1200	59285.1226	**99276.092**	1200	65920.6089
40	10	c	122458.033	1200.01	47822.8832	120429.041	1200	57946.6647	**113993.7**	1200	48491.3104
40	10	d	119683.455	1200.01	53286.7278	111468.367	1200	60837.3646	**108571.892**	1200	60617.07
40	10	e	125856.314	1200.01	59393.729	**113765.938**	1200	64433.5163	114981.315	1200	63755.946
40	15	a	131773.905	1200.01	46041.9617	120308.511	1200	64776.4997	**118616.149**	1200	68063.7176
40	15	b	134219.252	1200.02	56047.19	119651.536	1200	59765.74	**115073.484**	1200	64621.7112
40	15	c	140265.136	1200.01	44557.838	130414.542	1200	61762.1593	**129096.208**	1200	66835.0345
40	15	d	146380.564	1200.02	41215.3877	131133.606	1200	60178.8307	**122658.985**	1200	76030.2443
40	15	e	140628.768	1200.02	59837.4527	**124133.105**	1200	67310.9443	128210.022	1200	70540.3421
40	20	a	145588.954	1200.03	45381.1279	131032.493	1200	58492.4796	**129228.716**	1200	63152.74
40	20	b	150451.509	1200.03	55255.6183	138119.18	1200	67368.2722	**132814.682**	1200	73736.288
40	20	c	151663.468	1200	46488.6247	**137150.486**	1200	69603.3252	137332.117	1200	70124.4045
40	20	d	149824.028	1200.03	46638.7442	**136156.123**	1200	62562.5494	137847.661	1200	59878.5047
40	20	e	145202.595	1200.01	56629.6143	134971.289	1200	79182.6207	**128210.022**	1200	71185.3062
50	2	a	19887.782	1200	0	**15754.624**	1200.01	0	16728.958	1200	0
50	2	b	21976.891	1200	0	**20910.568**	1200	912.9805	24097.44	1200	876.1139
50	2	c	26225.25	1200.02	0	**25603.137**	1200	2671.2556	25993.239	1200	0
50	2	d	**22257.064**	1200.1	0	25704.163	1200	5755.1157	**22257.064**	1200	837.7672
50	2	e	33584.421	1200	0	27853.679	1200	782.0849	**27363.463**	1200	0
50	3	a	44239.826	1200	32.081	**38043.698**	1200	0	39665.196	1200	7825.4595
50	3	b	44754.343	1200	0	**33498.948**	1200	6118.3287	44449.317	1200	5317.0184
50	3	c	43398.314	1200	0	**37625.522**	1200	3150.7003	**37625.522**	1200	4135.379
50	3	d	42897.824	1200	0	**41026.155**	1200	0	41395.325	1200	185.0204
50	3	e	**44240.373**	1200	0	48846.45	1200	6835.5449	48556.236	1200	364.334
50	5	a	78475.738	1200.02	21456.4425	73245.33	1200.02	24682.1791	**72237.17**	1200	24329.8635
50	5	b	82478.916	1200	20111.8367	**72514.065**	1200	20498.1474	74205.39	1200	22746.5568
50	5	c	82264.173	1200.01	18979.6141	72267.307	1200	18260.3769	**70436.881**	1200	20096.1113
50	5	d	84582.134	1200.01	19152.6696	**74828.235**	1200	24362.9521	82797.698	1200	20997.2415
50	5	e	86142.625	1200	28326.1902	**79741.239**	1200	27254.8959	80309.402	1200	25811.8336
50	10	a	120386.351	1200.01	33784.0164	112041.226	1200	40499.3424	**107742.195**	1200	40429.4105
50	10	b	131922.899	1200.01	36196.9346	129780.111	1200	39067.015	**115328.168**	1200	38302.5298
50	10	c	127505.107	1200.01	29633.4083	121755.175	1200	38498.5548	**117584.639**	1200	38751.018
50	10	d	129510.776	1200.01	30826.7637	**118961.583**	1200	32213.8052	124205.25	1200	32018.3999
50	10	e	130852.066	1200.01	29572.5083	123938.002	1200	41056.0084	**117219.18**	1200	47620.4586
50	15	a	154255.682	1200.01	17702.3009	**134172.272**	1200	39577.553	140880.108	1200	50057.183
50	15	b	168875.902	1200.02	35317.4167	**134632.299**	1200	57751.6412	138942.574	1200	58458.5485
50	15	c	151550.059	1200.02	34086.4567	140002.541	1200	43298.9065	**139857.909**	1200	52033.5577
50	15	d	152174.91	1200.01	37794.7988	151642.527	1200	45066.433	**141824.138**	1200	55206.4462
50	15	e	162258.21	1200.02	33042.2542	141728.7	1200	48883.0709	**137799.942**	1200	50507.2102
50	20	a	167512.789	1200.04	35114.2741	164525.842	1200	43567.0849	**152283.617**	1200.01	41323.0018
50	20	b	173047.023	1200.04	24949.1913	**157362.791**	1200	48979.6052	159637.004	1200.01	49441.0666
50	20	c	182541.362	1200.04	27195.4954	**156806.508**	1200	61331.0497	166030.514	1200	49916.7115
50	20	d	167946.7	1200.04	29512.4493	**154673.491**	1200	45330.337	156495.701	1200	42254.0024
50	20	e	171997.321	1200.02	40596.9162	**163483.939**	1200	54960.2017	168360.853	1200	52566.3262

