Interactive Job Scheduling with Partially Known Personnel Availabilities

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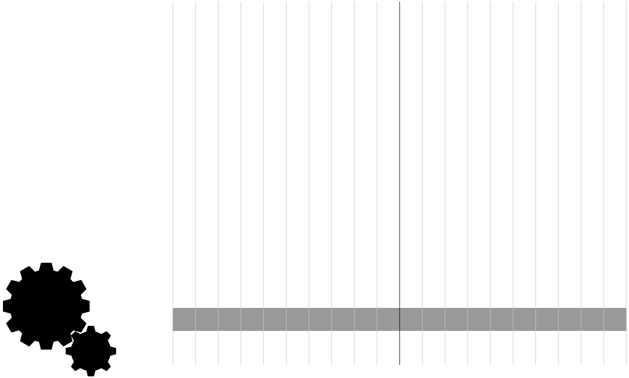
Problem



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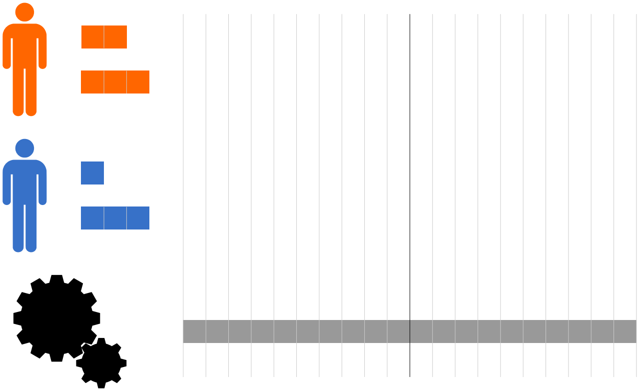

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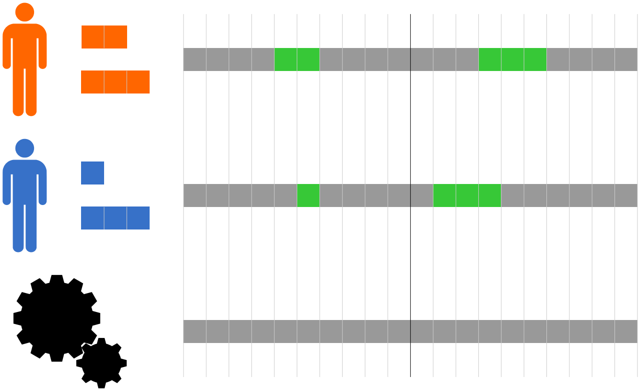

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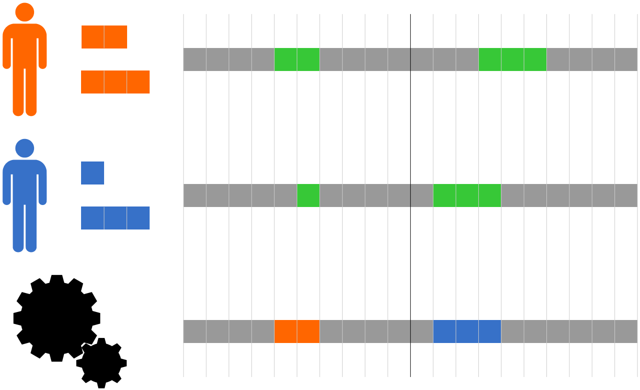

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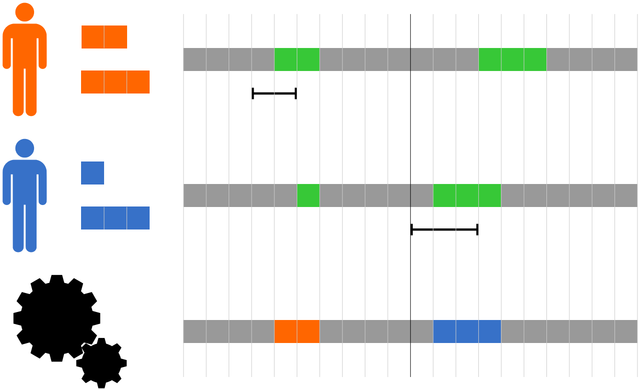

