Decision Diagram Based Limited Discrepancy Search for a Job Sequencing Problem*

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In this work we consider the *Price-Collecting Job Sequencing with One Common and Multiple Secondary Resources* (PC-JSOCMSR) introduced from [2, 3]. The task is to feasibly schedule a subset of jobs from a given set of jobs. Each job needs two resources: a common resource for a part of the job's execution time and a secondary resource for the whole execution time. In addition, each job has one or more time windows and an associated prize. A feasible schedule requires that there is no resource used by more than one job at the same time and each job is scheduled within one of its time windows. Due to the time windows it may not be possible to schedule all jobs. Therefore we aim to maximize the total prize over the actually scheduled jobs.

There are at least two applications of the PC-JSOCMSR problem. The first is in the field of the daily scheduling of particle therapies for cancer treatments. Here proton or carbon particles get accelerated to almost the speed of light in a particle accelerator, corresponding to the common resource, and are then redirected to one of several treatment rooms, the corresponding secondary resources, in which a patient gets radiated. The task is to find a feasible schedule of the cancer treatments for one day. The second application can be found in the field of hard real time scheduling of electronics within an aircraft, called avionics, where the PC-JSOCMSR appears as a subproblem. Since the scheduled jobs of hard real time systems have to compute the right results on the right time, such systems are usually not scheduled dynamically, but rather pre-runtime scheduled such that the schedule of jobs is determined in advance. An avionic system consists of a set of nodes and each node contains one communication module, corresponding to the common resource, and a set of applications modules, corresponding to the secondary resources, where jobs have to be scheduled. The PC-JSOCMSR problem addresses the partial schedules for the modules in a node. The considered scenario involves tasks which are executed on the application modules and need to communicate with the communication module and tasks which are executed only on the communication module by using a dummy secondary resource.

The PC-JSOCMSR was tackled on the exact side by Horn et al. [2], suggesting an A* based algorithm which is able to solve instances up to 40 jobs to proven

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optimality. On the heuristic side, Maschler and Raidl [3] applied *decision diagrams* (DDs) on the PC-JSOCMSR to obtain lower and upper bounds for large problem instances with up to 300 jobs. DDs are rooted weighted directed acyclic graphs and provide graphical representations of the solution spaces of combinatorial optimization problems. In particular relaxed DDs represent a superset of the feasible set of solutions and are therefore a *discrete* relaxation of the solution space, providing possible strong upper bounds on the objective value. The counterparts are restricted DDs which represent a subset of feasible solutions and provide therefore heuristic solutions. Both types of DDs were investigated in [3] and were compiled with the adapted standard methods from the literature. For further details on DDs we refer to [1].

In particular for the avionic system scenario it often appears that some jobs need to be finished before other jobs may start. To address this aspects, we consider in this work in addition *precedence constraints* in order to get a more realistic model of the real world scenarios. Thus, there are given relationships between pairs of jobs as additional input such that one job can only be scheduled if the other job is already be scheduled earlier. However, this new constraints require an adaption of the algorithmic side to incorporate the new precedence constraints. The goal is to solve large problem instances of the PC-JSOCMSR with precedence constraints heuristically. Our solution approach consists of an adapted *limited discrepancy search* (LDS) that exploits the structural information contained in a relaxed DD. The usage of the relaxed DD is three-folded: (1) as search guidance for the LDS, (2) to speed-up computation time of the LDS and (3) to provide besides a heuristic solution also an upper bound on the total prize objective. While we demonstrate the method's usefulness specifically for the PC-JSOCMSR, the general approach also appears promising for other combinatorial optimization problems.

References

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