Table 17: Results for $n = 100,\ 300,\ 500$ and $k = 20$

n	m	$type$	FdSdS r	FdSdS $t[s]$	FdSdS LB	New-1 r	New-1 $t[s]$	New-1 LB	New-2 r	New-2 $t[s]$	New-2 LB
100	2	a	18738.429	1200	0	15516.653	1200	0	**13968.951**	1200	0
100	2	b	22482.807	1200	0	**14280.509**	1200	0	16529.734	1200	0
100	2	c	20931.487	1200	0	**15651.607**	1200.01	0	16841.451	1200.01	0
100	2	d	22423.517	1200	0	13297.349	1200	0	**11349.463**	1200	0
100	2	e	18936.176	1200	0	17302.711	1200	0	**12142.291**	1200	0
100	3	a	**38539.352**	1200.01	0	42219.365	1200	0	42290.631	1200	0
100	3	b	48966.463	1200.16	0	**34711.638**	1200	0	38552.843	1200	0
100	3	c	37061.202	1200	0	**27669.828**	1200.01	0	35463.502	1200	0
100	3	d	38293.773	1200.06	0	35149.967	1200	0	**31769.314**	1200	0
100	3	e	44034.531	1200.01	0	40812.74	1200.01	0	**38022.886**	1200	0
100	5	a	85175.801	1200.01	0	**72734.617**	1200	0	76251.315	1200	0
100	5	b	73562.488	1200	0	75816.315	1200	0	**73066.94**	1200	0
100	5	c	74433.624	1200	0	67896.23	1200	0	**66365.87**	1200	0
100	5	d	80762.599	1200.01	0	**72826.599**	1200	0	74850.3	1200	0
100	5	e	82132.841	1200.01	0	80883.605	1200	0	**75694.989**	1200	0
100	10	a	140150.015	1200.02	0	**114288.196**	1200	0	128020.62	1200.01	0
100	10	b	**125837.246**	1200.02	0	126686.255	1200	0	131197.641	1200	0
100	10	c	143351.956	1200.04	0	126472.458	1200	0	**126305.426**	1200	0
100	10	d	128809.194	1200.02	0	118599.329	1200	0	**114156.828**	1200	0
100	10	e	123222.608	1200.04	0	**119845.514**	1200	0	128603.237	1200.01	0
100	15	a	161438.792	1200.04	0	**153791.97**	1200.01	0	157723.852	1200	0
100	15	b	171172.854	1200.05	0	160026.464	1200	0	**148072.478**	1200.01	0
100	15	c	182801.335	1200.05	0	166100.529	1200	0	**158536.992**	1200	0
100	15	d	175926.889	1200.06	0	158554.382	1200.01	0	**152198.384**	1200.01	0
100	15	e	167921.388	1200.06	0	159087.922	1200.01	0	**146134.198**	1200	0
100	20	a	206616.65	1200.05	0	181955.407	1200	0	**174036.686**	1200.01	0
100	20	b	188682.409	1200.06	0	**173547.332**	1200.01	0	190261.027	1200	17473.5647
100	20	c	186804.281	1200	0	**174736.896**	1200.01	0	184190.876	1200	318.1066
100	20	d	182366.634	1200.02	0	**177981.761**	1200	0	179675.345	1200.01	0
100	20	e	193672.78	1200.05	0	**174727.396**	1200	0	178553.107	1200	0
300	2	a	13587.961	1200	0	18249.454	1201.59	0	**12683.912**	1200.67	0
300	2	b	12067.991	1200	0	14771.328	1201.65	0	**11413.931**	1200.2	0
300	2	c	12374.791	1200.01	0	14991.223	1201.41	0	**12165.247**	1200.55	0
300	2	d	**12669.965**	1200.01	0	16331.898	1201.59	0	14328.549	1200.01	0
300	2	e	14511.679	1200	0	12289.724	1200.04	0	**11948.574**	1200.23	0
300	3	a	43371.486	1200.01	0	**41354.562**	1200.78	0	42708.605	1200.01	0
300	3	b	33068.883	1200.01	0	**30456.56**	1200.31	0	31405.245	1200	0
300	3	c	**27891.642**	1200	0	39082.941	1202.11	0	30118.851	1200.19	0
300	3	d	**28995.472**	1200.01	0	37457.181	1201.73	0	29447.739	1200.01	0
300	3	e	43838.736	1200.01	0	30726.751	1200.24	0	**24770.976**	1200.01	0
300	5	a	76799.488	1200.01	0	75654.393	1200	0	**58054.826**	1200	0
300	5	b	77768.979	1200.03	0	70451.769	1200.01	0	**64432.516**	1200.01	0
300	5	c	70455.526	1200.03	0	70580.092	1200	0	**55092.772**	1200	0
300	5	d	83507.45	1200	0	73918.947	1200.01	0	**67979.615**	1200	0
300	5	e	88925.771	1200.01	0	74748.139	1200.01	0	**71910.067**	1200	0
300	10	a	149973.728	1200.05	0	131002.034	1200	0	**129525.751**	1200.01	0
300	10	b	140379.075	1200.03	0	**116491.107**	1200.01	0	128580.775	1200.01	0
300	10	c	143263.489	1200.04	0	137868.56	1200.01	0	**127390.549**	1200.01	0
300	10	d	135928.482	1200.01	0	132887.424	1200	0	**111383.207**	1200.01	0
300	10	e	137790.4	1200.02	0	126339.207	1200.01	0	**118232.513**	1200.01	0
300	15	a	193113.661	1200	0	164736.055	1200.01	0	**158698.701**	1200.02	0
300	15	b	170506.182	1200.06	0	174727.987	1200	0	**159429.053**	1200.02	0
300	15	c	172581.584	1200.01	0	183649.719	1200	0	**158557.11**	1200.01	0
300	15	d	158292.211	1200.15	0	171475.576	1200.01	0	**146995.382**	1200	0
300	15	e	192356.707	1200.01	0	159919.247	1200	0	**158134.215**	1200.01	0
300	20	a	268211.104	1200.13	0	208952.404	1200.03	0	**198913.805**	1200	0
300	20	b	219532.397	1200.72	0	276850.834	1200.02	0	**198926.558**	1200	0
300	20	c	271096.716	1200.11	0	**171208.929**	1200.02	0	212901.36	1200	0
300	20	d	251449.518	1200.23	0	**206163.316**	1200.01	0	209085.073	1200.01	0
300	20	e	294829.803	1200.14	0	244949.129	1200.02	0	**201683.49**	1200.01	0
500	2	a	**9954.374**	1200.01	0	26828.492	1204.84	0	10059.817	1200.18	0
500	2	b	12221.512	1200.01	0	11540.78	1200.98	0	**10027.877**	1200.07	0
500	2	c	9295.492	1200.01	0	12183.718	1200.17	0	**8658.34**	1200.23	0
500	2	d	**7356.759**	1200.01	0	12640.917	1202.27	0	7599.385	1200.1	0
500	2	e	12801.393	1200.03	0	18758.339	1200.24	0	**9212.484**	1200.01	0
500	3	a	31792.661	1200.01	0	33352.587	1200.83	0	**22172.324**	1200.02	0
500	3	b	**29255.438**	1200.02	0	42497.881	1200.02	0	31875.831	1200.02	0
500	3	c	**24283.547**	1200.01	0	29758.057	1201.94	0	32296.877	1200.01	0
500	3	d	38227.767	1200.01	0	**30631.055**	1200.01	0	31297.855	1200.02	0
500	3	e	34892.869	1200.01	0	**27419.459**	1200.06	0	28231.101	1200.02	0
500	5	a	59371.544	1200.01	0	**59134.465**	1200.02	0	60015.311	1200.01	0
500	5	b	72332.802	1200.01	0	85817.005	1200.01	0	**56845.908**	1200.01	0
500	5	c	83072.692	1200	0	63884.144	1200.01	0	**59507.246**	1200	0
500	5	d	79184.811	1200.12	0	**61281.576**	1200.02	0	62866.185	1200.01	0
500	5	e	79862.741	1200.04	0	78251.929	1200.01	0	**61570.585**	1200.01	0
500	10	a	149033.44	1200	0	132495.076	1200.01	0	**124370.817**	1200.01	0
500	10	b	124501.653	1200.03	0	138451.282	1200.01	0	**124378.157**	1200.01	0
500	10	c	147562.537	1200.03	0	135919.164	1200.01	0	**116329.42**	1200.01	0
500	10	d	118326.033	1200.02	0	**111500.249**	1200.01	0	118299.614	1200.01	0
500	10	e	136702.733	1200.07	0	148120.66	1200.01	0	**123108.452**	1200.01	0
500	15	a	190791.348	1200.03	0	196594.78	1200.01	0	**157829.031**	1200.01	0
500	15	b	**165006.578**	1200.11	0	177963.637	1200.02	0	178485.957	1200.02	0
500	15	c	172750.811	1200.03	0	169224.357	1200.01	0	**142510.496**	1200.01	0
500	15	d	221455.113	1200.15	0	**156657.234**	1200.01	0	174144.6	1200.01	0
500	15	e	182074.971	1200.06	0	**151514.146**	1200.03	0	157913.943	1200.01	0
500	20	a	287740.487	1200.03	0	276852.224	1200.02	0	**184309.406**	1200.02	0
500	20	b	290779.134	1200.1	0	**194627.379**	1200.01	0	225194.695	1200.01	0
500	20	c	328909.753	1200.11	0	228837.383	1200.01	0	**211188.935**	1200.01	0
500	20	d	255382.67	1200.23	0	288371.475	1200.02	0	**194686.481**	1200.01	0
500	20	e	328848.552	1200.34	0	**198908.516**	1200.02	0	216699.094	1200.01	0

