A Large Neighborhood Search for the Directed Feedback Vertex Set Problem

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Diploma Thesis

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Outlook

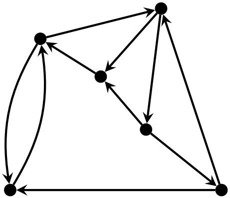
1. Directed Feedback Vertex Set Problem
2. Literature
3. MILP Formulations
4. Preprocessing
5. Construction Heuristics
6. Classical local search
7. Large Neighborhoods
8. Discussion

Problem Definition

Definition (Directed Feedback Vertex Set Problem)

Given: Directed graph $G = (V, A)$
Task: Find $F \subseteq V$ of minimum cardinality, s.t. $G[V \setminus F]$ is acyclic.

F: Directed Feedback Vertex Set (DFVS)



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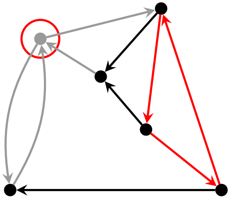

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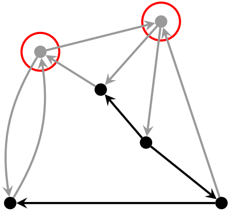


Variants:

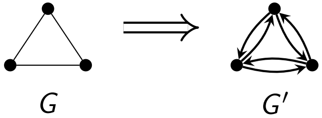
- undirected vs. directed
- vertex sets vs. arc sets

NP-Hardness

Definition (Minimum Vertex Cover Problem)

Given: Undirected graph $G = (V, E)$
Task: Find $C \subseteq V$ of minimum cardinality, s.t. $v \in C \vee w \in C$ for each edge $vw \in E$

$G' = (V, A)$ directed, $A := \{(v, w) \mid \{v, w\} \in E\}$



$C \subseteq V$ is *Vertex Cover* in $G \Leftrightarrow C$ is DFVS in G'

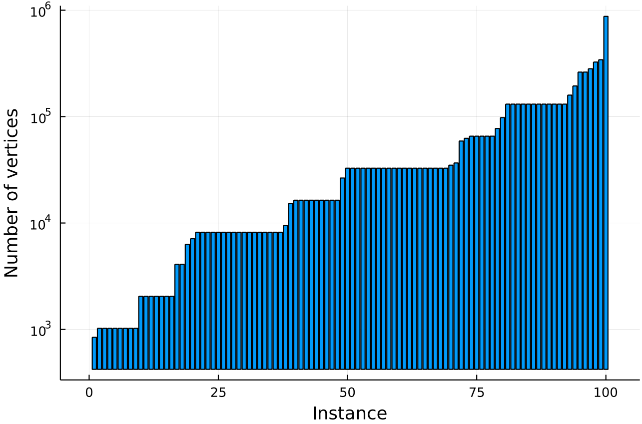
PACE Challenge 2022

2 Tracks:

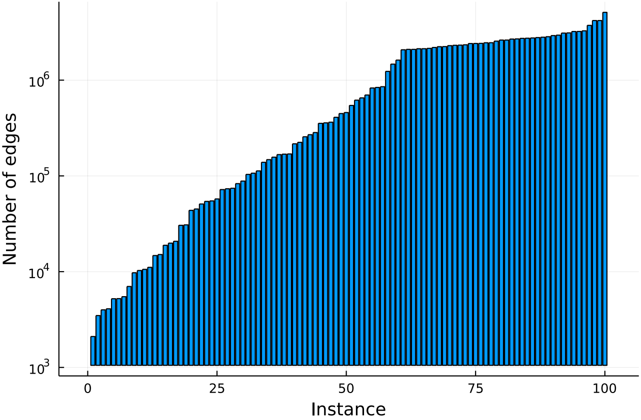
- Exact
- Heuristic

100 public instances
100 private instances
Timelimit: 10 minutes
Submission deadline: June 1, 2022

