A Learning Bilevel Optimization Approach for the Demand Maximizing Battery Swapping Station Location Problem

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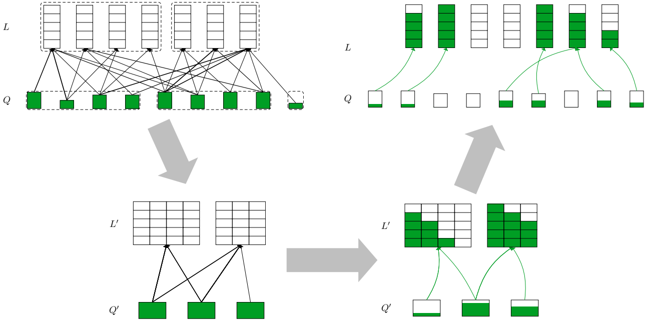
Bilevel Optimization (BLO) Approach

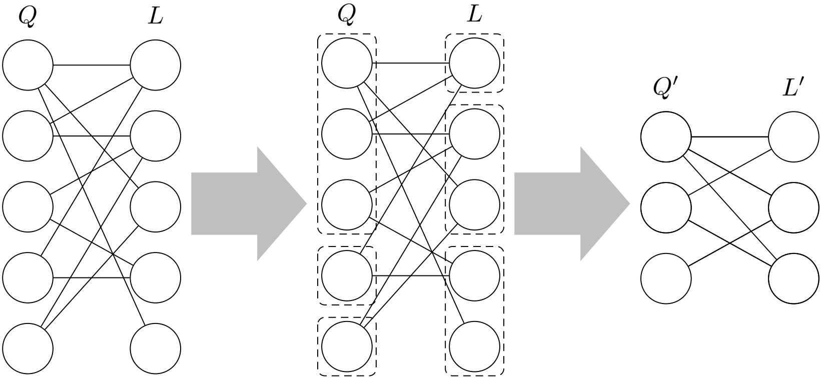
1. coarsening: reduce problem size
2. solve coarsest problem
3. projection: project solution to less coarse graphs



Bilevel Optimization (BLO) Approach

Coarsening

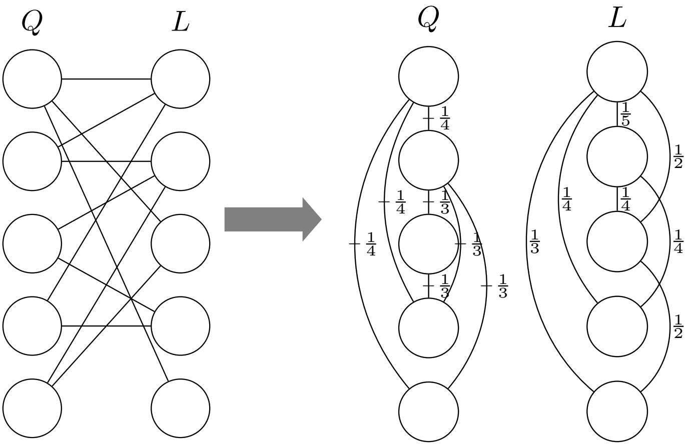
- partitioning: partition nodes of G
- contracting: aggregate nodes in each partition $\rightarrow G'$
- vertex Sets (Q, L) partitioned separately of each other



Learning Bilevel Optimization (LBLO) Approach

Similarity Calculation

- learned model: $S(v, v') = \mathrm{model}(v, v')\ldots$ machine learning similarity for L set
- heuristic approach: $S(v, v') = -\frac{|N(v) \cap N(v')|}{|N(v) \cup N(v')|}\ldots$ Jaccard similarity for Q set



Partitioning Strategies

- **k-medoids** (PAM)
  Greedy construction algorithm with 1-swap local search
- **minimum spanning tree clustering**
  Clustering by creating an mst and deleting edges of connected components until a desired size is reached
- **agglomerative clustering** with different **linkage methods**.
  clustering by partitioning the most promising nodes together
  - **minimum linkage**: use minimum weight of partitioned nodes
  - **maximum linkage**: use maximum weight of partitioned nodes
  - **mean linkage**: use mean weight of partitioned nodes
  - **dominant linkage**: use weight of the dominant node in the partitioning
- **Multi Level**: only allow pairs of nodes in a cluster. Cluster over Multiple Levels

