A Beam Search Approach to the Traveling Tournament Problem

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Nikolaus Frohner, Bernhard Neumann, and Günther R. Raidl

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ALGORITHMS AND
COMPLEXITY GROUP

Traveling Tournament Problem

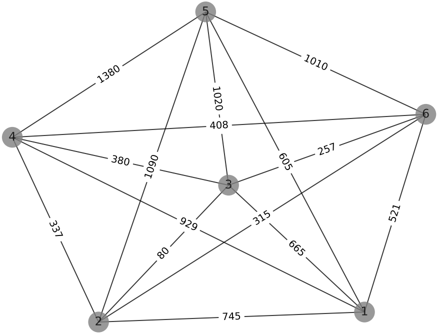
(Easton, Nemhauser, and Trick, 2001)

Given n teams and a distance function d, the goal is to schedule a double round robin tournament T, so that the total travel distance over all teams

$$f(T) = \sum_{i=1}^{n} \left(d(i, x_i^1) + \sum_{r=2}^{2n-2} d(x_i^{r-1}, x_i^r) + d(x_i^{2n-2}, i) \right) \quad (1)$$

is minimized, where each team i has its home venue at i and is at position x_i^r in round r.

Example Instance and Solution



$$
\begin{pmatrix}
5 & -3 & 2 & 6 & -1 & -4 \\
-3 & 6 & 1 & 5 & -4 & -2 \\
-2 & 1 & -4 & 3 & -6 & 5 \\
-5 & 3 & -2 & -6 & 1 & 4 \\
4 & -5 & 6 & -1 & 2 & -3 \\
2 & -1 & 4 & -3 & 6 & -5 \\
6 & -4 & 5 & 2 & -3 & -1 \\
-4 & 5 & -6 & 1 & -2 & 3 \\
-6 & 4 & -5 & -2 & 3 & 1 \\
3 & -6 & -1 & -5 & 4 & 2
\end{pmatrix}
$$

Constraints

A game i at j in round r is denoted as $i \to^r j$.

- no-repeat: $i \to^r j$ disallows $j \to^{r+1} i$.
- at-most: not more than U games are allowed to be played by a team consecutively away or consecutively at home.

Hardness

Problem gained a lot of attention, also with focus on exact approaches, e.g., combining IP/CP via branch and price, iterative deepening A*.

Theoretical: NP-completeness shown for decision variant of TTP in 2011 by Thielen and Westphal.

Empirical: Standard benchmark instances solved to optimality for ten teams but not for twelve.

Previous Work with Metaheuristics

Therefore: much previous work applying metaheuristics to the problem, since it is a nice benchmark. Relevant subset for us:

- Simulated annealing (TTSA) by Anagnostopoulos, Laurent, Van Hentenryck, and Vergados in 2006.
- Composite-neighborhood tabu search (CNTS) by Di Gasparo and Schaerf in 2007.
- Population-based Simulated annealing (PBSA) by Van Hentenryck and Vergados in 2007.
- Ant-Colony Optimization (AFC-TTP) by Uthus, Riddle, and Guesgen in 2009.

Best solutions found for NL*n* instances by PBSA, see leaderboard at `https://mat.tepper.cmu.edu/TOURN/` maintained by Michael Trick.

Outline of our Approach

- State space formulation to construct solutions.
- Layer-wise state graph traversal.
- Beam search guided by lower bounds derived from state.
- Memory-limited beam search variant to crank up beam width.
- Randomized multi-start beam search to diversify search.