PACE Challenge 2022: Instances



PACE Challenge 2022: Instances



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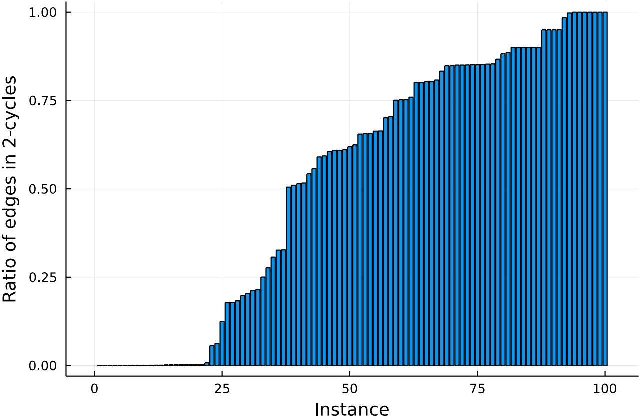


Related Literature

- Fleischer et al., 2009: Reduction rules, construction heuristics
- Galinier et al., 2013
  - Local Search + Simulated Annealing
  - Solution representation: Topological ordering of $V \setminus F$
- Tang et al., 2017
  - Extends (Galinier et al., 2013)
  - Prioritize promising neighbors
- Baharev et al., 2015 (arc sets)
  - 3 ILP formulations
  - Iterated Local Search
- Melo et al., 2021 (undirected)
  - 2 ILP formulations
  - Iterated Local Search

MILP-Formulation: MTZ

Find maximum induced DAG

$$
\begin{aligned}
\max \quad & \sum_{v \in V} y_v && && (1)\\
s.t. \quad & \Phi_w \geq \Phi_v + 1 - M(1 - x_{vw}) && \forall vw \in A && (2)\\
& y_v \leq \Phi_v \leq y_v n && \forall v \in V && (3)\\
& x_{vw} \leq y_v && \forall vw \in A && (4)\\
& x_{vw} \leq y_w && \forall vw \in A && (5)\\
& x_{vw} \geq y_v + y_w - 1 && \forall vw \in A && (6)\\
& y_v \in \{0, 1\} && \forall v \in V && (7)\\
& 0 \leq \Phi_v \leq n && \forall v \in V && (8)\\
& 0 \leq x_{vw} \leq 1 && \forall vw \in A && (9)
\end{aligned}
$$

MILP-Formulation: CEC

Find maximum induced DAG

$$\max \quad \sum_{v \in V} y_v \tag{10}$$

$$s.t. \quad \sum_{v \in C} y_v \leq |C| - 1 \qquad \forall C \subseteq V \text{ cycle in } G \tag{11}$$

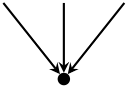
- Cutting-plane approach vs. lazy constraints
- We use MTZ-formulation

MILP-Formulations

Other formulations:

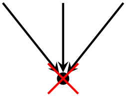
- Total ordering with transitivity constraints
- Reachability variables

Reduction Rules 1 (Fleischer et al., 2009)



$d^+(v) = 0$

Reduction Rules 1 (Fleischer et al., 2009)



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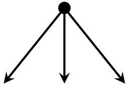
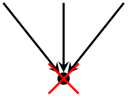
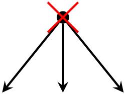


$d^-(v) = 0$

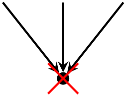


$d^+(v) = 0$

Reduction Rules 1 (Fleischer et al., 2009)



$d^{-}(v) = 0$



$d^{+}(v) = 0$

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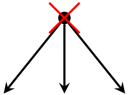
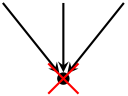


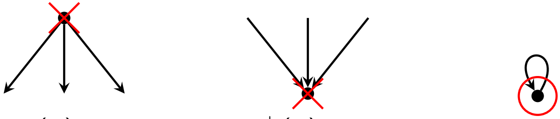
$d^{-}(v) = 0$



$d^{+}(v) = 0$

self-loop

Reduction Rules 1 (Fleischer et al., 2009)



