

# Selecting User Queries in Interactive Job Scheduling<sup>1</sup>

Johannes Varga<sup>a</sup>, Günther R. Raidl<sup>a</sup>, Tobias Rodemann<sup>b</sup>

<sup>a</sup>Institute of Logic and Computation, TU Wien, Vienna, Austria

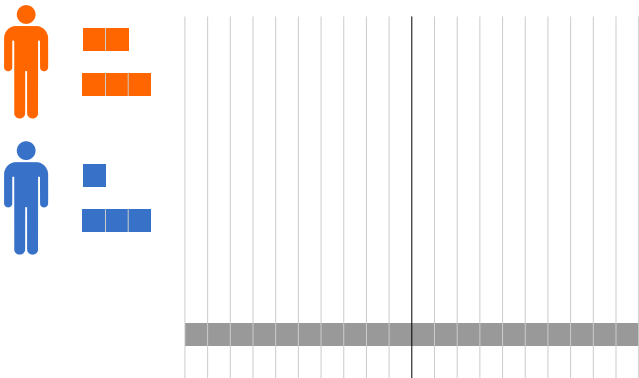
<sup>b</sup>Honda Research Institute Europe, Offenbach, Germany

Feb 27, 2024

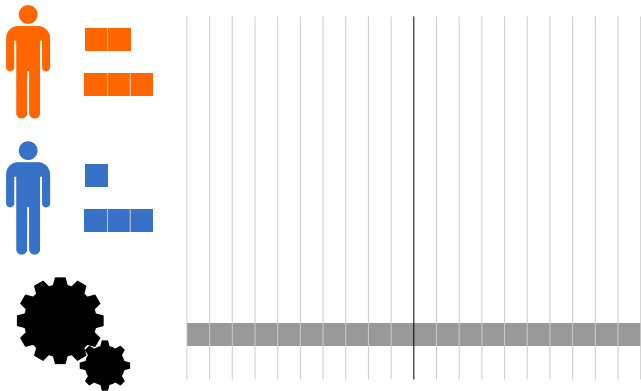
---

<sup>1</sup>J. Varga acknowledges the financial support from Honda Research Institute Europe.