Example Transition

Partial Solution

$$\begin{pmatrix} 5 & -3 & 2 & 6 & -1 & -4 \\ -3 & 6 & 1 & 5 & -4 & -2 \\ -2 & 1 & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \end{pmatrix}$$

State

$$P^* = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

$$\mathbf{x} = \begin{pmatrix} 2 \\ 2 \\ 3 \\ 4 \\ 4 \\ 2 \end{pmatrix} \mathbf{o}^* = \begin{pmatrix} 1 \\ 3 \\ 3 \\ 3 \\ 1 \\ 1 \end{pmatrix} \mathbf{h}^* = \begin{pmatrix} 3 \\ 1 \\ 1 \\ 1 \\ 3 \\ 3 \end{pmatrix}$$

Example Transition

Partial Solution

$$\begin{pmatrix} 5 & -3 & 2 & 6 & -1 & -4 \\ -3 & 6 & 1 & 5 & -4 & -2 \\ -2 & 1 & -4 & 3 & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \end{pmatrix}$$

State

$$P^* = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

$$\mathbf{x} = \begin{pmatrix} 2 \\ 2 \\ 4 \\ 4 \\ 4 \\ 2 \end{pmatrix} \mathbf{o}^* = \begin{pmatrix} 1 \\ 3 \\ 2 \\ 3 \\ 1 \\ 1 \end{pmatrix} \mathbf{h}^* = \begin{pmatrix} 3 \\ 1 \\ 3 \\ 0 \\ 3 \\ 3 \end{pmatrix}$$

Beam Search

Apply beam search on the state graph to keep the number of expanded nodes polynomially bounded $\mathcal{O}(n^2\beta)$, where β is the beam width.

Guided by f-value of each state s, the sum of the currently shortest path length plus a lower bound function depending on the state:

$$f(s) = g(s) + b(s) \tag{2}$$

Randomized Beam Search

To diversify search, we also consider a randomized multi-start beam search:

- Random team ordering.
- Add Gaussian noise to each f-value: $\tilde{f}(s) = f(s) + \mathcal{N}(0, \sigma)$.

Variance determined by tunable σ_{rel} scaling factor for the lower bound value of the root state s^{r}:

$$\sigma = \sigma_{\mathrm{rel}} \cdot b(s^{\mathrm{r}}) \tag{3}$$

Independent Lower Bound (ILB)

Already introduced in initial paper by Easton et al., extended to general states.

For a given state s, consider each team independently and its away games, away streak, and current position, and solve a corresponding CVRP problem (demand of each customer 1, capacity of each truck U).

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Sum independent bounds over all teams:

$$b^{\mathrm{CVRP}}(s) = \sum_{i=1}^{n} b_i^{\mathrm{CVRP}}(s) \qquad (4)$$

CVRPH bound

(Uthus et al., 2012)

Have to have the bound for each state occurring in our beam search, so it is not enough to calculate an optimal solution for the root state.

Constraints occur for the number of "trucks" we have to use at least/at most, due to required/available home stands → CVRPH bound.

Example: $\mathcal{HHHAHHH}$

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Example: $\mathcal{HHHAHHH}|\mathcal{AAHAAHAA}$

Optimal completion for CVRP.

CVRPH bound

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Example: $\mathcal{HHHAHHH} \mid \mathcal{AAHAA}\not{\mathcal{H}}\mathcal{AA}$

But we do not have enough home games left.

CVRPH bound

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Tightening the bound.

CVRPH bound

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Improvement: We precalculate CVRPH by constructing for each team an exact DD for the CVRP, do a backward sweep to calculate constrained shortest path lengths acting as lower bounds for a given state, and finally store the lower bounds into a lookup table.

Memory Demand & Runtimes Lookup Tables

Table: Memory demand in GB

n	TSP	CVRP	CVRPH
14	0.003	0.009	0.127
16	0.016	0.047	0.75
18	0.079	0.237	4.27
20	0.39	1.172	23.43

Table: Runtimes in minutes

	14	16	18
NLn	25	169	-
CIRCn	25	173	903

CVRPH bounds = number of teams × number of subsets of away teams × number of positions × possible streak values × number of home stands × bytes for bound value $= n2^{n-1}nUn \times 2 = \mathcal{O}(n^3 2^n)$.

Computational Study

- Single-threaded Intel Xeon E5-2640 with 2.40 GHz
- 32GB memory limit
- Python 3.7
- 180 randomly generated instances on 1000×1000 grid for comparing bounds and tuning.
- NL and CIRC classical benchmark instances up to 18 teams for comparison with other approaches.
- 30 multiple runs in parallel for noisy beam search variant.