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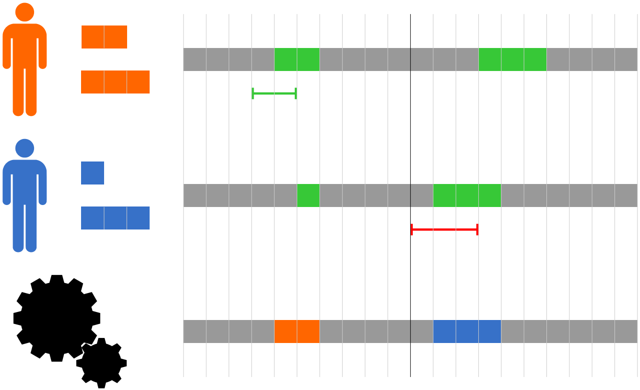

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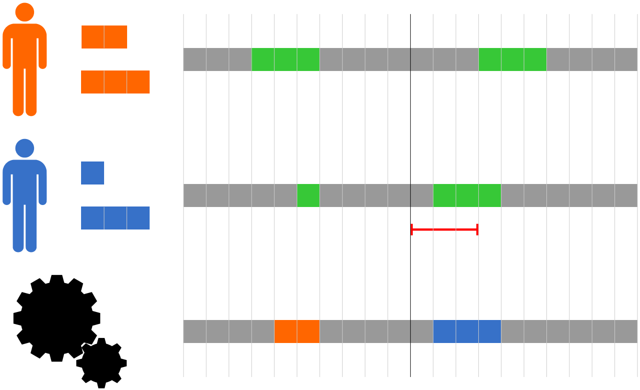

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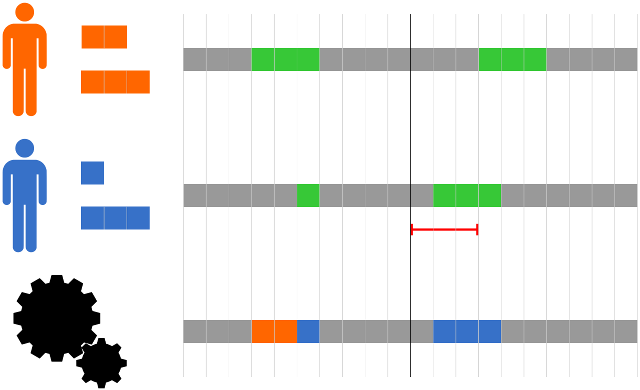

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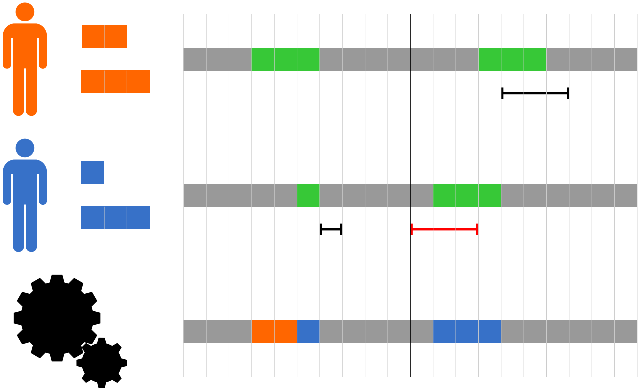

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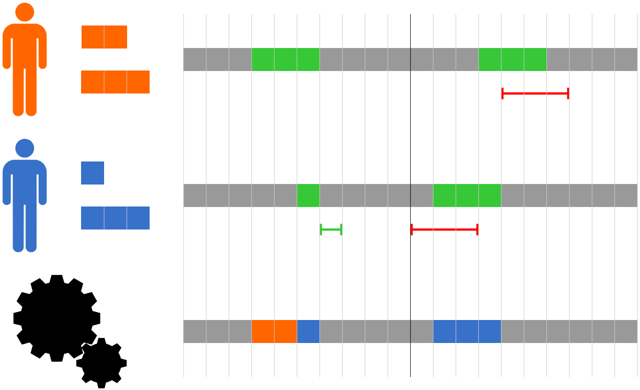

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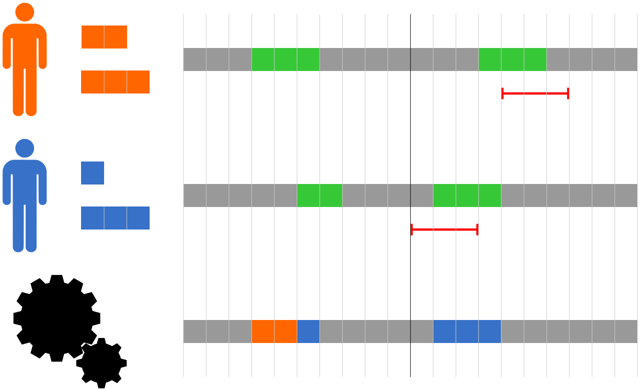