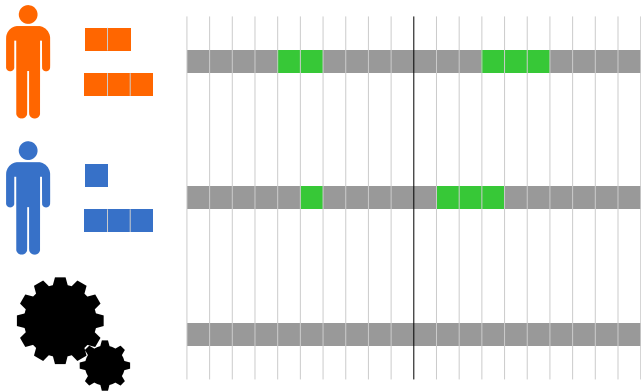
# Problem Setting



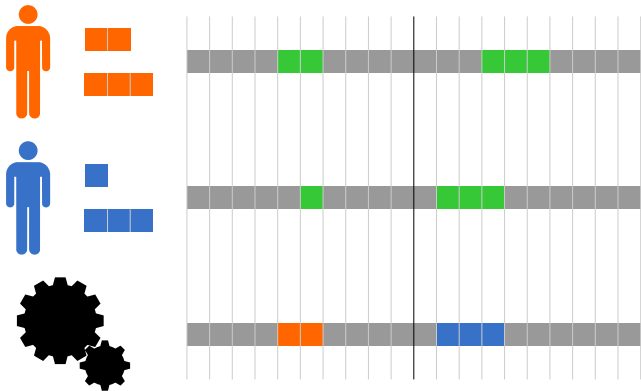
# Problem Setting



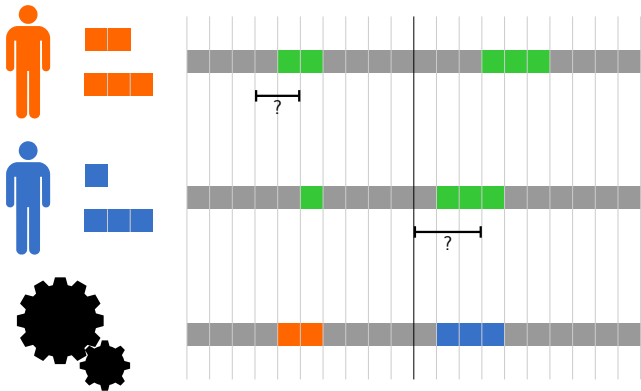
# Problem Setting



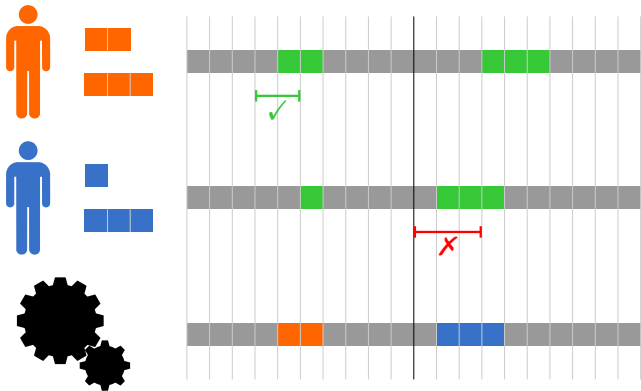
# Problem Setting



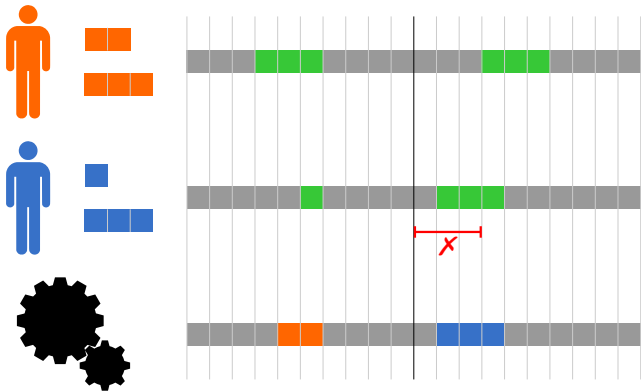
# Problem Setting



# Problem Setting

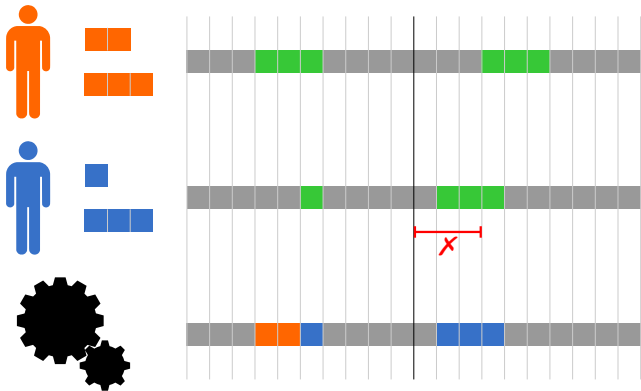


# Problem Setting

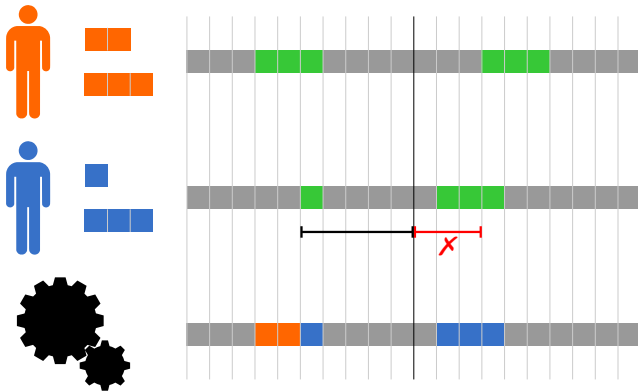




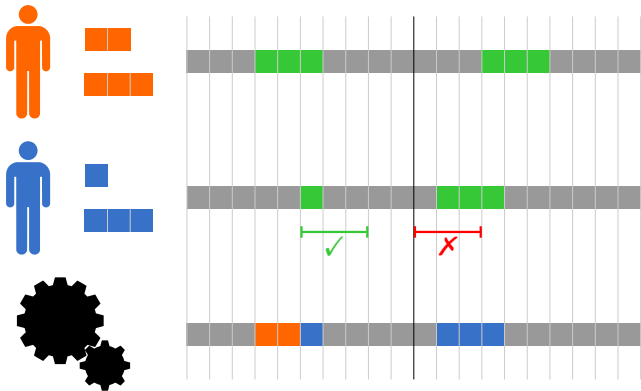
# Problem Setting



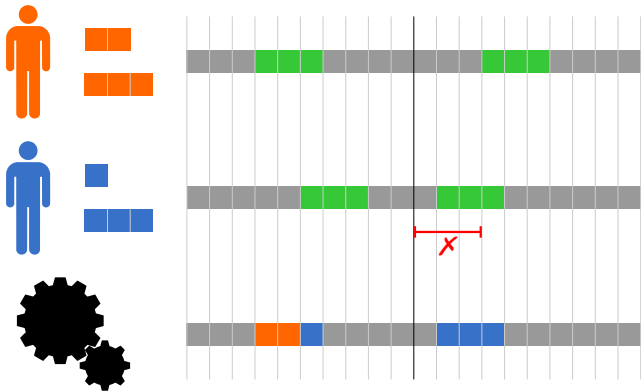
# Problem Setting



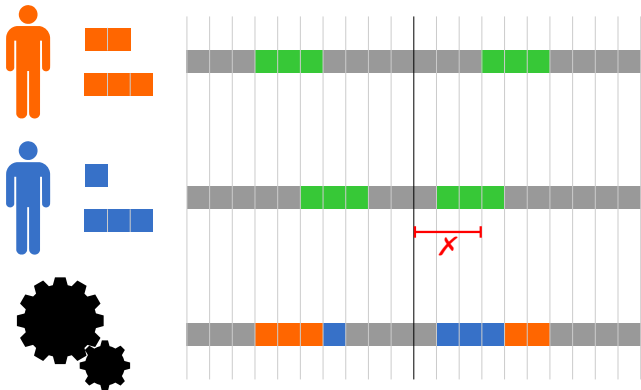
# Problem Setting



# Problem Setting



# Problem Setting



# Problem

Core problem:

- Discrete time planning horizon of multiple days
- Multiple users
- Multiple jobs per user
- one machine
- Schedule jobs non-preemptively on machine

Objective:

- time dependent costs  $c_t$  for using the machine at timestep  $t$
- penalty  $q_j$  for not scheduling a job  $j$
- not scheduling a job is more expensive than scheduling it

# Problem

## User availabilities:

- Limit job running times
- Only **partially known**
- Complement knowledge with **interaction**

## Interaction:

- **$B$  rounds** of interaction
- each with up to  **$b$  queries**
- Query types: **Yes/No** and **Timeframe**

## Application examples:

- Human resource planning (e.g. scheduling of lectures)
- EV Charging

# Literature

Considered problem setting in **previous work**<sup>2</sup>

Neglect users: Job Scheduling Problem  $Pm||TEC$

- MILP and various **heuristics** (greedy, genetic algorithm, local search) for  $Pm||C_{max}, TEC$ <sup>3</sup>
- MILP and **Dantzig-Wolfe decomposition** for  $Rm||TEC$ <sup>4</sup>
- Improved MILP<sup>5</sup>

---

<sup>2</sup>Varga et al. 2023; Varga et al. 2024.

<sup>3</sup>Wang et al. 2018; Anghinolfi, Paolucci, and Ronco 2021.

<sup>4</sup>Ding et al. 2016.

<sup>5</sup>Cheng, Chu, and Zhou 2018; Saberi-Aliabad, Reisi-Nafchi, and Moslehi 2020.



# Probabilistic User Models

Criteria for queries:

1. Good response likely → Model users in probabilistic way
2. Improve the schedule → Optimize

Given:

- Known availabilities  $\mathcal{T}^{\text{avail}}$
- Rejected time intervals  $I^{\text{rej}}$

Compute:

- Acceptance probability of queries
- Reasonable user availabilities

Single user, single day

# Markov-Model<sup>6</sup>

Model availabilites with **Markov process**

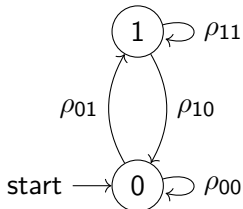


Figure: Two-state Markov Chain

---

<sup>6</sup>Varga et al. 2023.

## Advanced model<sup>7</sup>

Assumptions:

- User available in  $\leq 2$  intervals
- Morning interval: start 9am $\pm$ 1h, duration 4h $\pm$ 1h, probability 90%
- Afternoon interval: start 1pm $\pm$ 1h, duration 5h $\pm$ 1h, probability 90%

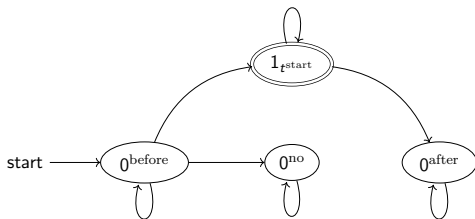


Figure: State diagram for single interval.

---

<sup>7</sup>Varga et al. 2024.

# Probability Threshold<sup>8</sup>

Query candidates

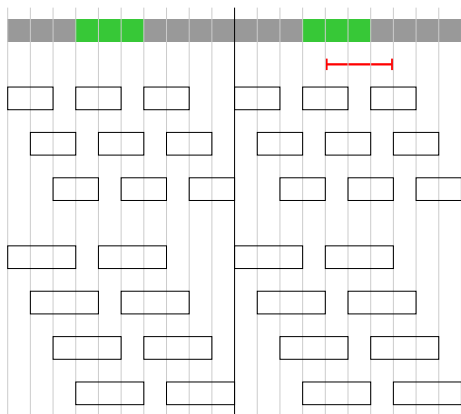


Figure: Procedure in each round

---

<sup>8</sup>Varga et al. 2023.

# Probability Threshold<sup>8</sup>

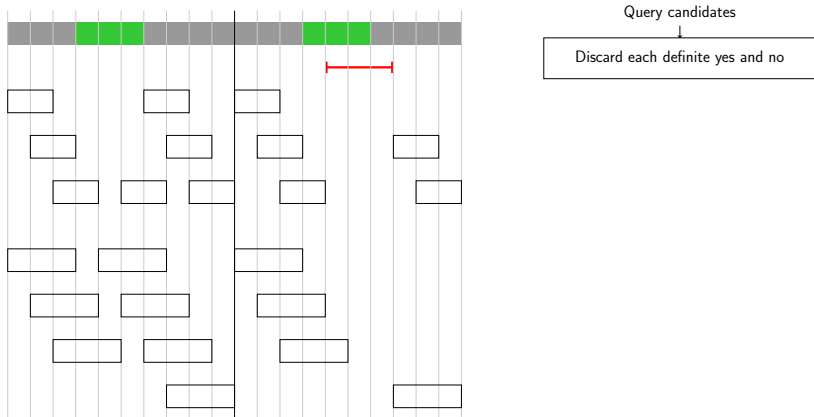


Figure: Procedure in each round

<sup>8</sup>Varga et al. 2023.