Table 18: Results for $n = 200,\ 400$ and $k = 20$

n	m	$type$	FdSdS			New-1			New-2		
			r	$t[s]$	LB	r	$t[s]$	LB	r	$t[s]$	LB
200	2	a	19826.397	1200	0	21284.665	1200.71	0	**18610.559**	1200	0
200	2	b	18001.888	1200	0	19039.985	1200.76	0	**14811.683**	1200.3	0
200	2	c	**16196.436**	1200.01	0	17239.176	1200.64	0	16303.185	1200	0
200	2	d	13589.955	1200	0	13827.483	1200.05	0	**10706.078**	1200.44	0
200	2	e	**14274.714**	1200	0	22266.618	1200.62	0	18363.904	1200.53	0
200	3	a	40273.383	1200.01	0	49475.016	1200.48	0	**37468.928**	1200	0
200	3	b	41403.101	1200.01	0	**38028.13**	1200.26	0	41354.665	1200	0
200	3	c	36121.35	1200.01	0	38184.915	1200.32	0	**33962.429**	1200.01	0
200	3	d	34015.349	1200	0	**32887.175**	1200.17	0	35403.953	1200	0
200	3	e	44769.353	1200	0	41919.895	1204.48	0	**39032.858**	1200.01	0
200	5	a	75217.684	1200.02	0	**61211.907**	1200	0	72876.448	1200	0
200	5	b	79134.478	1200	0	**65231.08**	1200	0	81203.054	1200	0
200	5	c	74603.333	1200.01	0	**72095.284**	1200	0	72993.909	1200	0
200	5	d	73924.891	1200.03	0	**60503.396**	1200	0	63366.867	1200	0
200	5	e	85564.382	1200.01	0	**77420.53**	1200	0	81900.136	1200	0
200	10	a	147948.77	1200	0	**130968.825**	1200	0	140788.058	1200.01	0
200	10	b	133269.354	1200.04	0	**128829.767**	1200	0	131295.814	1200.01	0
200	10	c	129398.19	1200.06	0	137847.036	1200	0	**128739.005**	1200.01	0
200	10	d	120410.04	1200.05	0	121622.588	1200	0	**114383.895**	1200.01	0
200	10	e	146231.893	1200.02	0	**127402.105**	1200	0	132516.882	1200.01	0
200	15	a	180522.923	1200.06	0	**160715.931**	1200	0	167277.556	1200.01	0
200	15	b	**159209.998**	1200.05	0	169214.855	1200	0	171196.175	1200.02	0
200	15	c	175244.823	1200.04	0	177968.774	1200.01	0	**155955.988**	1200.01	0
200	15	d	185230.055	1200.45	0	164745.719	1200	0	**161738.768**	1200.01	0
200	15	e	177103	1200.06	0	169147.955	1200.01	0	**162750.618**	1200.01	0
200	20	a	**193519.545**	1200.07	0	196378.633	1200.01	0	194538.074	1200.01	0
200	20	b	298022.152	1200.16	0	**177593.054**	1200.01	0	181157.079	1200.02	0
200	20	c	237566.267	1200.17	0	186462.698	1200.01	0	**157664.823**	1200.02	0
200	20	d	228537.007	1200.1	0	206487.394	1200.02	0	**177485.602**	1200.01	0
200	20	e	**179759.4**	1200.11	0	197196.694	1200	0	189278.635	1200.02	0
400	2	a	15412.153	1200.01	0	**11634.815**	1206.37	0	16650.924	1202.17	0
400	2	b	**12065.167**	1200.01	0	13766.591	1209.06	0	15241.48	1200.01	0
400	2	c	10635.921	1200.01	0	14579.975	1206.65	0	**8200.965**	1200.01	0
400	2	d	14279.323	1200.01	0	15220.883	1200.02	0	**9508.295**	1200.68	0
400	2	e	16515.233	1200.01	0	18271.816	1206.49	0	**9409.827**	1205.63	0
400	3	a	**28018.98**	1200.01	0	29072.249	1200.99	0	35629.978	1200.01	0
400	3	b	35481.258	1200.02	0	**33303.467**	1201.82	0	36812.317	1201.67	0
400	3	c	**27320.715**	1200.02	0	31157.791	1202.91	0	30513.281	1200.01	0
400	3	d	36744.359	1200.01	0	**31123.067**	1203.98	0	32737.294	1200.28	0
400	3	e	40335.8	1200.01	0	34918.536	1200.01	0	**34377.87**	1200.02	0
400	5	a	80228.475	1200.01	0	84377.946	1200.01	0	**70385.478**	1200	0
400	5	b	61953.995	1200	0	61834.167	1200.01	0	**60124.146**	1200.01	0
400	5	c	72586.122	1200.01	0	**62069.923**	1200.01	0	80517.669	1200.01	0
400	5	d	84600.553	1200.04	0	**64803.389**	1200.01	0	67573.793	1200.01	0
400	5	e	79576.122	1200.01	0	72462.818	1200	0	**70139.172**	1200.01	0
400	10	a	**124572.868**	1200.02	0	135976.471	1200.01	0	141913.366	1200.02	0
400	10	b	**125295.709**	1200.11	0	127575.343	1200	0	134043.391	1200.01	0
400	10	c	**126180.106**	1200.03	0	138930.709	1200	0	139132.533	1200	0
400	10	d	131704.112	1200.05	0	**113592.482**	1200.01	0	115569.289	1200.01	0
400	10	e	122094.124	1200.02	0	127540.923	1200.01	0	**118084.099**	1200.01	0
400	15	a	181326.871	1200.04	0	**172160.575**	1200.01	0	176194.496	1200.02	0
400	15	b	163829.189	1200.07	0	**149565.739**	1200.01	0	180396.059	1200.01	0
400	15	c	**155286.165**	1200.01	0	156115.461	1200	0	167093.368	1200.01	0
400	15	d	169874.918	1200.19	0	166414.212	1200.01	0	**156488.151**	1200.01	0
400	15	e	180057.054	1200.06	0	**143670.723**	1200.01	0	158648.328	1200.02	0
400	20	a	289792.015	1200.34	0	220295.339	1200.02	0	**185344.989**	1200	0
400	20	b	303587.845	1200.02	0	303676.412	1200.01	0	**197993.438**	1200.02	0
400	20	c	237757.881	1200.14	0	207829.938	1200.02	0	**204500.671**	1200.02	0
400	20	d	313170.396	1200.13	0	210612.452	1200.01	0	**208856.222**	1200.02	0
400	20	e	281467.302	1200.22	0	209707.366	1200	0	**189999.644**	1200.01	0