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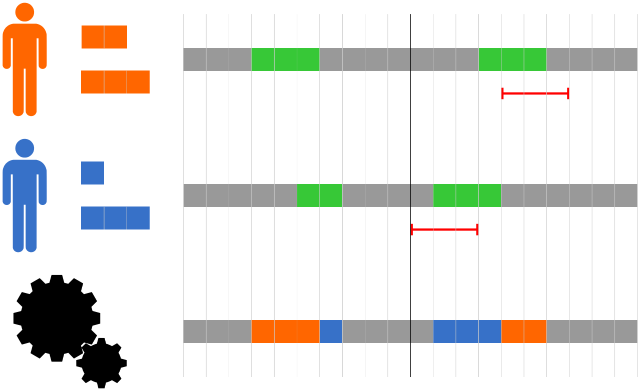


Problem

Objective:

- time dependent costs c_{it} for using a machine i at timestep t
- penalty q_j for not scheduling a job j
- not scheduling a job is more expensive than scheduling it

Interaction:

- B rounds of interaction
- each with up to b queries

Symbols:

- Machines M
- Jobs J
- Time horizon T

MILP

$$
\begin{aligned}
\text{ILP}(\mathcal{T}) \quad \min \quad & \sum_{j\in J}\sum_{i\in M}\sum_{t\in\mathcal{T}_j}\sum_{t'\in T_j[t]} c_{it'}x_{jit} + \sum_{j\in J} q_j\left(1-\sum_{i\in M}\sum_{t\in\mathcal{T}_j} x_{jit}\right) && \\
\text{s.t.} \quad & \sum_{i\in M}\sum_{t\in\mathcal{T}_j} x_{jit} \leq 1 && j\in J \\
& \sum_{j\in J}\sum_{t\in\mathcal{T}_j \mid t'\in T_j[t]} x_{jit} \leq 1 && i\in M,\ t'\in T \\
& \sum_{j\in J_u}\sum_{i\in M}\sum_{t\in\mathcal{T}_j \mid t'\in T_j[t]} x_{jit} \leq 1 && u\in U,\ t'\in T \\
& x_{jit}\in\{0,1\} && j\in J,\ i\in M,\ t\in\mathcal{T}_j
\end{aligned}
$$

$\mathcal{T}_j$ . . . allowed starting times for job j

x_{jit} . . . 1, iff job j starts at timestep t on machine i

$T_j[t]$ . . . timesteps job j runs in when started at timestep t

Literature

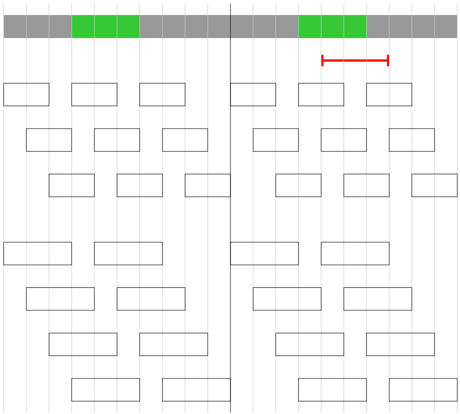
Exact problem not yet considered

Neglect users: Job Scheduling Problem $Pm||TEC$

- MILP and various heuristics (greedy, genetic algorithm, local search) for $Pm||C_{max}$,TEC (Wang et al.,2018), (Anghinolfi et al.,2021)
- MILP and Dantzig-Wolfe decomposition for $Rm||TEC$ (Ding et al.,2016)
- Improved MILP (Cheng et al.,2018), (Saberi et al.,2020)

Interactive optimization with multiple users: (Jatschka et al.,2021)