$d^{-}(v) = 0$ $d^{+}(v) = 0$ self-loop

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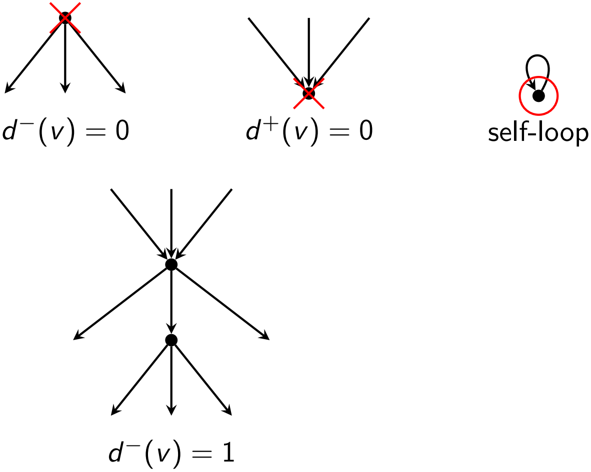

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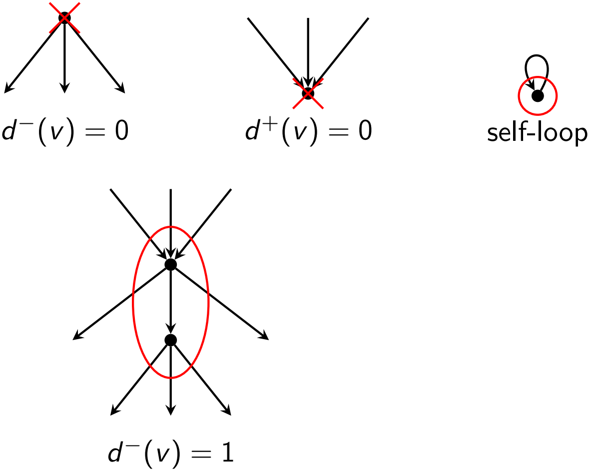

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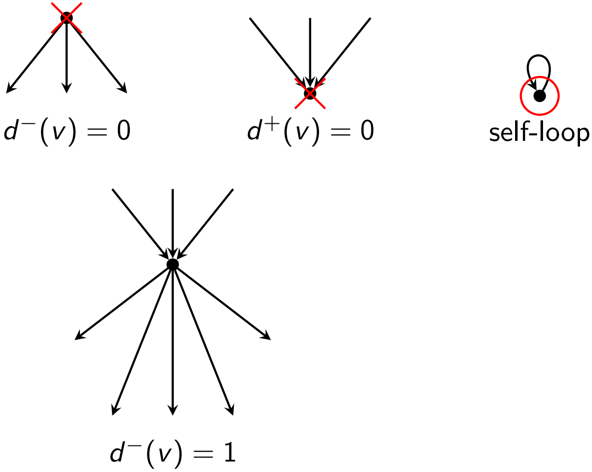

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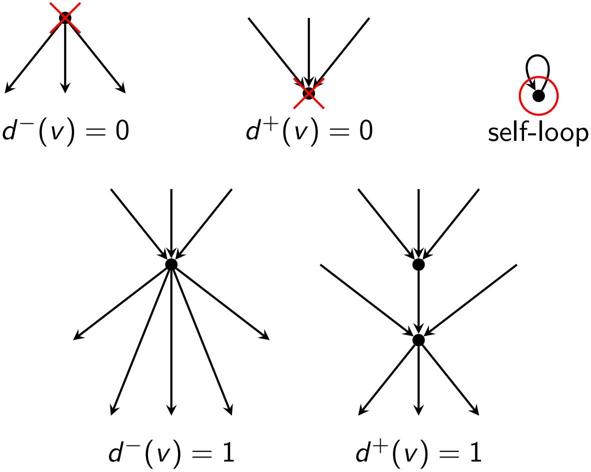

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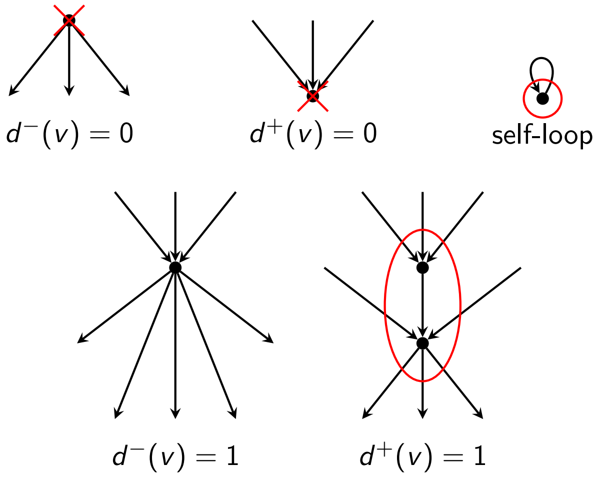