Table 19: Results for $n = 30,\ 40,\ 50$ and $k = 30$

n	m	$type$	FdSdS			New-1			New-2		
			r	$t[s]$	LB	r	$t[s]$	LB	r	$t[s]$	LB
30	2	a	**94160.9**	7.16	94160.9	**94160.9**	1.2	94160.9	**94160.9**	0.2	94160.9
30	2	b	**94629.75**	7.31	94629.75	**94629.75**	0.93	94629.75	**94629.75**	0.37	94629.75
30	2	c	**97381.886**	7.42	97381.886	**97381.886**	6.52	97381.886	**97381.886**	0.78	97381.886
30	2	d	**96557.486**	5.67	96557.486	**96557.486**	1.35	96557.486	**96557.486**	0.29	96557.486
30	2	e	**99440.688**	23.44	99440.688	**99440.688**	1.56	99440.688	**99440.688**	0.51	99440.688
30	3	a	**94160.9**	11.5	94160.9	**94160.9**	1.45	94160.9	**94160.9**	0.47	94160.9
30	3	b	**94629.75**	14.27	94629.75	**94629.75**	2.86	94629.75	**94629.75**	0.51	94629.75
30	3	c	**97381.886**	12.88	97381.886	**97381.886**	1.72	97381.886	**97381.886**	0.29	97381.886
30	3	d	**96557.486**	11.23	96557.486	**96557.486**	1.93	96557.486	**96557.486**	0.6	96557.486
30	3	e	**99440.688**	13.67	99440.688	**99440.688**	3.67	99440.688	**99440.688**	0.55	99440.688
30	5	a	**95895.434**	48.78	95895.434	**95895.434**	14.42	95895.434	**95895.434**	1.21	95895.434
30	5	b	**97263.902**	43.17	97263.902	**97263.902**	5.56	97263.902	**97263.902**	1.9	96290.031
30	5	c	**97381.886**	29.69	97381.886	**97381.886**	3.51	97381.886	**97381.886**	1.06	97381.886
30	5	d	**98658.481**	20.65	98658.481	**98658.481**	3.55	98658.481	**98658.481**	0.57	98658.481
30	5	e	**99440.688**	23.6	99440.688	**99440.688**	16.82	99440.688	**99440.688**	1.04	99440.688
30	10	a	**95973.224**	60.95	95973.224	**95973.224**	6.44	95973.224	**95973.224**	2.47	95973.224
30	10	b	**97396.324**	65.25	97396.324	**97396.324**	15.95	97396.324	**97396.324**	4.39	97396.324
30	10	c	**98602.841**	187.8	98602.841	**98602.841**	19.71	98602.841	**98602.841**	2.54	98602.841
30	10	d	**98658.481**	115.9	98658.481	**98658.481**	19.55	98658.481	**98658.481**	3.94	98658.481
30	10	e	**99440.688**	129.89	99440.688	**99440.688**	18	99440.688	**99440.688**	3.86	99440.688
30	15	a	**96736.519**	165.86	96736.519	**96736.519**	91.7	96736.519	**96736.519**	4.67	96736.519
30	15	b	**98089.894**	223.03	98089.894	**98089.894**	21.71	98089.894	**98089.894**	5.27	98089.894
30	15	c	**98602.841**	195.27	98602.841	**98602.841**	27.54	98602.841	**98602.841**	2.49	98350.451
30	15	d	**98658.481**	427.49	98658.481	**98658.481**	25.42	98658.481	**98658.481**	3.55	98658.481
30	15	e	**99440.688**	272.71	99440.688	**99440.688**	25.56	99440.688	**99440.688**	10.69	99440.688
30	20	a	**98548.27**	794.49	98548.27	**98548.27**	37.5	98548.27	**98548.27**	9.22	96736.519
30	20	b	**98462.28**	463.2	98462.28	**98462.28**	38.89	98462.28	**98462.28**	11.52	98462.28
30	20	c	**98602.841**	541.18	98602.841	**98602.841**	36.04	98602.841	**98602.841**	15.07	98350.451
30	20	d	**98658.481**	628.8	98658.481	**98658.481**	31.73	98658.481	**98658.481**	5.04	98658.481
30	20	e	**99440.688**	334.2	99440.688	**99440.688**	37.38	99440.688	**99440.688**	14.14	99440.688
40	2	a	**59599.299**	1200	55197.3368	**59599.299**	155.56	59599.299	**59599.299**	59.22	59599.299
40	2	b	**70356.887**	1200	61407.2553	**70356.887**	74.95	70356.887	**70356.887**	43.72	70356.887
40	2	c	**69257.532**	1200.01	62640.2375	**69257.532**	1200	63055.4814	**69257.532**	922.77	69257.532
40	2	d	**71682.73**	1200.01	63359.1667	**71682.73**	51.52	71682.73	**71682.73**	35.18	71682.73
40	2	e	**75388.455**	1200	66889.046	**75388.455**	544.38	75388.455	**75388.455**	107.86	75388.455
40	3	a	**78396.442**	1200.01	72988.3663	**78396.442**	347.72	78396.442	**78396.442**	108.72	78396.442
40	3	b	**82199.457**	1200	66725.0827	**82199.457**	1200	69338.382	**82199.457**	1010.61	82199.457
40	3	c	**78347.277**	1200.01	58850.5191	**78347.277**	1200	67539.8036	**78347.277**	1200	68087.6653
40	3	d	**79856.257**	1200	71153.1492	**79856.257**	1200	70477.6541	**79856.257**	117.74	79856.257
40	3	e	88590.194	1200	80708.3671	**88089.2**	1200	81340.0972	**88089.2**	1200	87729.0936
40	5	a	105832.033	1200.01	51128.7963	**102479.974**	1200	72484.1883	**102479.974**	1200	70465.1475
40	5	b	106745.774	1200.01	79038.4157	**105617.486**	1200	68954.1184	**105617.486**	1200	80639.2285
40	5	c	93326.94	1200.01	51817.6328	**90350.566**	1200	54697.2752	**90350.566**	1200	86071.9348
40	5	d	106396.002	1200.01	69656.0088	**98292.727**	1200	76471.6995	**98292.727**	1200	76619.8475