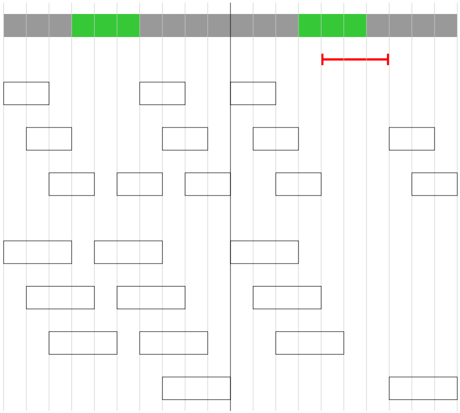
Solving Approach

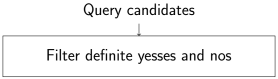


Query candidates

Figure: Procedure in each round

Solving Approach





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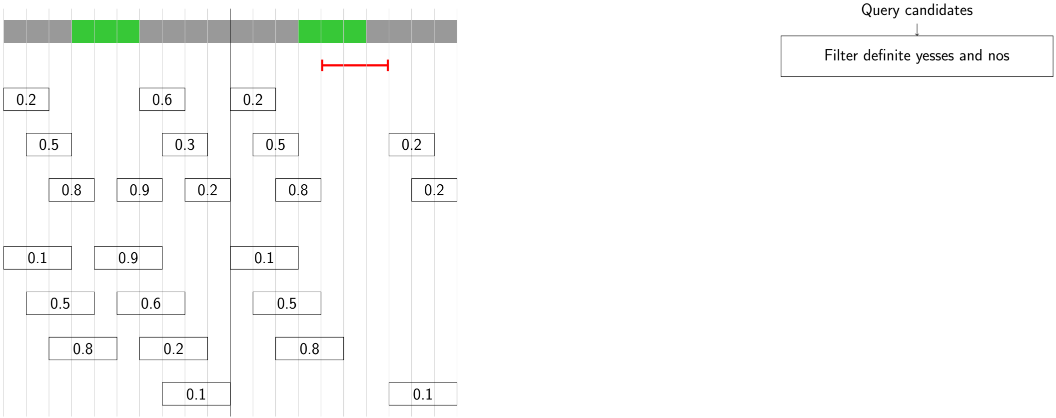

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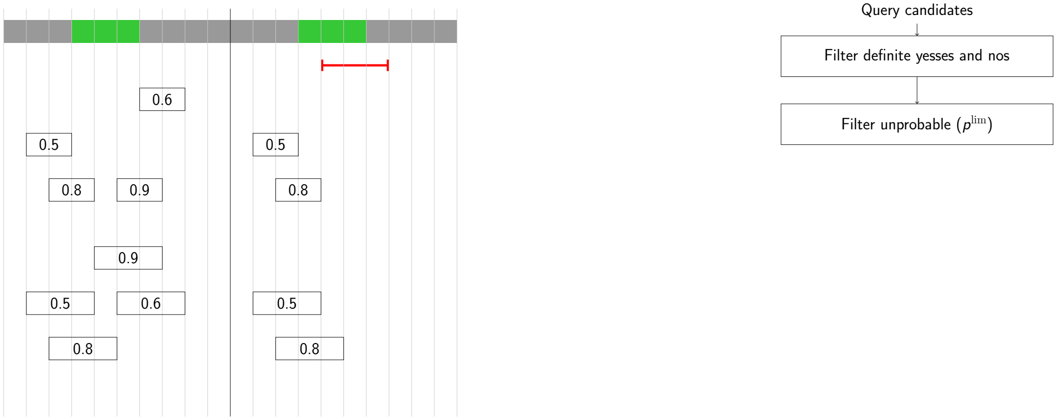

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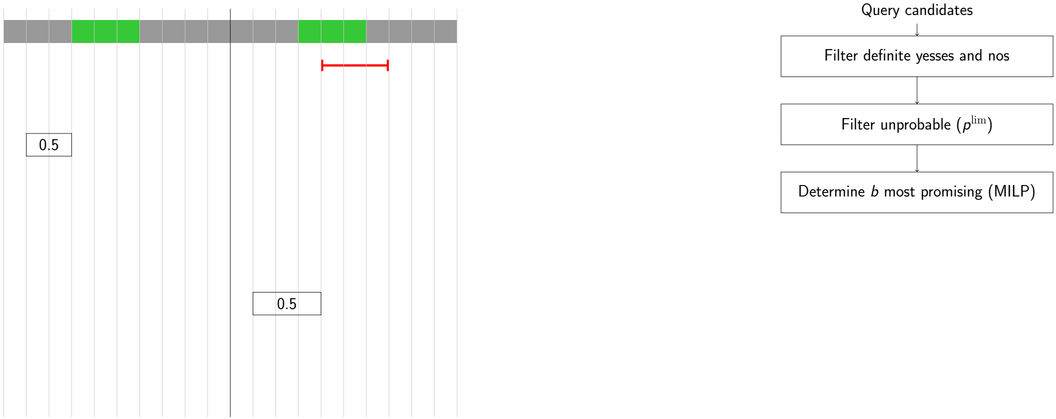