Related Work

- Jatschka et al. (2023), EvoCOP23: Multilevel Optimization for MBSSLP
- Jatschka et al. (2020), OPTIMA20: Large Neighborhood Search for MBSSLP
- Walshaw (2002), Operations Research: Multilevel Optimization for traveling salesman problem
- Valejo et al. (2020), ACM Computing Surveys: Overview of MLO approaches

Problem Motivation

- adoption of electric vehicles steadily increasing
- major hindrance for customers: long charging times
- alternative approach: battery swapping stations
  - users replace depleted batteries with fully charged ones
  - depleted batteries are recharged and provided again later
- **Goal:** optimal setup of battery swapping station

  → The Demand Maximizing Battery Swapping Station Location Problem (DMBSSLP)

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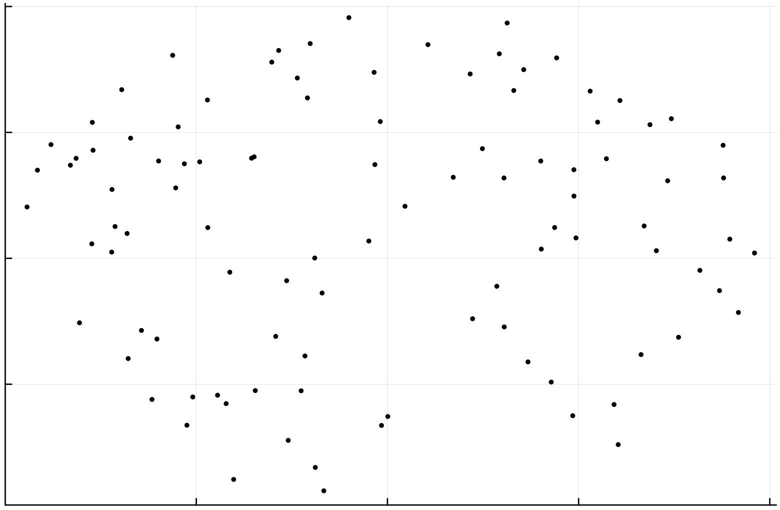
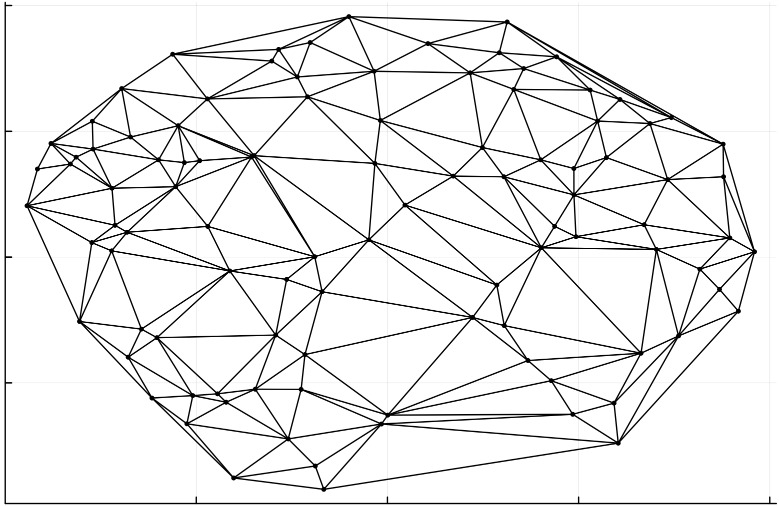
Instance and Solution Representation



Instance and Solution Representation



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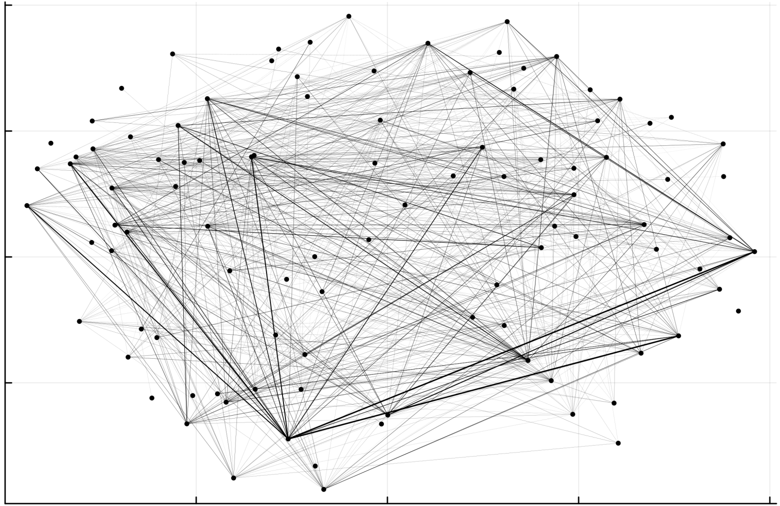