# Probability Threshold<sup>8</sup>

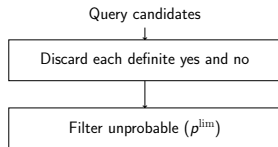
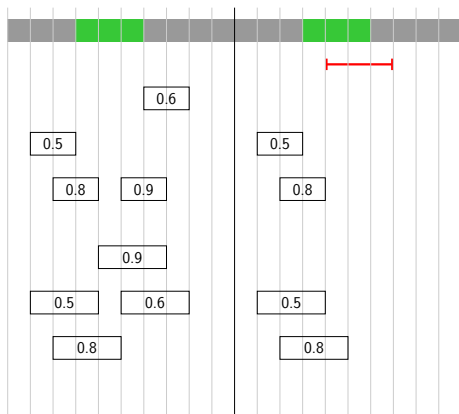


Figure: Procedure in each round

<sup>8</sup>Varga et al. 2023.

# Probability Threshold<sup>8</sup>

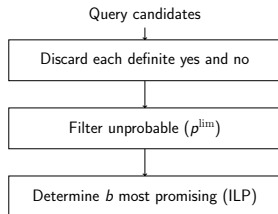
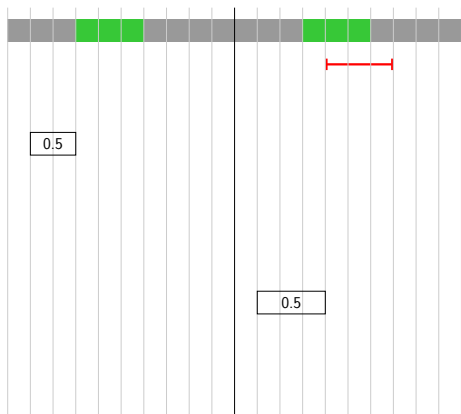


Figure: Procedure in each round

---

<sup>8</sup>Varga et al. 2023.



# Probability Threshold<sup>8</sup>

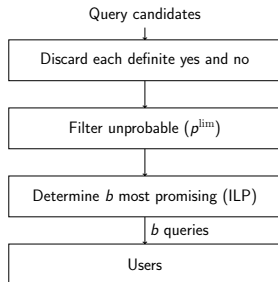
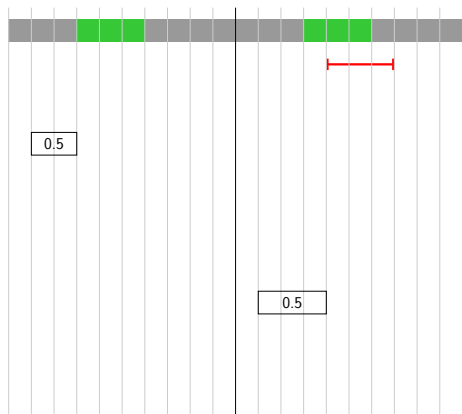


Figure: Procedure in each round

<sup>8</sup>Varga et al. 2023.

# Probability Threshold<sup>8</sup>

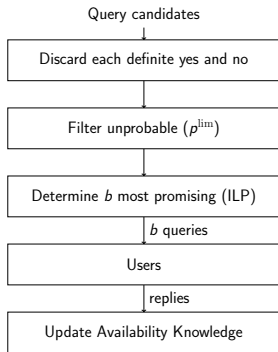
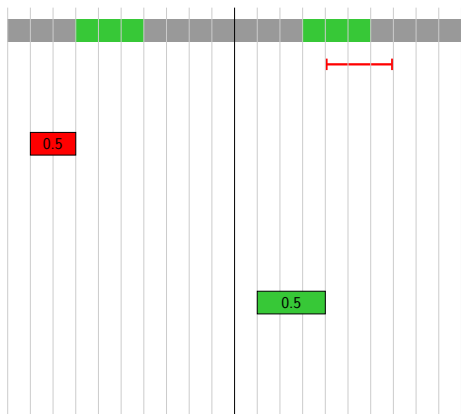


Figure: Procedure in each round

<sup>8</sup>Varga et al. 2023.

# Probability Threshold<sup>8</sup>

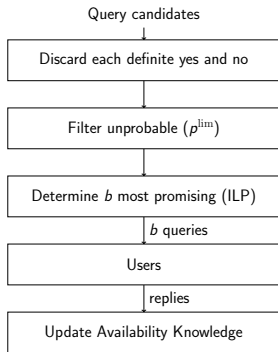
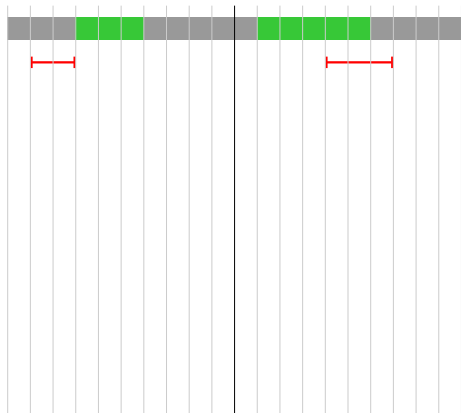


Figure: Procedure in each round

<sup>8</sup>Varga et al. 2023.