40	5	e	111673.586	1200.01	72781.0343	**109073.526**	1200	79333.8355	**109073.526**	1200	84173.5721
40	10	a	126595.114	1200	51157.6815	**118493.449**	1200	64857.8937	118822.794	1200	66251.7604
40	10	b	134584.636	1200.03	77051.6363	132090.983	1200	78274.391	**131606.339**	1200	89295.2131
40	10	c	127608.067	1200.02	74523.4833	123889.492	1200	77987.8182	**122835.311**	1200	76005.673
40	10	d	139358.568	1200.02	55271.821	**126483.665**	1200	83142.4416	127055.296	1200	88268.461
40	10	e	**126976.189**	1200	58273.7492	**126976.189**	1200	75941.7298	**126976.189**	1200	80978.1379
40	15	a	159524.845	1200.06	47191.1934	137438.798	1200	70533.782	**135778.161**	1200	84630.555
40	15	b	154688.16	1200.07	41516.1214	142619.822	1200	91114.5809	**139717.219**	1200	91314.4459
40	15	c	156508.747	1200.01	59948.6305	135329.141	1200	85695.0206	**132638.426**	1200	75262.5445
40	15	d	155834.933	1200.01	78912.2935	**136892.519**	1200	83497.0777	**136892.519**	1200.01	77414.0106
40	15	e	165718.822	1200	50292.6871	138864.052	1200	94448.2509	**137059.666**	1200.01	80641.5541
40	20	a	263795.958	1200.16	211.213	**144504.175**	1200	73197.6688	**144504.175**	1200	71539.575
40	20	b	364351.538	1200.18	218.9343	146346.468	1200.01	97802.7001	**143862.081**	1200	80572.0252
40	20	c	665331.391	1200.13	251.3194	**140792.68**	1200.01	79745.4884	141347.44	1200.01	78199.6221
40	20	d	167943.89	1200.09	62785.1164	141984.338	1200	89413.7276	**139146.865**	1200	78033.3746
40	20	e	328050.578	1200.11	43450.2513	147138.476	1200	91072.1926	**143208.561**	1200	91381.3884
50	2	a	44038.524	1200.01	18269.491	46121.941	1200	14715.5913	**41790.384**	1200	21189.9737
50	2	b	**46906.656**	1200	27594.1435	**46906.656**	1200	33637.4017	**46906.656**	1200	37987.115
50	2	c	47492.538	1200.01	25402.1795	**46728.578**	1200	33265.3462	46954.195	1200	31348.1609
50	2	d	53796.88	1200	40521.8152	**52412.181**	1200	43536.3754	**52412.181**	409.8	52412.181
50	2	e	42740.866	1200.01	28068.8009	**42621.504**	1200	33300.1179	**42621.504**	1200	29362.3917
50	3	a	70573.677	1200	39418.9706	**68320.363**	1200	42034.6117	**68320.363**	1200	44638.5789
50	3	b	72583.313	1200	30825.6339	**72180.859**	1200	26958.0696	**72180.859**	1200	37424.6444
50	3	c	68545.294	1200	38233.0965	**66476.403**	1200	38415.4152	**66476.403**	1200	43243.2062
50	3	d	74127.786	1200	36994.3169	**71001.682**	1200	37388.2572	**71001.682**	1200	40968.0831
50	3	e	81466.739	1200.01	45967.7125	**66647.433**	1200	32793.4744	68197.434	1200	50919.4102
50	5	a	111911.158	1200.02	42715.6481	**94105.31**	1200	45243.4161	99117.746	1200.01	46229.1332
50	5	b	104810.22	1200	52223.4928	95640.693	1200	53252.5675	**94336.443**	1200	58936.9436
50	5	c	114922.188	1200.01	44401.6886	**93785.741**	1200	47082.4716	99118.254	1200	46066.0562
50	5	d	114144.67	1200.01	40758.0377	97589.071	1200	45543.4623	**96798.509**	1200	57305.9588
50	5	e	117526.411	1200.01	38050.4846	107848.345	1200	42924.0363	**103507.577**	1200	46385.7182
50	10	a	131644.913	1200.03	35484.0848	126934.454	1200	50260.868	**125454.434**	1200	64063.8249
50	10	b	136810.053	1200.04	53466.0705	131653.61	1200	59456.2638	**129899.7**	1200	65476.065
50	10	c	140229.245	1200.03	55941.5775	129864.033	1200	58565.151	**126603.906**	1200.01	57922.9736
50	10	d	141722.013	1200.05	46575.4685	129078.172	1200	59575.2977	**126527.477**	1200	56442.7203
50	10	e	149685.648	1200	39651.6763	143038.016	1200	48102.8958	**125686.149**	1200	66127.6289
50	15	a	157897.026	1200.06	17650.2299	138712.607	1200	55809.4933	**136204.826**	1200.01	64950.637
50	15	b	156728.626	1200.08	36554.0247	**145213.388**	1200	70496.3711	145269.872	1200	72246.782
50	15	c	168341.93	1200.08	40310.7899	146816.437	1200	60985.5758	**144227.83**	1200	62037.1268
50	15	d	157074.072	1200.08	33230.2372	146556.579	1200	66627.7095	**145081.588**	1200.01	59073.2001
50	15	e	169580.945	1200.09	41192.3978	**137083.219**	1200	65386.3642	138135.247	1200	64374.6512
50	20	a	879304.004	1200.07	19223.5462	153373.083	1200	65332.0658	**151382.038**	1200	60100.8581
50	20	b	230964.8	1200.17	31656.5855	**151825.352**	1200	73771.0762	155334.547	1200	72526.4574
50	20	c	263435.795	1200.11	74.3912	**150757.776**	1200.01	66790.966	151001.377	1200.01	62756.1479
50	20	d	907439.851	1200.14	32478.7937	151118.72	1200	70465.2498	**150532.516**	1200.01	59561.1838
50	20	e	482080.833	1200.23	175.3652	**150495.029**	1200	73324.7582	151836.601	1200.01	82636.7756