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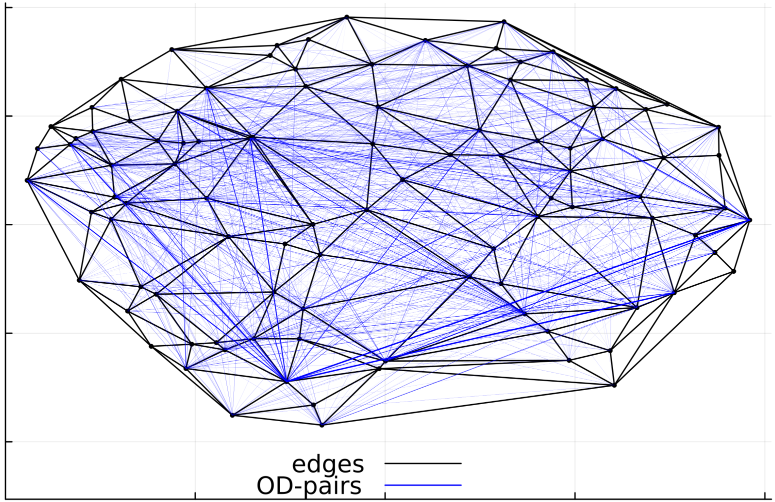

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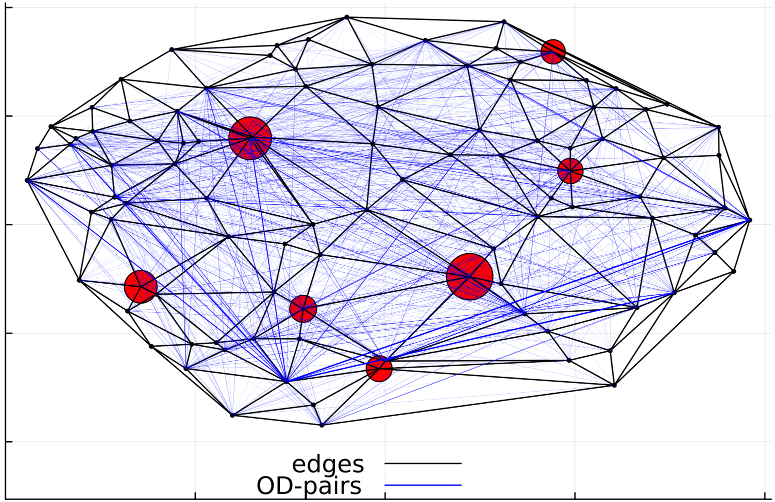
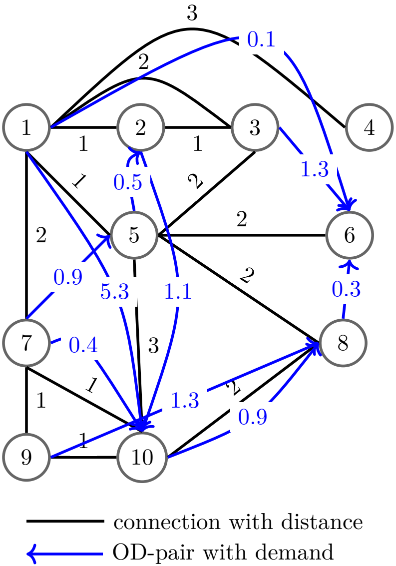


The Demand Maximizing Battery Swapping Station Location Problem (DMBSSLP)



- edge weights are distances
- nodes are possible areas for a station (#nodes = n)
- origin destination pairs (OD-pairs) constitute demand (#OD-pairs = m)
- detours impact user acceptance

Goal: decide area and capacity of a station in order to maximize satisfied demand

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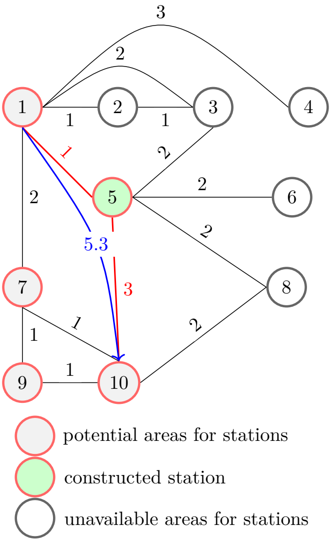
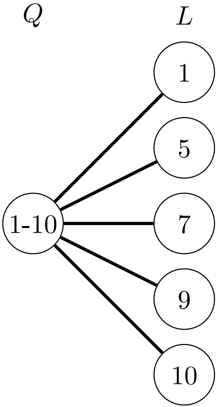