# Probability Threshold<sup>8</sup>

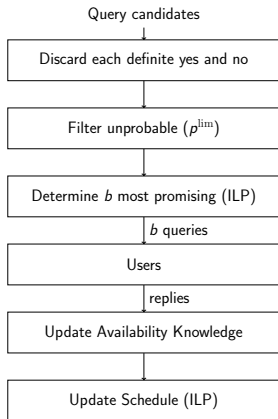
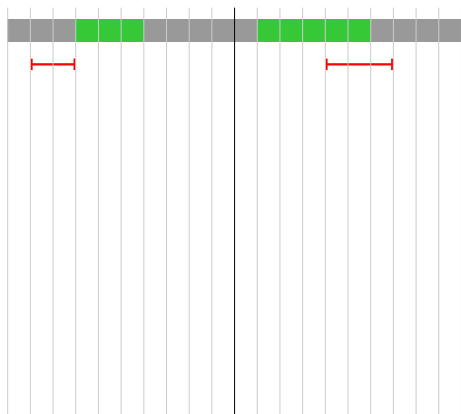


Figure: Procedure in each round

---

<sup>8</sup>Varga et al. 2023.

# Stochastic Programming

Possible queries for each job:

- New starting time
- Half-day timeframe (6am-2pm, 2pm-10pm)

Two stages:

1. Select queries
2. Expected costs

$$\begin{aligned} \min \quad & \mathbb{E}_{T^{\text{avail}^*}}(\text{ILP}(\mathcal{T}(T^{\text{avail}^*}, s^{\text{time}}, s^{\text{frame}}))) \\ \text{s.t.} \quad & \sum_{j \in J} \left( \sum_{t \in T} s_{jt}^{\text{time}} + \sum_{f \in F} s_{jf}^{\text{frame}} \right) \leq b \\ & s_{jt}^{\text{time}} \in \{0, 1\} \quad j \in J, t \in T \\ & s_{jf}^{\text{frame}} \in \{0, 1\} \quad j \in J, f \in F \end{aligned}$$

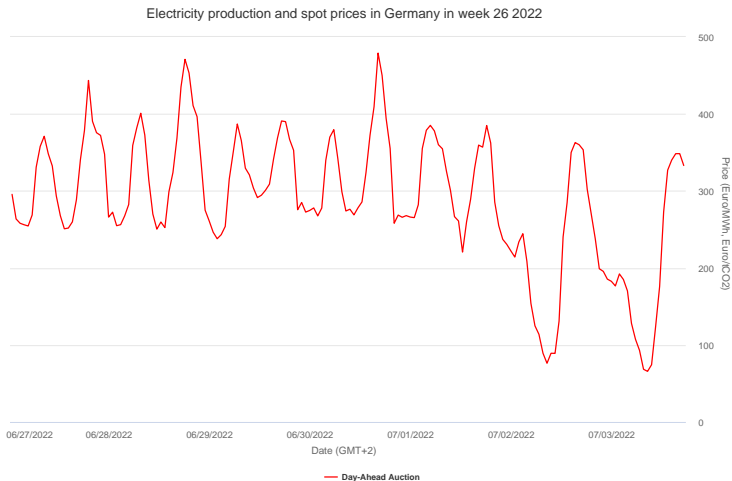
Solve: ILP for sample average approximation (70 samples)

## Instance Generation

- one machine, 24 jobs, 6 users
- 30 instances
- five rounds, each with 6 user queries
- five days (6am to 10pm), 4 timesteps per hour
- Job duration: uniformly random from [0.5h,4h]
- Users are simulated and available
  - from 9am $\pm$ 1h for 4h $\pm$ 1h with probability 0.9 and
  - from 1pm $\pm$ 1h for 5h $\pm$ 1h with probability 0.9
- Known availabilities: one random starting time chosen for each job

# Cost Function: Energy Prices in Germany

Job penalty: 40 Euro per timestep



Energy-Charts.info; Data Source: 50 Hertz, Amprion, Tennet, TransnetBW, EEX, EPEX SPOT; Last Update: 01/04/2023, 2:08 PM GMT+1