Results Tuning Instances

	$\beta = 1000$		
class	SHORT	CVRP	CVRPH
$\mathcal{I}_{L^1}^{8}$	42532 ± 5384	40530 ± 5214	40405 ± 5030
$\mathcal{I}_{L^1}^{10}$	70049 ± 7280	65483 ± 6886	64760 ± 6689
$\mathcal{I}_{L^1}^{12}$	99086 ± 7991	92838 ± 8089	91728 ± 7726
$\mathcal{I}_{L^2}^{8}$	34412 ± 5088	33034 ± 5109	32965 ± 5071
$\mathcal{I}_{L^2}^{10}$	55019 ± 5872	51723 ± 5988	51269 ± 5808
$\mathcal{I}_{L^2}^{12}$	79699 ± 7293	74231 ± 6933	73700 ± 6456

Wilcoxon signed rank sum test shows that CVRPH is significantly better than CVRP with a significance level of $\alpha = 1\%$.

Results Benchmark Instances

inst	RBS-CVRPH		RBS-CVRPH-RTO		AFC-TTP		PBSAFS		PBSAHQ	
	min	mean	min	mean	min	mean	min	mean	min	mean
nl12	112680	113594.6	112791	113581.5	112521	114427.4	**110729**	**112064.0**	n/a	n/a
nl14	192625	198912.6	196507	199894.8	195627	197656.6	**188728**	190704.6	**188728**	**188728.0**
nl16	266736	271367.1	265800	270925.9	280211	283637.4	**261687**	265482.1	262343	**264516.4**
circ12	410	415.7	410	414.6	430	436.0	**404**	418.2	408	**414.8**
circ14	632	641.0	**630**†	**640.7**	674	692.8	640	654.8	632	645.2
circ16	918	933.8	**910**†	931.6	1034	1039.6	958	971.8	916	**917.8**
circ18	1300	1322.0	1296	1320.4	1486	1494.8	1350	1371.6	**1294**	**1307.0**

Comparison with state of randomized beam search approach with $\beta = 10^5$ and reported solutions lengths of state of the art methods. †New best feasible solutions.

Runtimes

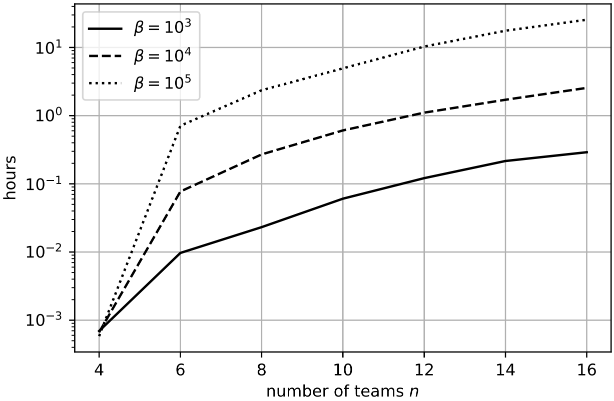

Figure: Runtimes in hours for deterministic beam search runs on NL instances with $\beta \in \{10^3, 10^4, 10^5\}$.

Conclusion

Using beam search on a state graph with large beam widths and guidance by the independent lower bound is a viable option to construct good solutions for the TTP.

Contributions (to the best of our knowledge):

- Novel state-space formulation for the TTP.
- Memory-limited randomized beam search approach to the TTP.
- Memory-demand reduction for the CVRPH bound to tackle instances up to 18 teams using decision diagrams.
- New best feasible solutions for the circ14 and circ16 instances (last improvement 2007).

Outlook

- Fast and memory-efficient implementation in a compiled language to go for even higher beam widths of 10^6.
- Other beam search guidance approaches, to tackle larger instances up to 40 teams, since there is no chance to exactly enumerate whole state space for CVRPH lower bounds, recall $\mathcal{O}(n^3 2^n)$, possibly using relaxed decision diagrams.
- Share information between parallel beam search runs to make avoid redundancies in state space exploration.
- Hybridize with local search based approaches.

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Thank you

Feel free to contact me:
`nfrohner@ac.tuwien.ac.at`.