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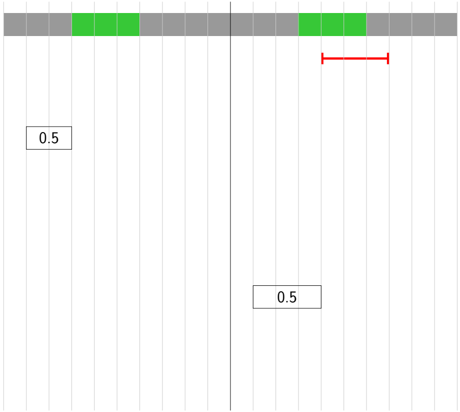

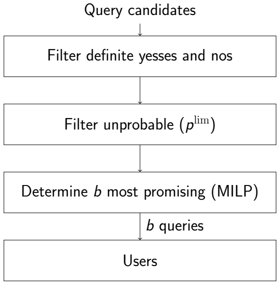

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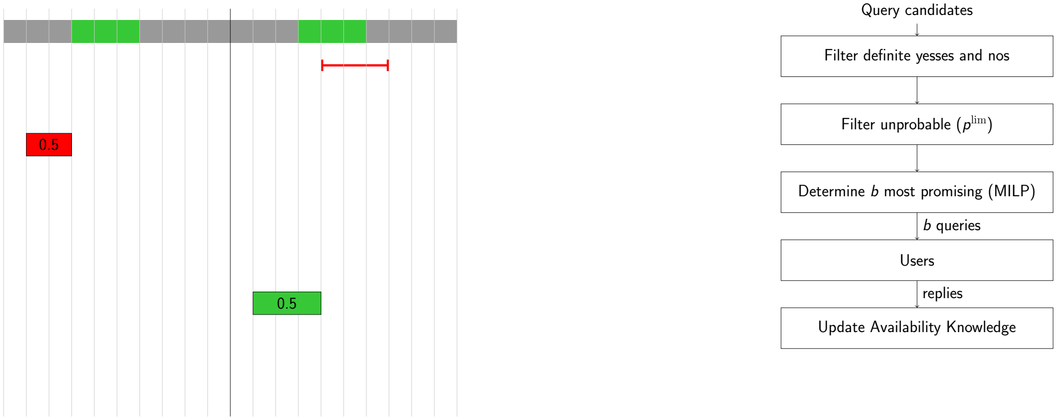

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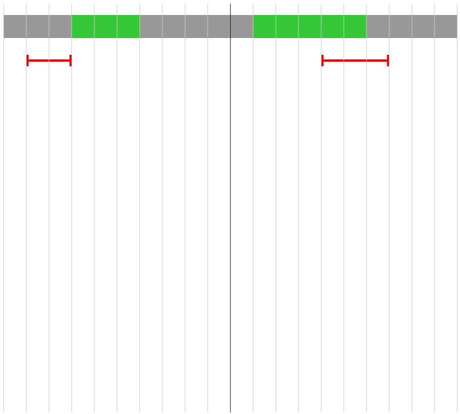

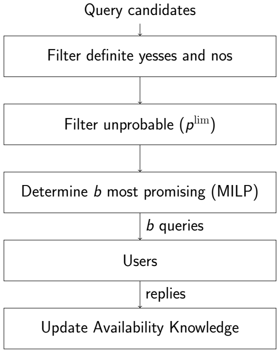

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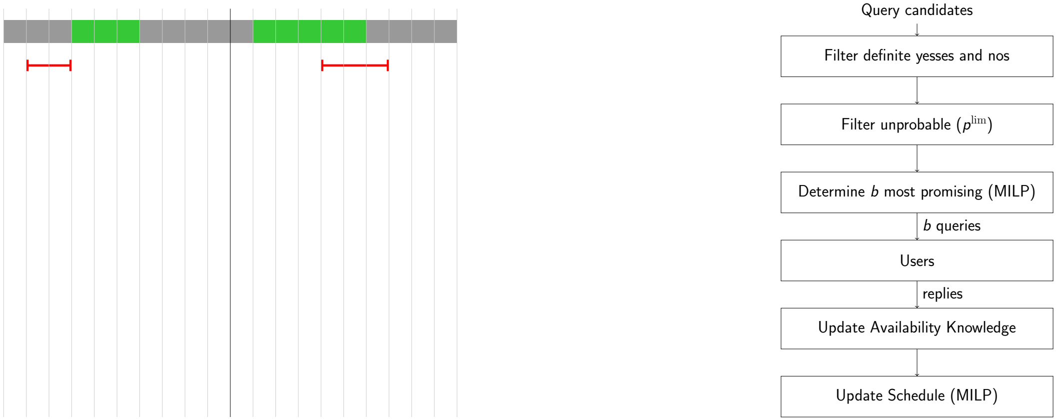