# Results 1

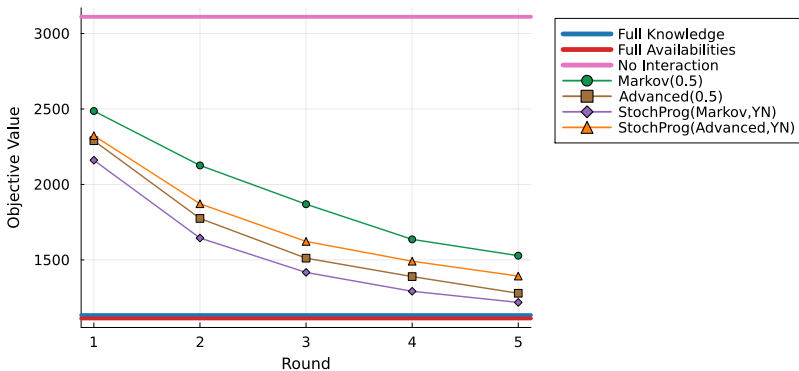


Figure: Mean objective values, 24 jobs, 6 users



## Results 2

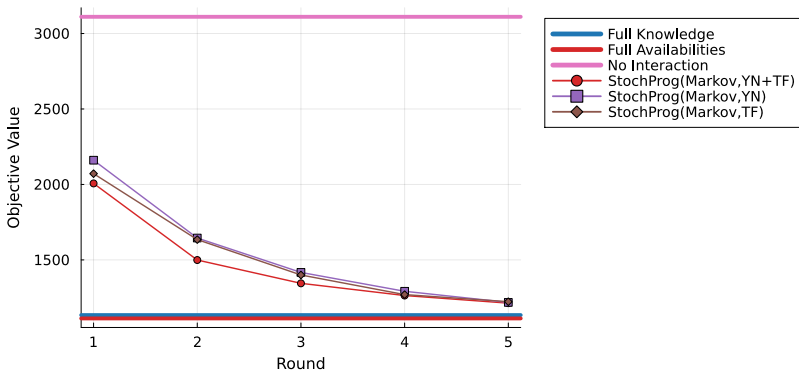


Figure: Mean objective values, 24 jobs, 6 users

## Results 3

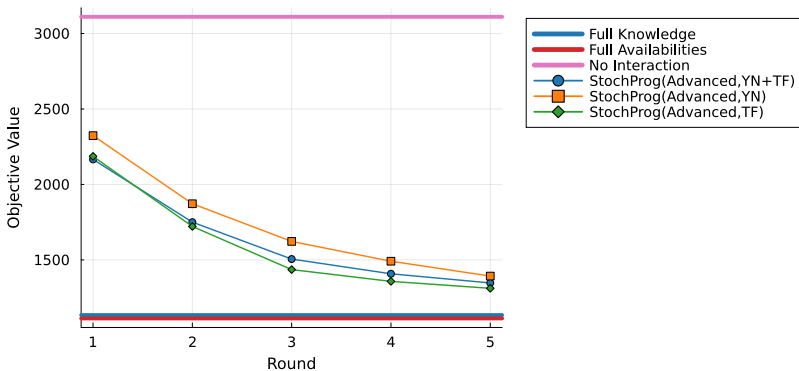


Figure: Mean objective values, 24 jobs, 6 users

## Conclusion and Future Work

### Conclusions:

- Stochastic programming significantly better than probability threshold approach
- But: harder to solve
- Query type: small influence
- Markov model better for stochastic programming

### Future Work:

- Learn user model from historic availability data
- Heuristic and/or Benders decomposition → solve larger instances

Thank you!

## ILP (Interactive Job Scheduling)

$$\begin{aligned}
 \text{ILP}(\mathcal{T}) \quad \min \quad & \sum_{j \in J} \sum_{t \in \mathcal{T}_j} \sum_{t' \in \mathcal{T}_j[t]} c_{t'} x_{jt} + \sum_{j \in J} q_j \left( 1 - \sum_{t \in \mathcal{T}_j} x_{jt} \right) \\
 \text{s.t.} \quad & \sum_{t \in \mathcal{T}_j} x_{jt} \leq 1 && j \in J \\
 & \sum_{j \in J} \sum_{t \in \mathcal{T}_j | t' \in \mathcal{T}_j[t]} x_{jt} \leq 1 && t' \in \mathcal{T} \\
 & \sum_{j \in J_u} \sum_{t \in \mathcal{T}_j | t' \in \mathcal{T}_j[t]} x_{jt} \leq 1 && u \in U, t' \in \mathcal{T} \\
 & x_{jt} \in \{0, 1\} && j \in J, t \in \mathcal{T}
 \end{aligned}$$

$J$  ... Jobs       $T$  ... Time horizon

$\mathcal{T}_j$  ... allowed starting times for job  $j$

$x_{jt}$  ... 1, iff job  $j$  starts at timestep  $t$

$\mathcal{T}_j[t]$  ... timesteps job  $j$  runs in when started at timestep  $t$

## Second Stage of Probabilistic Program

$n^{\text{samples}}$  instances of ILP( $T^{\text{job}}$ ) with variables  $x_{jt}^{(k)}$  and additional constraints

$$x_{jt}^{(k)} = 0 \quad j \in J, t \in T \setminus T_j^{\text{pos},(k)}$$

$$s_{jt}^{\text{time}} + \sum_{f \in F_j^{\text{pos},(k)} \mid t_{jf}^{\text{reply},(k)} = t} s_{jf}^{\text{frame}} \geq x_{jt}^{(k)}$$

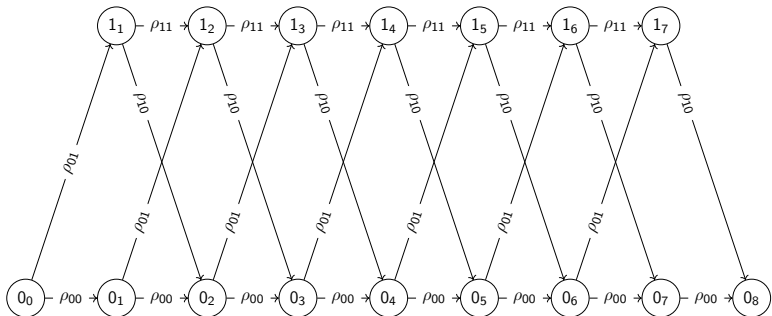
$$u \in U, j \in J_u, t \in T, T_j[t] \not\subseteq T_u^{\text{avail}}$$

