Solving a Weighted Set Covering Problem for Improving Algorithms for Cutting Stock Problems with Setup Costs by Solution Merging

Benedikt Klocker, Günther R. Raidl

Institute of Computer Graphics and Algorithms, TU Wien, Vienna, Austria
{klocker|raidl}@ac.tuwien.ac.at

1 Introduction

There are many different kinds of cutting stock problems (CSPs) occurring in practice and in theory having in common that they ask for a set of patterns, where each pattern is a collection of elements, to satisfy given element demands while minimizing the costs of the patterns. The classical CSP only considers fixed costs for each individual pattern, but in many practical applications an additional cost factor are setup costs arising whenever the machine has to be set up to cut a different pattern. Thus, finding a solution involving a small number of different types of patterns is often preferred.

Most approaches to solve CSPs generate many candidate solutions yielding, if collected, a large and diverse set of patterns. We formalize an extension of the weighted set covering problem which exploits all these collected patterns by deriving an optimal combination of a subset of them resembling a feasible, possibly new incumbent solution. Solving this subproblem can be seen as a kind of solution merging. It can be applied either as a post-processing or as an intermediate step to also lead the pattern construction in a more promising direction. We investigate this extension specifically on K-staged two-dimensional CSPs with setup costs.

The merging problem is defined as follows. Given is a set of elements $E = \{1, \dots, n\}$ with a demand vector $(d_i)_{i=1}^n \in \mathbb{N}^n$ and the set of collected patterns P. The actual structure of the patterns is not relevant here, but each pattern $p \in P$ has an associated element vector $(e_i^p)_{i=1}^n \in \mathbb{N}^n$ indicating how often an element $i \in E$ occurs in p. Every pattern $p \in P$ has associated production costs c_p^{P} and setup costs c_p^{S}. The goal is to find amounts $a = (a_p)_{p \in P} \in \mathbb{N}^{|P|}$ such that

$$c(a) := \sum_{p \in P} c_p^{\mathrm{P}} \cdot a_p + \sum_{p \in P\,:\,a_p > 0} c_p^{\mathrm{S}}$$

is a minimum and the demands are satisfied, i.e. $\sum_{p \in P} e_i^p \cdot a_p \geq d_i,\ i = 1, \dots, n$.

2 Related Work

In [1] a similar approach is considered for the one dimensional CSP, where each pattern has the same production and setup costs. An integer linear programming

(ILP) model is proposed and solved by CPLEX. In an older work Foerster and Wascher [2] present a two phase approach with a pattern reduction in the second phase. A more theoretical analysis of a general weighted set covering problem is done in [3], where no concept like setup costs were considered. We use the methods described in [4] as base algorithm yielding the collection of patterns.

3 Solution Approaches

Similarly as in [1], we can solve our problem directly with an ILP solver. Since this exact approach does not scale very well we also consider a greedy heuristic.

This greedy approach starts with no selected patterns and selects promising patterns until all demands are satisfied. To decide in a greedy manner which pattern we choose next we keep track of the unsatisfied demands $(u_i)_{i=1}^n$ which get initialized with $u_i = d_i$. To rate the quality of a pattern we use a size value $v_i \in \mathbb{R}$ for each element $i \in E$. In the one-dimensional case this is the length, in the two-dimensional case the area of the element. In each step, we select a best pattern p together with an amount a according to the following rating

$$r(p,a) := \frac{\sum_{i=1}^n \max\left(a \cdot e_i^p, u_i\right) \cdot v_i}{a \cdot c_p^{\mathrm{P}} + c_p^{\mathrm{S}} \cdot \delta_p}$$

where $\delta_p = 1$ if pattern p is not already in the solution and $\delta_p = 0$ if p was already added in a previous step. We add a pattern p with a maximal rating $r(p,a)$ with amount a to our current solution and recalculate the u_i values. We stop when $u_i = 0$ for all i.

We compare the greedy algorithm with the exact ILP approach on real-world instances. Results indicate that the greedy approach is substantially faster, scales much better, and nevertheless yields solutions of almost equal quality.

The greedy approach is further extended with the preferred iterative lookahead technique (PILOT) resulting in better solution qualities for some instances, however, at the cost of longer running times. Furthermore some considered extensions are disallowing overproduction of elements, i.e. satisfying demands exactly, limiting the number of patterns for one setup, and limiting the amount of different sheet types.

References

1. Cui, Y., Zhong, C., Yao, Y.: Pattern-set generation algorithm for the one-dimensional cutting stock problem with setup cost. European Journal of Operational Research **243**(2) (2015) 540–546
2. Foerster, H., Wascher, G.: Pattern reduction in one-dimensional cutting stock problems. International Journal of Production Research **38**(7) (2000) 1657–1676
3. Yang, J., Leung, J.Y.T.: A generalization of the weighted set covering problem. Naval Research Logistics **52**(2) (2005) 142–149
4. Dusberger, F., Raidl, G.R. In: A Scalable Approach for the K-Staged Two-Dimensional Cutting Stock Problem with Variable Sheet Size. Volume 9520 of LNCS. Springer (2015) 384–392