Figure: Procedure in each round

Markov-Model

- Single user
- Model availabilites with Markov process
- Given:
  - Knowledge about availabilities T^{avail}
  - Rejected time intervals I^{rej}

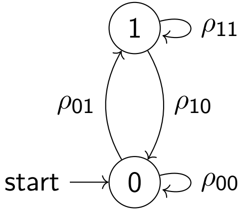
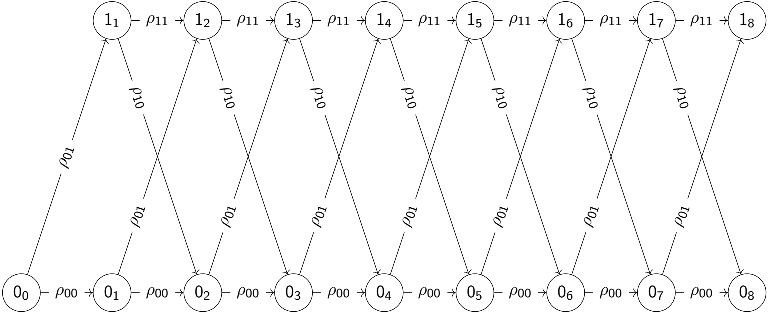


Figure: Two-state Markov Chain

Probability Calculation: Graph

E.g. day with 7 timesteps, $T^{\text{avail}} = \{5, 6\}$, $I^{\text{rej}} = \{[1, 5], [3, 7]\}$



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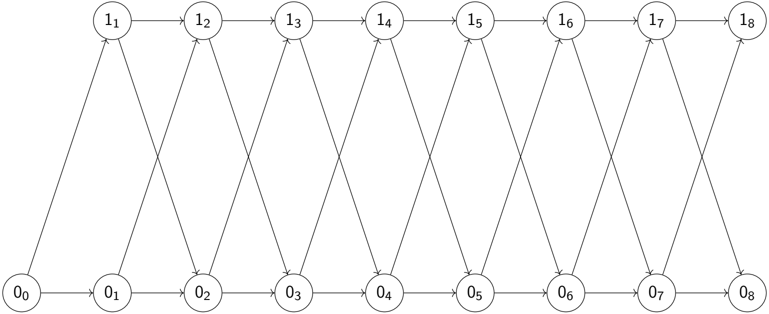

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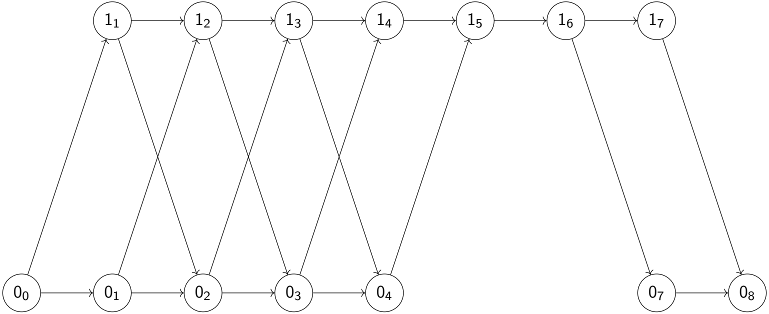

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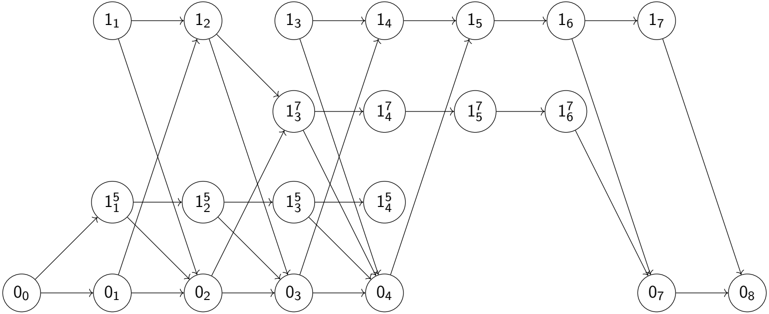