# Computing Environment

- Julia 1.9.3
- Gurobi 10.0.3 (single-threaded) via JuMP
- Single core of Intel Xeon E5-2640 v4
- 60min timelimit for Gurobi

# Probability Calculation: Graph

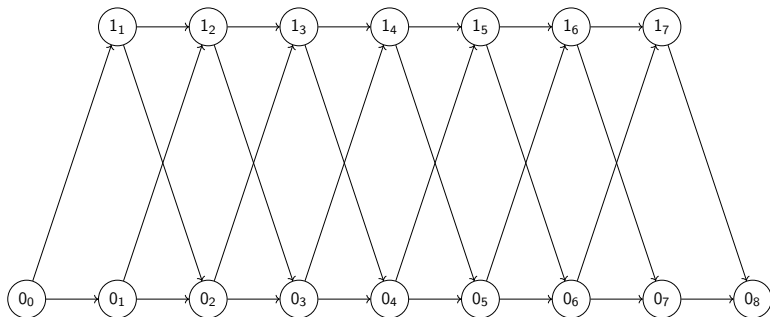
E.g. day with 7 timesteps,  $T^{\text{avail}} = \{5, 6\}$ ,  $I^{\text{rej}} = \{[1, 5], [3, 7]\}$





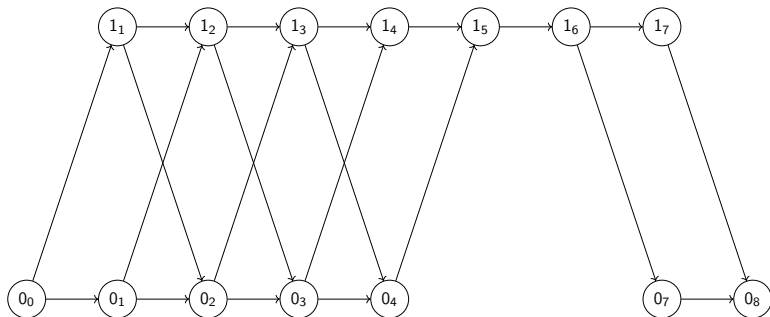
## Probability Calculation: Graph

E.g. day with 7 timesteps,  $T^{\text{avail}} = \{5, 6\}$ ,  $I^{\text{rej}} = \{[1, 5], [3, 7]\}$



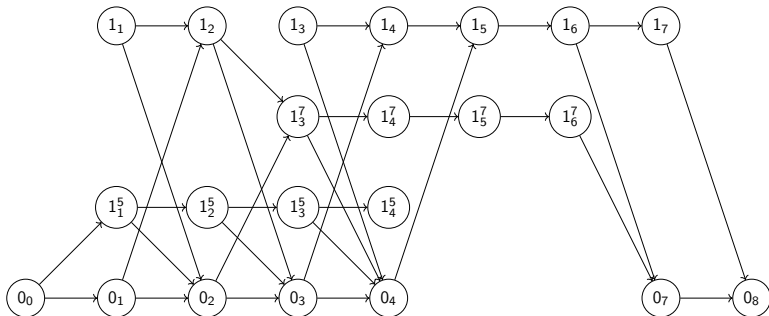
## Probability Calculation: Graph

E.g. day with 7 timesteps,  $T^{\text{avail}} = \{5, 6\}$ ,  $I^{\text{rej}} = \{[1, 5], [3, 7]\}$



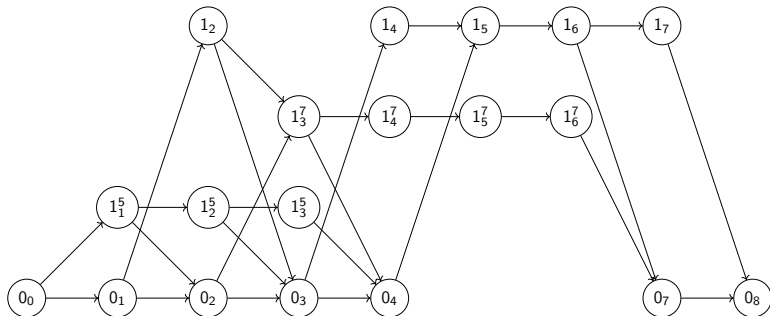
## Probability Calculation: Graph

E.g. day with 7 timesteps,  $T^{\text{avail}} = \{5, 6\}$ ,  $I^{\text{rej}} = \{[1, 5], [3, 7]\}$



## Probability Calculation: Graph

E.g. day with 7 timesteps,  $T^{\text{avail}} = \{5, 6\}$ ,  $I^{\text{rej}} = \{[1, 5], [3, 7]\}$



## Probability Calculation

$$\begin{aligned}
 p_{0_0, v}^{\text{path}} &= \sum_{P \in \text{Paths}(0_0, v)} \prod_{(w, w') \in P} \rho(w, w') \\
 &= \sum_{u \in N^-(v)} \sum_{P \in \text{Paths}(0_0, u)} \left( \prod_{(w, w') \in P} \rho(w, w') \right) \cdot \rho(u, v) \\
 &= \sum_{u \in N^-(v)} p_{0_0, u}^{\text{path}} \rho(u, v) \\
 p_{v, 0_{t_{\max}+1}}^{\text{path}} &= \sum_{w \in N^+(v)} p_{w, 0_{t_{\max}+1}}^{\text{path}} \rho(v, w)
 \end{aligned}$$