Table 20: Results for $n = 100,\ 300,\ 500$ and $k = 30$

n	m	$type$	FdSdS			New-1			New-2		
			r	$t[s]$	LB	r	$t[s]$	LB	r	$t[s]$	LB
100	2	a	33795.249	1200	0	**14946.226**	1200	0	17247.181	1200.01	0
100	2	b	30679.797	1200	0	**16519.746**	1200	0	22706.532	1200	0
100	2	c	27593.52	1200	0	**15428.117**	1200	0	16592.208	1200	0
100	2	d	31750.464	1200.01	0	17809.959	1200.01	0	**16975.308**	1200	0
100	2	e	31028.727	1200	0	15602.254	1200	0	**11915.383**	1200	0
100	3	a	69926.921	1200.02	0	45175.218	1200	0	**44669.574**	1200	0
100	3	b	65166.53	1200	0	43653.064	1200	0	**42615.909**	1200	0
100	3	c	55469.589	1200	0	**45297.399**	1200	0	48864.373	1200.01	0
100	3	d	67757.193	1200.02	0	45675.517	1200.01	0	**42710.83**	1200	0
100	3	e	77455.808	1200.01	0	**42184.533**	1200	0	48217.645	1200.01	0
100	5	a	113341.855	1200.01	0	82491.694	1200	0	**73532.576**	1200	0
100	5	b	104807.285	1200.03	0	**78813.043**	1200	0	84570.21	1200.01	0
100	5	c	114360.583	1200.01	0	**78525.801**	1200	0	79415.537	1200	0
100	5	d	98944.047	1200.02	0	**76850.513**	1200	0	87010.239	1200.01	0
100	5	e	112734.198	1200.03	0	**76083.384**	1200	0	85377.188	1200	0
100	10	a	168138.262	1200.02	0	132853.897	1200.01	0	**125633.84**	1200.01	0
100	10	b	148209.045	1200.04	0	147085.768	1200	9693.4025	**136909.213**	1200.01	13213.7606
100	10	c	163707.598	1200.3	0	141531.935	1200	0	**127955.426**	1200	0
100	10	d	147351.055	1200.06	0	**126780.78**	1200	0	140420.74	1200.01	0
100	10	e	159186.162	1200.06	0	139643.938	1200	11766.9693	**133132.31**	1200.01	12729.3958
100	15	a	324834.121	1200.18	8913.381	179997.495	1200	16546.969	**161377.343**	1200.01	20161.7951
100	15	b	268465.224	1200.18	10017.1618	176692.953	1200.01	18076.1511	**159666.805**	1200.01	18688.9421
100	15	c	314493.485	1200.07	5947.2333	184502.477	1200	3924.1864	**173343.972**	1200.01	20312.0173
100	15	d	335394.086	1200.15	7322.3815	180218.087	1200.01	7291.755	**155478.361**	1200.02	15876.5348
100	15	e	198293.901	1200.08	6440.8593	**170147.755**	1200	11607.3173	173638.042	1200.01	15290.2138
100	20	a	293435.513	1200.16	1727.9619	310242.587	1200.03	0	**180746.374**	1200.02	14835.6736
100	20	b	319472.361	1200.2	0	189267.936	1200.01	16461.5144	**172987.098**	1200.01	20471.3104
100	20	c	295039.109	1200.23	0	202384.51	1200.03	10424.7118	**179049.591**	1200	15116.9964
100	20	d	328222.772	1200.22	0	201238.651	1200.01	11463.7216	**185212.95**	1200.01	20865.6854
100	20	e	381863.596	1200.26	13818.1073	**191164.597**	1200.01	13598.8174	191371.739	1200.01	25551.6852
300	2	a	30073.813	1200.02	0	**14425.998**	1200.37	0	17603.693	1201.89	0
300	2	b	23665.193	1200.01	0	18123.032	1200.8	0	**13573.363**	1200.02	0
300	2	c	24118.019	1200.02	0	24590.699	1201.7	0	**15385.168**	1201.81	0
300	2	d	22681.587	1200.01	0	16089.573	1200.25	0	**11513.436**	1200.54	0
300	2	e	22469.267	1200	0	23127.793	1201.84	0	**19313.25**	1200.01	0
300	3	a	60110.288	1200.01	0	**44704.783**	1200.18	0	45574.891	1200.01	0
300	3	b	50730.267	1200.05	0	34866.872	1200.36	0	**34789.825**	1200.02	0
300	3	c	53790.266	1200.02	0	**33227.888**	1200.01	0	37026.153	1200.01	0
300	3	d	58664.787	1200.03	0	49915.651	1200.01	0	**35064.137**	1200.01	0
300	3	e	48186.639	1200.01	0	**38317.32**	1200.31	0	44247.806	1200.02	0
300	5	a	120988.226	1200.39	0	90736.794	1200.01	0	**72962.087**	1200	0
300	5	b	119236.781	1200.46	0	**77008.855**	1200.01	0	88254.853	1200.01	0
300	5	c	111963.587	1200.41	0	78949.492	1200.01	0	**70068.396**	1200.01	0
300	5	d	122842.843	1208.72	0	92496.88	1200.01	0	**70080.3**	1200.01	0
300	5	e	134523.856	1200.54	0	100726.443	1200.01	0	**78316.321**	1200.01	0
300	10	a	176436.976	1200.33	0	**126135.058**	1200.01	0	142530.183	1200.01	0
300	10	b	188365.144	1200.19	0	167499.322	1200.01	0	**141577.266**	1200.01	0
300	10	c	189743.993	1200.08	0	**134255.519**	1200.01	0	154596.745	1200	0
300	10	d	**139233.278**	1200.17	0	174671.36	1200	0	148433.74	1200.01	0
300	10	e	172531.112	1200.1	0	**138893.352**	1200.01	0	150829.842	1200.01	0
300	15	a	218408.604	1200.21	0	192727.749	1200.02	0	**190766.322**	1200.02	0
300	15	b	218319.221	1200.16	0	221682.919	1200.02	0	**171087.529**	1200.01	0
300	15	c	312960.063	1200.1	0	239985.006	1200.02	0	**197420.8**	1200.02	0
300	15	d	253433.959	1200.48	0	297166.182	1200.01	0	**194624.941**	1200.01	0
300	15	e	258128.965	1200.35	0	266542.492	1200.02	0	**182665.387**	1200.03	0
300	20	a	344294.436	1200.69	0	342831.375	1200.04	0	**203788.99**	1200.03	0
300	20	b	403083.812	1200.76	0	364102.209	1200.03	0	**177420.249**	1200.02	0
300	20	c	346626.593	1200.26	0	362893.21	1200.03	0	**202894.119**	1200.02	0
300	20	d	368032.543	1200.87	0	358903.964	1200.04	0	**231428.932**	1200.04	0
300	20	e	377355.084	1200.74	0	261697.819	1200.03	0	**209873.225**	1200.01	0
500	2	a	20460.434	1200.02	0	23296.327	1200.03	0	**14424.059**	1200.31	0
500	2	b	20275.721	1200.02	0	28145.243	1200.04	0	**14304.523**	1200.03	0
500	2	c	16139.057	1200.03	0	**12604.384**	1200.82	0	13738.81	1200.12	0
500	2	d	19515.743	1200.05	0	**10556.88**	1201.09	0	13524.126	1200.03	0
500	2	e	**17783.455**	1200.06	0	22298.284	1200.13	0	21848.822	1200.03	0
500	3	a	91905.565	1200.38	0	56799.846	1200.02	0	**30330.474**	1200.02	0
500	3	b	69355.91	1200.35	0	62317.164	1200.02	0	**37089.192**	1200.02	0
500	3	c	62645.769	1200.95	0	**40998.396**	1200.79	0	52018.446	1200.02	0
500	3	d	70460.765	1200.36	0	49016.452	1200.04	0	**28417.351**	1200.03	0
500	3	e	75836.937	1200.39	0	**43047.631**	1200.04	0	53684.861	1200.04	0
500	5	a	103273.871	1200.93	0	78308.168	1200.01	0	**68628.48**	1200.01	0
500	5	b	109283.714	1200.63	0	77968.566	1200.02	0	**76051.29**	1200.01	0
500	5	c	126480.115	1200.57	0	**73553.368**	1200.02	0	85317.895	1200.01	0
500	5	d	108885.954	1200.02	0	86018.063	1200.03	0	**70931.701**	1200.02	0
500	5	e	122069.934	1200.02	0	**60297.402**	1200.04	0	69240.317	1200.01	0
500	10	a	219468.074	1200.43	0	148863.063	1200.01	0	**143356.292**	1200	0
500	10	b	175880.532	1210.71	0	173764.929	1200.01	0	**138372.188**	1200.01	0
500	10	c	167988.004	1203.35	0	163494.664	1200.02	0	**130006.924**	1200.01	0
500	10	d	200656.406	1200.08	0	**128463.227**	1200.01	0	147517.656	1200.01	0
500	10	e	166645.678	1203.92	0	157657.885	1200.02	0	**147617.121**	1200.01	0
500	15	a	266014.437	1200.55	0	271017.152	1200.03	0	**191771.568**	1200.04	0
500	15	b	296581.767	1200.14	0	240301.606	1200.01	0	**165587.041**	1200.03	0
500	15	c	275012.367	1200.56	0	199740.974	1200.02	0	**182125.167**	1200.04	0
500	15	d	358607.455	1200.58	0	**202168.406**	1200.02	0	208769.829	1200.03	0
500	15	e	291411.457	1200.56	0	435631.215	1200.06	0	**176933.188**	1200.04	0
500	20	a	404633.233	1200.81	0	295695.838	1200.03	0	**225641.363**	1200.17	0
500	20	b	418527.599	1200.19	0	339849.83	1200.05	0	**223993.495**	1200.01	0
500	20	c	331494.553	1200.69	0	236013.161	1200.03	0	**221685.662**	1200.17	0
500	20	d	401011.827	1200.81	0	334659.671	1200.02	0	**198875.414**	1200.03	0
500	20	e	461161.234	1200.77	0	324974.307	1200.01	0	**224112.052**	1200.18	0