- edge weights are distances
- nodes are possible areas for a station (#nodes = n)
- origin destination pairs (OD-pairs) constitute demand (#OD-pairs = m)
- detours impact user acceptance

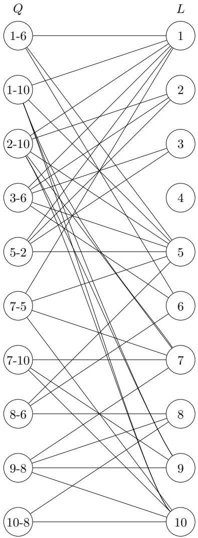
Goal: decide where to set up stations with which capacity in order to maximize satisfied demand

The Demand Maximizing Battery Swapping Station Location Problem (DMBSSLP)



- reformulation on a bipartite graph
- node set Q: OD-pairs
- node set L: areas for stations
- OD-pairs are connected to areas if a station in that area may satisfy part of the demand

The Demand Maximizing Battery Swapping Station Location Problem (DMBSSLP)



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Properties of DMBSSLP

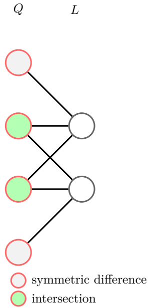
Bipartite Graph Properties

- properties of Q set
  - $\mathbf{d_q}$: demand of an OD-pair
- properties of L set
  - $\mathbf{r_l}$: number of allowed stations at area l
  - $\bar{d}_l$: The maximum demand that can be satisfied at an area l
- properties of an edge
  - e_{ql}: maximum demand that can be assigned form OD-pair q to area l
  - $\mathbf{g_{ql}}$: customer loss factor when satisfying demand of OD-pair q at area l
- global properties
  - $\mathbf{s}$: number of battery charging slots that can be built at a station
  - $\mathbf{c}$: cost of a station
  - $\mathbf{b}$: cost of a battery charging slot
  - $\mathbf{B}$: budget for the project

Model

Features

- some important features for the L neural network:
  - Δd_q: the sum of demands of OD-pairs in the symmetric difference of sets of neighboring OD-pairs
  - $\cap d_q$: the sum of demands of OD-pairs in the intersection of sets of neighboring OD-pairs
  - $\bar{d}_{l_1}, \bar{d}_{l_2}, \bar{d}_{l_3}$: the satisfiable demand at an area for the grouped nodes and the prospective merged node
  - $\frac{\sum_{q \in N(l_1)} e_{ql_1} g_{ql_1}}{\sum_{q \in N(l_1)} e_{ql_1}}, \frac{\sum_{q \in N(l_2)} e_{ql_2} g_{ql_2}}{\sum_{q \in N(l_2)} e_{ql_2}}, \frac{\sum_{q \in N(l_3)} e_{ql_3} g_{ql_3}}{\sum_{q \in N(l_3)} e_{ql_3}}$ demand weighted by the customer loss



Model

Data Collection

the trainingset contains 3000 instances of varying sizes;
1000 instances with $m = n = 1600$;
1000 instances with $m = n = 3200$;
1000 instances with $m = n = 6400$

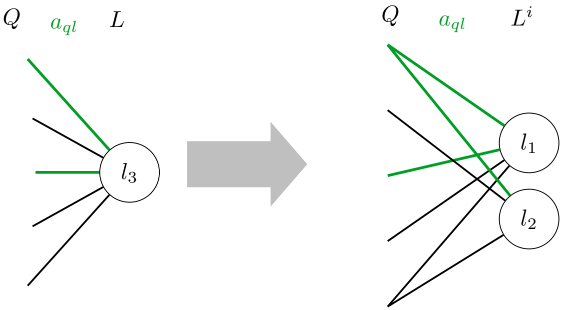
- coarsen instance by randomly merging neighboring L nodes.
- solve smaller instance using MILP
- project MILP solutions of small instance
- record features of merged nodes
- create dependent variable

Model

dependent variable

$$z_{l_1 l_2} = \left(\sum_{q \in N(l_3)} a_{ql_3}\right) - \left(\sum_{q \in N(l_3)} \tilde{a}_{ql_1} + \tilde{a}_{ql_2}\right)$$