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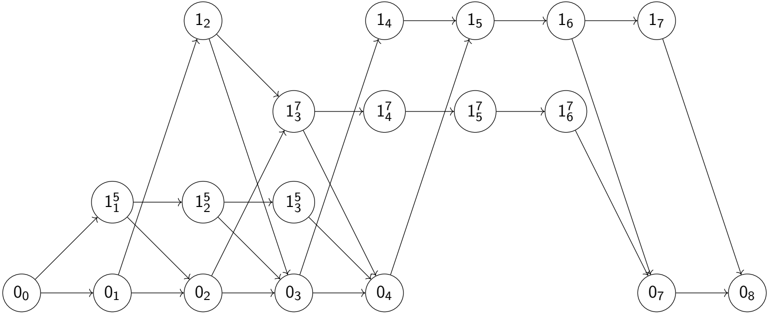


Probability Calculation

$$p^{\text{path}}_{0_0,v} = \sum_{P \in \text{Paths}(0_0,v)} \prod_{(u,u') \in P} \rho(u,u')$$

$$= \sum_{u \in N^-(v)} \sum_{P \in \text{Paths}(0_0,u)} \left(\prod_{(u,u') \in P} \rho(u,u') \right) \cdot \rho(u,v)$$

$$= \sum_{u \in N^-(v)} p^{\text{path}}_{0_0,u} \rho(u,v)$$

$$p^{\text{path}}_{v,0_{t\text{max}+1}} = \sum_{w \in N^+(v)} p^{\text{path}}_{w,0_{t\text{max}+1}} \rho(v,w)$$

Probability Calculation

Probability that $[\tau, \tau']$ will be accepted:

$$p^{\text{avail}}([\tau, \tau'] \mid T^{\text{avail}}, I^{\text{rej}}, 0_{t^{\max}+1}) = \frac{\sum_{P \in \text{1-Paths}(\tau,\tau')} p^{\text{path}}_{0_0, P_\tau} \cdot \rho_{11}^{\tau'-\tau} \cdot p^{\text{path}}_{P_{\tau'}, 0_{t^{\max}+1}}}{p^{\text{path}}_{0, t^{\max}+1}}$$

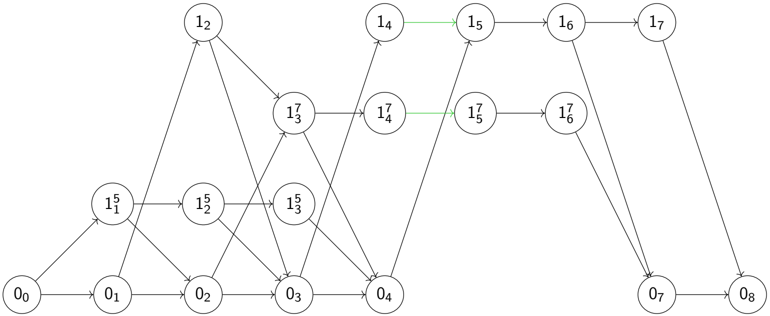


Figure: 1-Paths(4,5) for the example in green

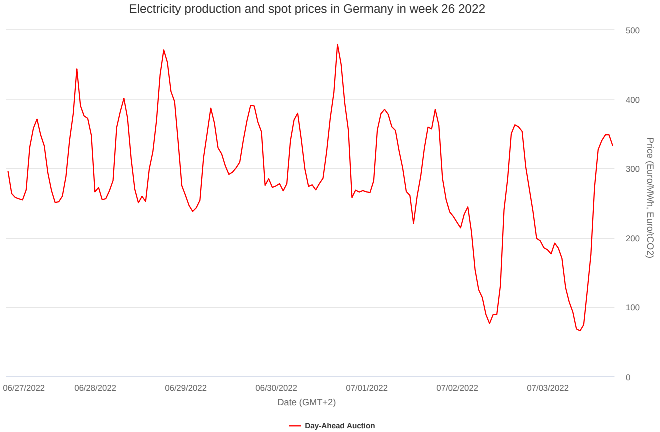
Instance Generation

- one to five machines
- 25 or 50 jobs per machine
- 30 instances per instance size
- five jobs per user
- seven rounds, each with $|U|$ user queries
- five days (6am to 10pm)
- Users are available
  - from 9am±1h for 4h±1h with probability 0.9 and
  - from 1pm±1h for 5h±1h with probability 0.9
- Job duration: uniformly random from [0.5h,4h]
- Known availabilities: one random starting time chosen for each job