Table 21: Results for $n = 200,\ 400$ and $k = 30$

n	m	$type$	FdSdS r	FdSdS $t[s]$	FdSdS LB	New-1 r	New-1 $t[s]$	New-1 LB	New-2 r	New-2 $t[s]$	New-2 LB
200	2	a	27142.123	1200	0	21721.241	1200.87	0	**20055.708**	1200.58	0
200	2	b	31561.089	1200	0	21234.1	1200.01	0	**18009.89**	1200.01	0
200	2	c	18245.553	1200.01	0	**16596.613**	1200.1	0	16716.786	1200.01	0
200	2	d	21295.207	1200.01	0	**17978.136**	1200.59	0	19149.396	1200.01	0
200	2	e	29751.618	1200.03	0	**19976.092**	1200.67	0	23481.054	1200.01	0
200	3	a	60886.156	1200.01	0	49085.41	1202.39	0	**47099.942**	1200.01	0
200	3	b	59479.384	1200.02	0	**46978.783**	1200.01	0	56987.7	1200.01	0
200	3	c	68067.445	1200.01	0	38529.439	1203.94	0	**38314.194**	1200.34	0
200	3	d	48720.442	1200.03	0	47547.679	1203.32	0	**37484.179**	1200.01	0
200	3	e	64861.358	1200.02	0	**42091.648**	1202.52	0	47069.376	1200.01	0
200	5	a	101893.955	1200.02	0	**82795.707**	1200.01	0	87281.3	1200.01	0
200	5	b	91939.812	1200.05	0	**69153.085**	1200.01	0	72738.565	1200.01	0
200	5	c	102521.436	1200	0	**84873.477**	1200	0	92128.657	1200	0
200	5	d	96670.849	1200.05	0	83700.824	1200.01	0	**79368.219**	1200	0
200	5	e	111026.462	1200.04	0	97045.612	1200	0	**75587.944**	1200.01	0
200	10	a	186456.854	1200.23	0	136736.205	1200.01	0	**129927.382**	1200.01	0
200	10	b	191897.337	1200.01	0	**141121.639**	1200.01	0	143435.808	1200	0
200	10	c	**144881.763**	1200.07	0	159805.174	1200.01	0	152779.562	1200.01	0
200	10	d	141149.666	1200.22	0	152666.957	1200.01	0	**126730.246**	1200.01	0
200	10	e	208143.886	1200.05	0	159944.125	1200.01	0	**139525.216**	1200.02	0
200	15	a	209492.081	1200.06	0	275837.417	1200.02	0	**176792.552**	1200.03	0
200	15	b	244182.356	1200.03	0	211201.055	1200	0	**186001.032**	1200.01	0
200	15	c	237833.014	1200.14	0	274164.956	1200.02	0	**172702.002**	1200.01	0
200	15	d	233225.946	1200.07	0	304057.436	1200.02	0	**169232.314**	1200.01	0
200	15	e	299695.466	1200.15	0	291915.434	1200.02	0	**191672.537**	1200.03	0
200	20	a	370435.96	1200.74	0	**198724.789**	1200.03	0	209022.086	1200.02	0
200	20	b	378449.861	1200.29	0	255023.035	1200	0	**181388.343**	1200.03	0
200	20	c	377734.108	1200.3	0	206412.886	1200.04	0	**202153.152**	1200.04	0
200	20	d	335722.962	1200.19	0	201151.438	1200.05	0	**192179.384**	1200.01	0
200	20	e	350278.73	1200.24	0	244776.506	1200.03	0	**198703.202**	1200.01	0
400	2	a	17903.731	1200.05	0	21231.765	1203.04	0	**13384.939**	1200.11	0
400	2	b	17835.496	1200.02	0	**14063.693**	1207.49	0	15761.954	1200.17	0
400	2	c	24408.702	1200.03	0	**18816.619**	1200.16	0	23345.858	1200.03	0
400	2	d	19079.41	1200.03	0	22044.303	1200.75	0	**14328.506**	1200.04	0
400	2	e	32164.011	1200.03	0	25514.88	1200.01	0	**16767.723**	1200.87	0
400	3	a	76500.831	1200.03	0	50098.699	1200.01	0	**47521.799**	1200.01	0
400	3	b	69897.618	1200.04	0	48129.803	1202.59	0	**43752.644**	1200.02	0
400	3	c	86046.87	1200.37	0	**30648.797**	1202.36	0	49587.578	1200.02	0
400	3	d	74116.867	1200.02	0	49152.67	1200.01	0	**43045.61**	1200.02	0
400	3	e	76396.694	1200.29	0	**33462.715**	1202.76	0	46378.14	1200.02	0
400	5	a	102350.709	1200.63	0	**84775.503**	1200.01	0	90284.267	1200.01	0
400	5	b	101424.225	1200.54	0	**75254.005**	1200.02	0	79370.322	1200.01	0
400	5	c	131755.977	1200.27	0	62497.997	1200.03	0	**62203.051**	1200.03	0
400	5	d	93173.214	1200.46	0	90133.798	1200.02	0	**71567.544**	1200.01	0
400	5	e	124530.206	1200.49	0	87065.397	1200.01	0	**82230.65**	1200.01	0
400	10	a	157416.293	1202.32	0	160893.06	1200.01	0	**154393.888**	1200.01	0
400	10	b	164721.96	1206.33	0	168545.428	1200.01	0	**141098.564**	1200.01	0
400	10	c	192125.462	1200.09	0	**140819.311**	1200.01	0	141719.199	1200.01	0
400	10	d	182752.591	1200.04	0	139539.673	1200.01	0	**129064.43**	1200.01	0
400	10	e	**147370.509**	1200.23	0	151272.489	1200.01	0	153404.239	1200.01	0
400	15	a	**186635.482**	1200.61	0	202358.096	1200	0	198905.838	1200.03	0
400	15	b	262613.021	1200.51	0	**193614.044**	1200.02	0	199343.226	1200.03	0
400	15	c	293718.029	1200.17	0	206083.786	1200	0	**171842.428**	1200.03	0
400	15	d	281342.361	1200.11	0	263662.225	1200.02	0	**213618.833**	1200.03	0
400	15	e	219797.407	1200.77	0	240635.998	1200.01	0	**192171.991**	1200.02	0
400	20	a	359727.164	1201.27	0	308544.548	1200.04	0	**195824.004**	1200.05	0
400	20	b	397111.02	1201.1	0	279531.291	1200.03	0	**231497.352**	1200.04	0
400	20	c	386961.468	1201.03	0	305364.634	1200.04	0	**215117.468**	1200.05	0
400	20	d	345029.688	1201	0	346388.905	1200.04	0	**222724.931**	1200.01	0
400	20	e	406917.962	1201.5	0	359173.207	1200.01	0	**235989.533**	1200.06	0