- record difference between coarse and projected solution for individual nodes



Model
Model Parameters

Fully connected MLP
ADAM Optimizer
MSE loss with symlog transformation

$$\mathrm{symlog}(x) := \mathrm{sign}(x) \log\left(|x| + 1\right)$$

- **architecture:**
  - **width:** [40,20]; [40,40]; [80,40]; **[80,80]**; [120,80]
  - **regularization:** dropout, **no dropout**
- **learning rate:** 0.001; 0.0005; **0.0001**
- **batch size:** 64; **256**
- **number of trainings instances:** **90**; 900; 7200

Experiments

Methodology

- 9 set of 30 benchmark instances with size of $n = m = \{100, 200, 400, 800, 1600, 3200, 6400, 12800, 25600\}$
- Programming Language: Julia 1.10
- AMD EPYC 7402, 2.80GHz
- Gurobi (single threaded) 10.0 for all MI(LP) solving

Experiments

Computational Results

Average solution quality using ML-similarity or Jaccard similarity on benchmark instances

size	ML	Jaccard	ML time	Jaccard time
100	0.9274	**0.9342**	1.38s	**1.15s**
200	**0.9211**	0.9127	2.39s	**1.79s**
400	**0.9000**	0.8750	4.67s	**3.03s**
800	**0.8860**	0.8571	13.1s	**9.45s**
1600	**0.8825**	0.8614	37.5s	**16.4s**
3200	**0.8881**	0.8593	81.2s	**74.8s**
6400	**0.8817**	0.8431	331s	**268s**
12800	**0.8809**	0.8435	1240s	**1120s**
25600	**0.8714**	0.8454	**6220s**	8060s

Average solution quality using ML-similarity or Jaccard similarity on benchmark instances grouped by different partitioning algorithms

	ML	Jaccard
mst-clustering	**0.9005**	0.8749
k-medoids	**0.9019**	0.8739
multi-level	0.8641	0.8439
max-linkage	0.8964	0.871
min-linkage	**0.9005**	0.876
mean-linkage	0.8954	0.876
dominant-linkage	0.8981	0.8756

Detailed results selected algorithms solution quality

	ML			Jaccard		
	mst-clustering	k-medoids	min-linkage	mst-clustering	k-medoids	min-linkage
100	0.9321	0.9304	**0.9352**	**0.9443**	0.9425	0.9420
200	0.9272	**0.9304**	0.9261	0.9227	0.9145	**0.9227**
400	0.9045	**0.9104**	0.9046	0.8765	**0.8812**	0.8802
800	0.8893	**0.8930**	0.8895	**0.8582**	0.8582	0.8613
1600	0.8956	**0.8977**	0.8916	**0.8625**	0.8579	0.8606
3200	0.8958	0.8924	**0.8986**	**0.8680**	0.8628	0.8682
6400	0.8886	0.8891	**0.8927**	0.8450	**0.8472**	0.8468
12800	0.8838	0.8896	**0.8897**	0.8487	0.8500	**0.8510**
25600	**0.8877**	0.8841	0.8773	0.8480	**0.8512**	0.8508

Detailed results selected algorithm times

	ML			Jaccard		
	mst-clustering	k-medoids	min-linkage	mst-clustering	k-medoids	min-linkage
100	**1.30s**	1.37s	1.41s	**1.19s**	1.19s	1.19s
200	**1.98s**	2.16s	2.36s	1.55s	**1.51s**	1.88s
400	**3.47s**	3.77s	4.50s	**2.40s**	2.42s	3.23
800	**10.1s**	10.6s	12.8s	6.51s	**6.41s**	9.81s
1600	**29.2s**	29.3s	36.5s	12.5s	**12.5s**	16.4s
3200	72.2s	**64.7s**	77.0s	**64.1s**	65.5s	76.1s
6400	290s	**281s**	297s	**231s**	277s	270s
12800	**1070s**	1130s	1150s	1060s	**1030s**	1150s
25600	**4640s**	5950s	6470s	**7370s**	8640s	7960

Conclusion and Future Work

- Machine Learning supported partitioning of BLO for DMBSSLP instances improves solution quality in comparison to simple heuristic by up to 4%
- Different clustering methods don't lead to significantly different solution qualities. Faster methods like mst-clustering are therefore preferable.