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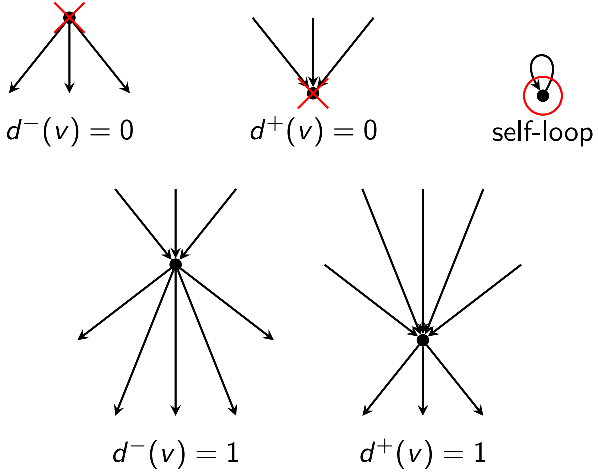


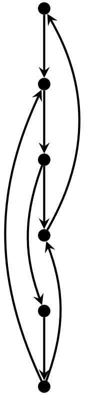
Reduction Rules 2 (Fleischer et al., 2009)

$\mathcal{C}$: Strongly connected components of G
$\rightarrow C'$ cycle $\Rightarrow C' \subseteq C$ for some $C \in \mathcal{C}$

G decomposes into $G[C], C \in \mathcal{C}$

Construction Heuristic

- Vertex ordering by DFS
- Select vertices with up-arcs



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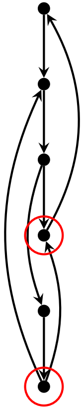


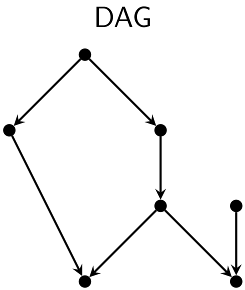
Local Search

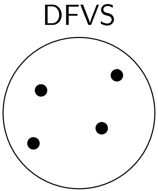
One-flip neighborhood

Problem: Check acyclicity often

$\rightarrow$ time-consuming

Large Neighborhood 1





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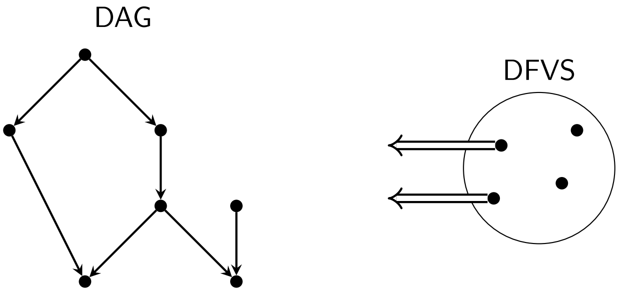
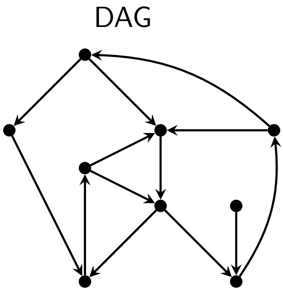
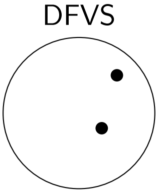


Destroy: Select k vertices from DFVS

Large Neighborhood 1





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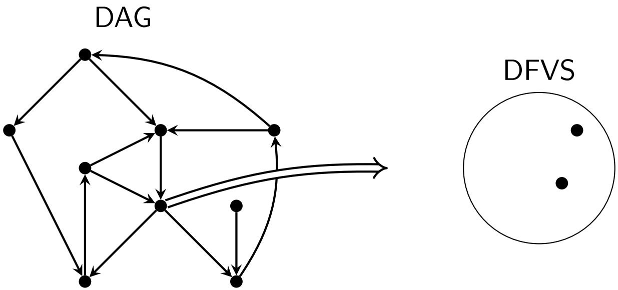
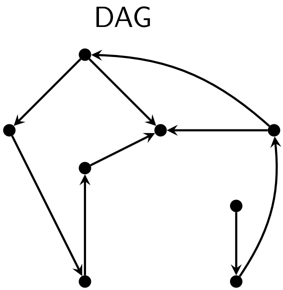
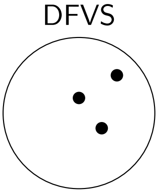