B CP models

B.1 CP model od Kojic

Following the notation of the model of Kojic, a CP model can be given by

$$\min r \in [0, +\infty) \tag{1}$$

s.t.

$$\bigwedge_{l=1}^{m} \left(-0.5 \cdot r + \sum_{i=1}^{n} v_{il} x_i \leq s_l \right) \tag{2}$$

$$\bigwedge_{l=1}^{m} \left(0.5 \cdot r + \sum_{i=1}^{n} v_{il} x_i \geq s_l \right) \tag{3}$$

$$(\forall i \in \{1, \ldots, n\})(x_i \in \{0, 1\}). \tag{4}$$

B.2 CP model of Faria

Following the notation of Faria model used in the paper, a CP model can be given by

$$\min r \in [0, +\infty) \tag{5}$$

s.t.

$$\bigwedge_{i=1}^{n} \left(\sum_{j=1}^{k} x_{ij} = 1 \right) \tag{6}$$

$$\bigwedge_{j=1}^{k} \left(\sum_{i=1}^{n} x_{ij} \geqslant 1 \right) \tag{7}$$

$$\bigwedge_{l=1}^{m} \bigwedge_{j_1=1}^{k-1} \bigwedge_{j_2=j_1+1}^{k} \left(\sum_{i=1}^{n} v_{il} (x_{ij_1} - x_{ij_2}) \leq r \right) \tag{8}$$

$$\bigwedge_{l=1}^{m} \bigwedge_{j_1=1}^{k-1} \bigwedge_{j_2=j_1+1}^{k} \left(\sum_{i=1}^{n} v_{il} (x_{ij_2} - x_{ij_1}) \leq r \right) \tag{9}$$

$$(\forall i \in \{1, \ldots, n\})(\forall j \in \{1, \ldots, k\})(x_{ij} \in \{0, 1\}). \tag{10}$$

B.3 CP model of New-1

Following the notation of New-1 model used in the paper, a CP model can be given by

$$\min \quad r \in [0, +\infty) \tag{11}$$

s.t.

$$\bigwedge_{i=1}^{n} \left(\sum_{j=1}^{k} x_{ij} = 1 \right) \tag{12}$$

$$\bigwedge_{j=1}^{k} \bigwedge_{l=1}^{m} \left(\sum_{i=1}^{n} v_{il} \cdot x_{ij} = t_{lj} \right) \tag{13}$$

$$\bigwedge_{j=1}^{k} \bigwedge_{l=1}^{m} (t_{lj} \leq y_l), \tag{14}$$

$$\bigwedge_{j=1}^{k} \bigwedge_{l=1}^{m} (t_{lj} \geq z_l), \tag{15}$$

$$\bigwedge_{l=1}^{m} (y_l - z_l \leq r), \tag{16}$$

$$(\forall i \in \{1, \ldots, n\})(\forall j \in \{1, \ldots, k\})(x_{ij} \in \{0, 1\}) \tag{17}$$

$$(\forall l \in \{1, \ldots, m\})(y_l, z_l \in \mathbb{R}) \tag{18}$$

$$(\forall l \in \{1, \ldots, m\})(\forall j \in \{1, \ldots, k\})(t_{lj} \in \mathbb{R}). \tag{19}$$

B.4 CP model of New-2

Following the notation of New-2 model used in the paper, and the corresponding CP model for New-1 model, instead of constraints (14)–(16), we introduced the following:

$$\bigwedge_{j=1}^{k} \bigwedge_{l=1}^{m} \left(\sum_{i=1}^{n} v_{il} \cdot x_{ij} \leq y_l \right), \tag{20}$$

$$\bigwedge_{j=1}^{k} \bigwedge_{l=1}^{m} \left(\sum_{i=1}^{n} v_{il} \cdot x_{ij} \geq z_l \right). \tag{21}$$

where variables $y_l, z_l \in \mathbb{R}$, $l \in \{1, \ldots, m\}$, are new variables as introduced in the MILP model New-2.