## Probability Calculation

Probability that  $[\tau, \tau']$  will be accepted:

$$p^{\text{avail}}([\tau, \tau'] \mid \mathcal{T}^{\text{avail}}, I^{\text{rej}}, 0_{t^{\text{max}+1}}) = \frac{\sum_{P \in \text{1-Paths}(\tau, \tau')} p_{0_0, P_\tau}^{\text{path}} \cdot \rho_{11}^{\tau' - \tau} \cdot p_{P_{\tau'}, 0_{t^{\text{max}+1}}}^{\text{path}}}{p_{0_0, 0_{t^{\text{max}+1}}}^{\text{path}}}$$

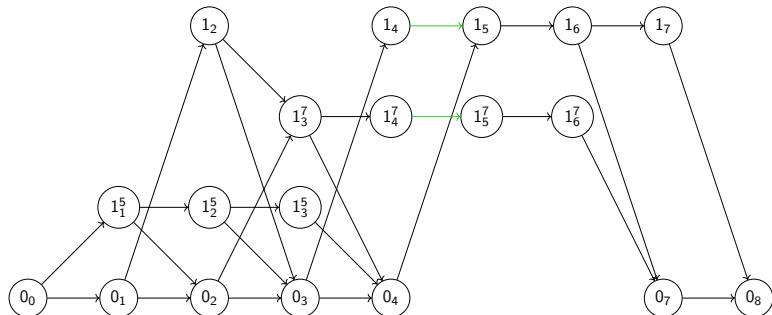


Figure: 1-Paths(4,5) for the example in green

## Results 4

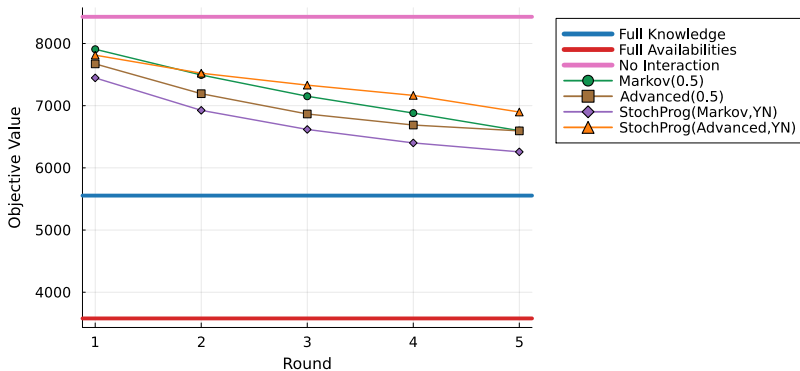


Figure: Mean objective values, 48 jobs, 12 users

## Results 5

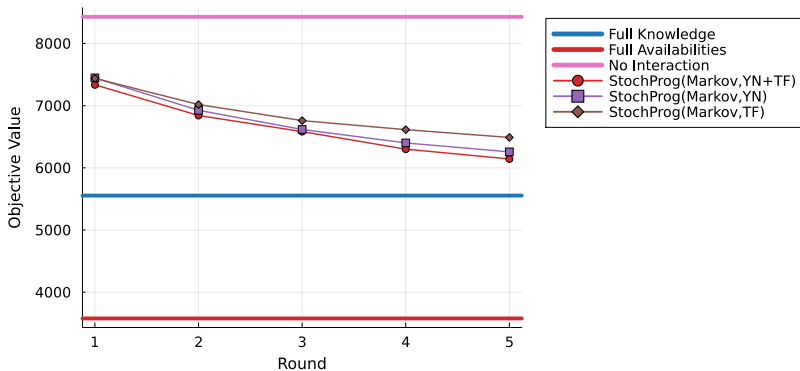


Figure: Mean objective values, 48 jobs, 12 users



## Results 6

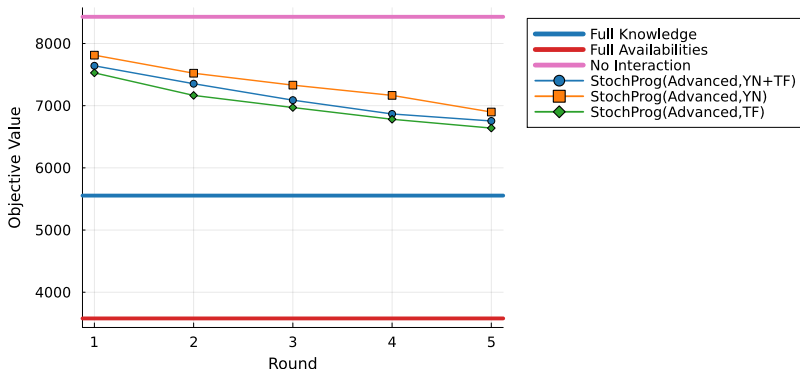


Figure: Mean objective values, 48 jobs, 12 users