Destroy: Select k vertices from DFVS
Repair: Solve with MILP (MTZ)

Large Neighborhood 1





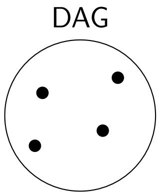
Destroy: Select *k* vertices from DFVS
Repair: Solve with MILP (MTZ)

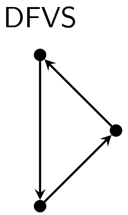
Large Neighborhood 1: Destroy

Select k vertices

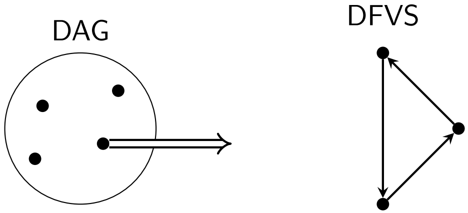
1. randomly
2. with highest (lowest) values of $h(v) := d^+(v) + d^-(v) - \lambda|d^+(v) - d^-(v)|$
3. tournament selection with $h(v)$

Large Neighborhood 2



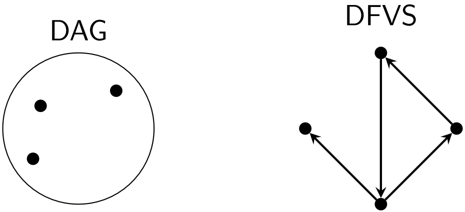


Large Neighborhood 2



Destroy: Similar to Large Neighborhood 1

Large Neighborhood 2



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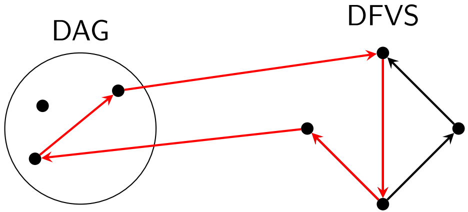

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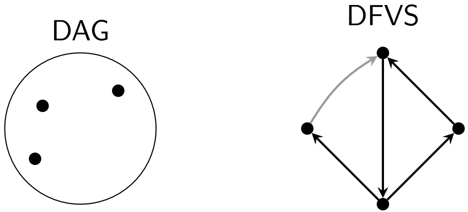
Large Neighborhood 2



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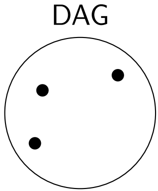

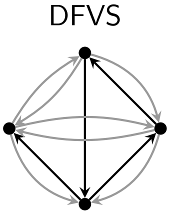

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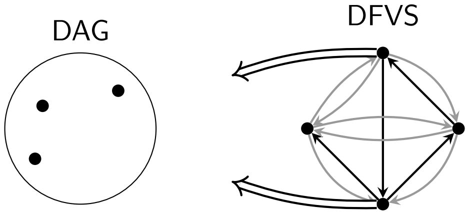
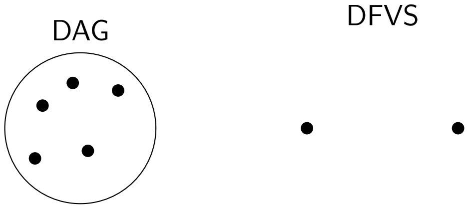
Large Neighborhood 2



Destroy: Similar to Large Neighborhood 1
Repair: MILP (MTZ)

Large Neighborhood 2



Destroy: Similar to Large Neighborhood 1
Repair: MILP (MTZ)

Large Neighborhood 2: Repair

Problem: Dense generated graph
Largest instance: $n = 875713$
$\rightarrow$ Preselect vertices from DFVS
Again: Tournament selection with $h(v)$

Discussion

- No results yet
- Current challenge: Local search takes too long
- Next steps:
  - Solve issue with local search
  - Parameter tuning
  - Submit to challenge

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