- show that this principle is applicable to other more complex problems
- expand the usage of ML-similarity to both Q and L set at the same time
- apply graph neural networks, with message passing

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Properties of DMBSSLP

Solution and Objective Function

decision variables for a solution:

- $\boldsymbol{x} = (x_l)_{l \in L} \in \{0, \ldots, r_l\}$ # of opened stations in l
- $\boldsymbol{y} = (y_l)_{l \in L} \in \{0, \ldots, s \cdot x_l\}$ # of battery slots in l
- $\boldsymbol{a} = (a_{ql})_{q \in Q, l \in L}$ s.t. $0 \leq a_{ql} \leq e_{ql}$: the part of the demand of OD-pair q assigned to stations at area l

objective function (maximize fulfilled demand):

$$\max \sum_{q \in Q} \sum_{l \in N(q)} a_{ql}$$

DMBSSLP MILP formulation

$$\max \sum_{q \in Q} \sum_{l \in N(q)} a_{ql}$$

$$sx_l \geq y_l \qquad l \in L$$

$$\sum_{l \in N(q)} a_{ql} \leq d_q \qquad q \in Q$$

$$\sum_{q \in N(l)} a_{ql} \leq y_l \qquad l \in L$$

$$\sum_{l \in L} (c_l x_l + b_l y_l) \leq B$$

$$x_l \in \{0, \ldots, r_l\} \qquad l \in L$$

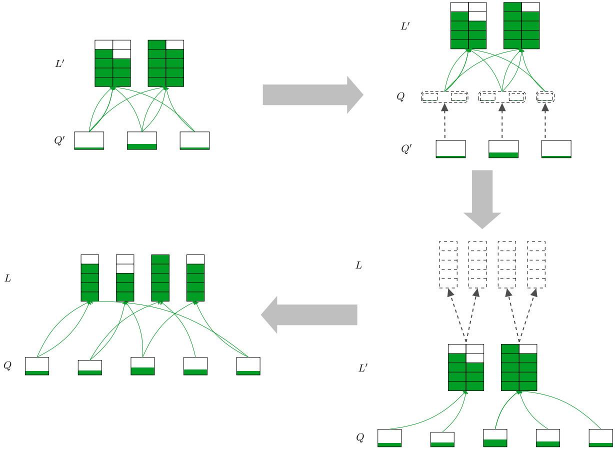
$$y_l \in \{0, \ldots, \lceil \bar{d}_l \rceil\} \qquad l \in L$$

$$0 \leq a_{ql} \leq e_{ql} \qquad q \in Q, \; l \in N(q)$$

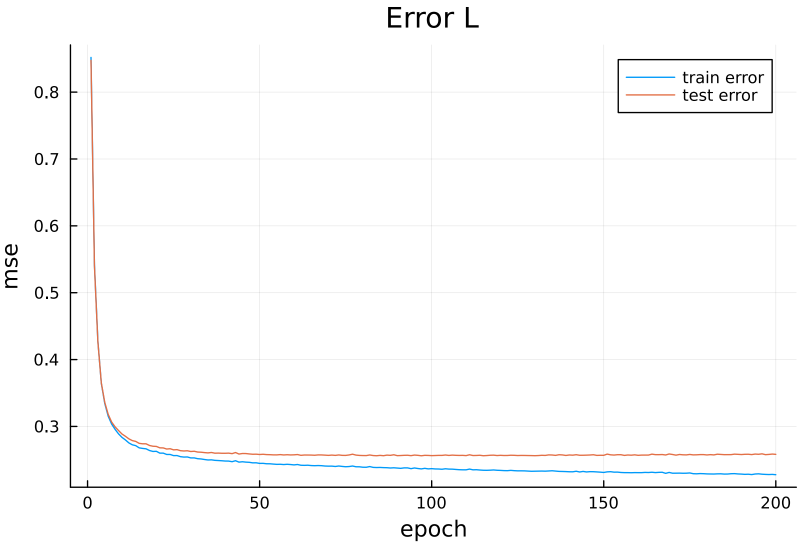
Bilevel Optimization (MLO) Approach

Projecting Solution

- vertex Sets (Q', L') projected independent of each other
- for each OD-pair and for each area a MI(LP) is solved
- MILP for area l are dependent of each other