Instance Generation

Costs:

- Random power consumption per machine from [50kW,150kW]
- Job penalty: 40 Euro per timestep



Computing Environment

- Julia 1.8.3
- Gurobi 10.0 via JuMP
- 15min timelimit for Gurobi

$\rightarrow$ $\leq$ 5% optimality gap

Results 1

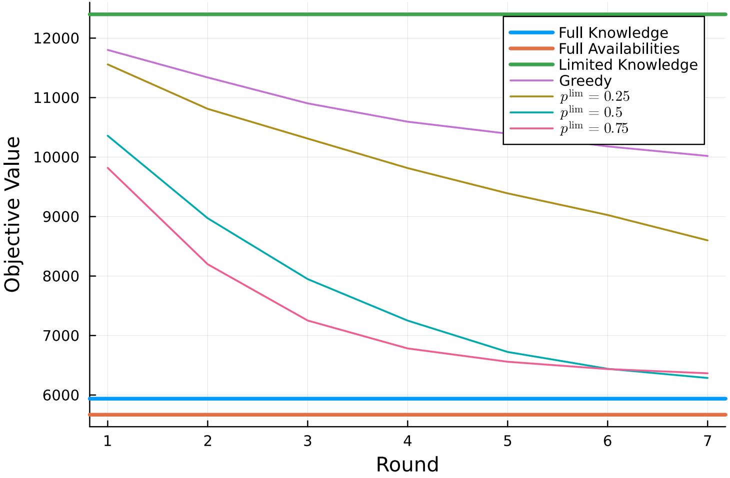


Figure: Objective over the rounds for 5 machines and 125 jobs

Results 2

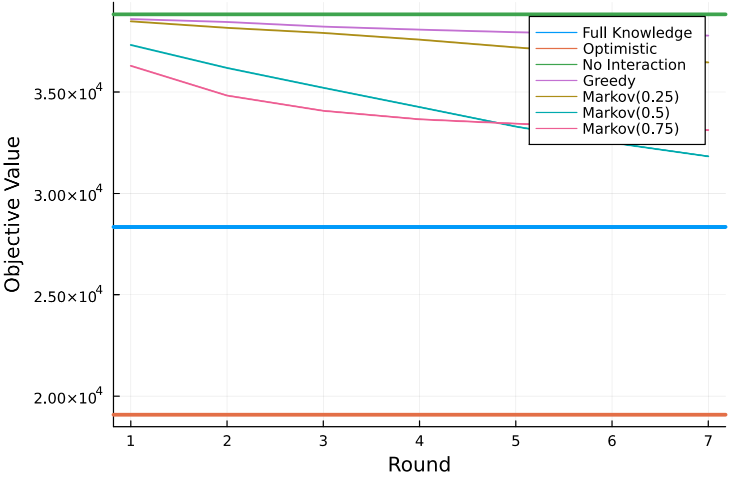


Figure: Objective over the rounds for 5 machines and 250 jobs

Results 3

Table: Mean %-gaps to the Full Knowledge case

m	n	Round 5				Round 7			
		Greedy	Markov(p^{lim})			Greedy	Markov(p^{lim})		
			0.25	0.5	0.75		0.25	0.5	0.75
1	25	77.7	54.5	**39.6**	40.0	70.7	36.9	**18.0**	24.6
2	50	95.2	74.5	27.0	**23.8**	87.0	52.6	**11.9**	13.3
3	75	77.5	62.3	18.2	**15.0**	69.1	48.8	**8.4**	9.3
4	100	79.5	64.3	15.7	**12.1**	71.8	47.9	**6.7**	7.9
5	125	77.8	60.4	13.4	**10.7**	71.5	46.7	**6.0**	7.3
1	50	40.2	33.9	**19.4**	22.6	37.1	29.5	**12.8**	21.7
2	100	36.8	31.9	**18.6**	19.2	35.6	29.1	**13.0**	18.2
3	150	34.4	31.6	**17.8**	18.0	33.4	28.7	**12.1**	17.0
4	200	35.0	32.2	**18.3**	18.8	34.0	29.2	**12.6**	17.6
5	250	34.4	31.8	**17.7**	18.3	33.8	29.1	**12.4**	17.2

Conclusion and Future Work

Conclusions:

- Quick convergence
- Exploring unknown times pays off on the long term

Future Work:

- More realistic users
- Probability computation from historic availability data (ML model)
- Exploit probabilties better

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