Further Results



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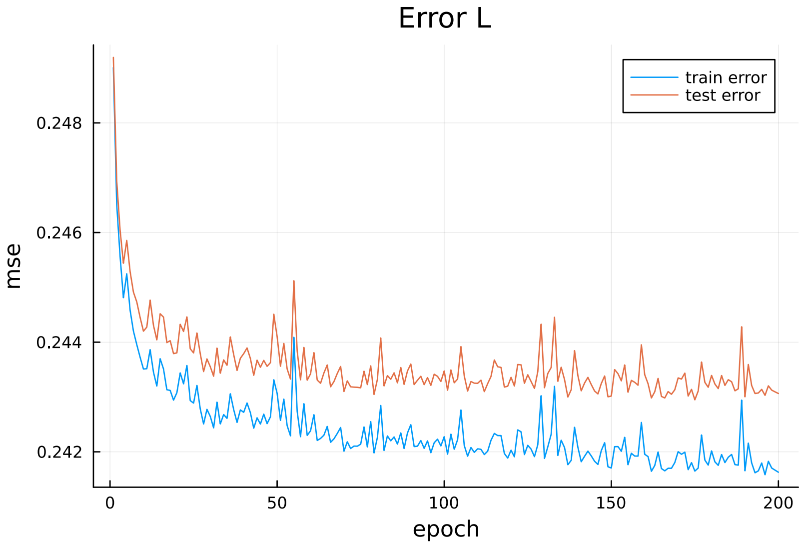

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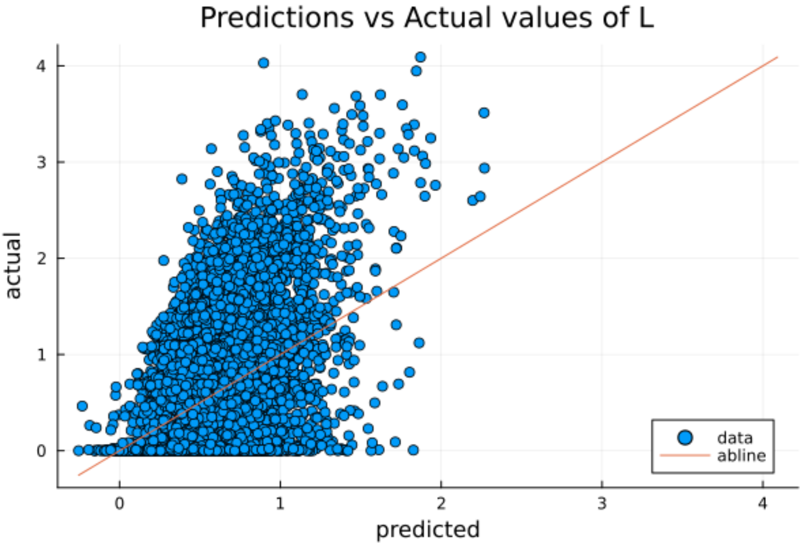
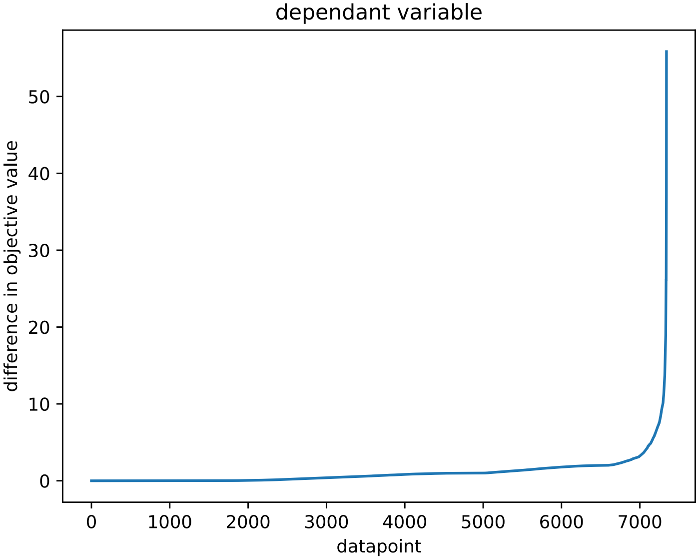


Further Results

Algorithm 5.2: Partitioning

```
Input : a set of nodes U
Output: a partitioning M(U)
S ←empty max heap;
for u ∈ U do
    for v ∈ neighborhood(u)  s.t.  (u, v) ∉ S do
        S(u, v) = similarity(u, v));
    end for
end for
M ← ∅;
unpartitioned ← U;
while S is not empty do
    (u, v) ← pop pair with highest similarity from S;
    if u, v ∈ unpartitioned then
        M ← M ∪ (u, v);
        remove u, v from unpartitioned;
    end if
end while
add all nodes in unpartitioned to M partitioned just to themselves ;
return M;
```

Further Results



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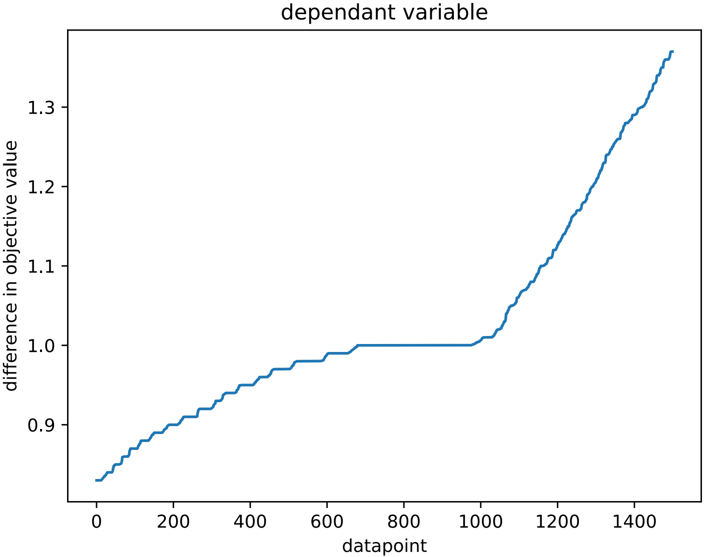

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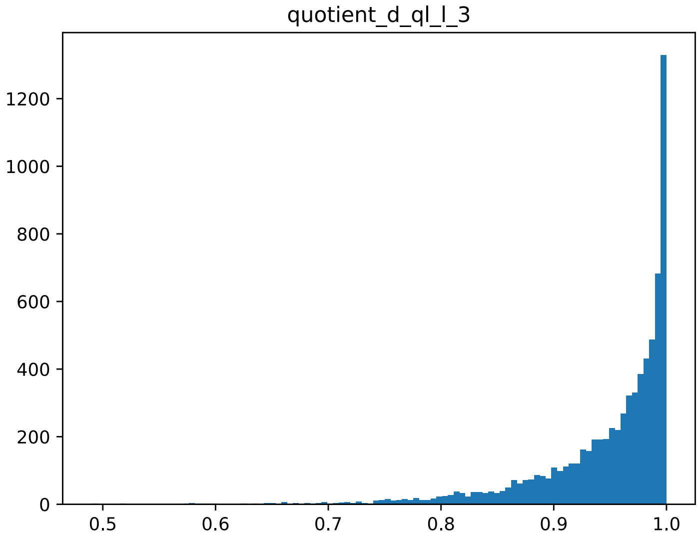

Further Results

MILP L projection

$$
\begin{aligned}
\max \quad & \sum_{l' \in L_l^{i-1}} \sum_{q \in N^{i-1}(l')} a_{ql'}^{i-1} && \\
& s x_{l'}^{i-1} \geq y_{l'}^{i-1} && l' \in L_l^{i-1} \\
& \sum_{l' \in L_l^{i-1} \cap N^{i-1}(q)} a_{ql'}^{i-1} \leq \delta_q && q \in N^{i-1}(l) \\
& \sum_{q \in N^{i-1}(l')} a_{ql'}^{i-1} \leq y_{l'}^{i-1} && l' \in L_l^{i-1} \\
& \sum_{l' \in L_l^{i-1}} (c x_{l'}^{i-1} + b y_{l'}^{i-1}) \leq c \tilde{x}_l^i + b \tilde{y}_l^i && \\
& x_{l'}^{i-1} \in \{0, \ldots, r_{l'}^{i-1}\} && l' \in L_l^{i-1} \\
& y_{l'}^{i-1} \in \{0, \ldots, \lceil \bar{d}_{l'}^{i-1} \rceil\} && l' \in L_l^{i-1} \\
& 0 \leq a_{ql'}^{i-1} \leq e_{ql'}^{i-1} && l' \in L_l^{i-1},\ q \in N^{i-1}(l')
\end{aligned}
$$

Further Results

LP Q projection

$$
\begin{aligned}
\max \quad & \sum_{q' \in Q_q^{i-1}} \sum_{l \in \tilde{N}^i(q')} \tilde{a}^i_{q'l} && \\
& \sum_{l \in \tilde{N}^i(q')} \tilde{a}^i_{q'l} \le \tilde{d}^i_{q'} && q' \in Q_q^{i-1} \\
& \sum_{q' \in \tilde{N}^i(l) \cap Q_q^{i-1}} \tilde{a}^i_{q'l} \le a^i_{ql} && l \in N^i(q) \\
& 0 \le \tilde{a}^i_{q'l} \le \tilde{e}_{q'l} && q' \in Q_q^{i-1},\ l \in \tilde{N}^i(q)
\end{aligned}
$$