Label Number Maximization in the Slider Model
(Extended Abstract)

Dietmar Ebner, Gunnar W. Klau, and René Weiskircher

Institute of Computer Graphics and Algorithms, Vienna University of Technology, http://www.ads.tuwien.ac.at,
ebner|gunnar|weiskircher@ads.tuwien.ac.at

Abstract. We consider the *NP*-hard label number maximization problem LNM: Given a set of rectangular labels, each of which belongs to a point feature in the plane, the task is to find a *labeling* for a largest subset of the labels. A labeling is a placement such that none of the labels overlap and each is placed so that its boundary touches the corresponding point feature.
The purpose of this paper is twofold: We present a new force-based simulated annealing algorithm to heuristically solve the problem and we provide the results of a very thorough experimental comparison of the best known labeling methods on widely used benchmark sets.
The design of our new method has been guided by the goal to produce labelings that are similar to the results of an experienced human performing the same task. So we are not only looking for a labeling where the number of labels placed is high but also where the distribution of the placed labels is good.
Our experimental results show that the new algorithm outperforms the other methods in terms of quality while still being reasonably fast and confirm that the simulated annealing method is well-suited for map labeling problems.

1 Introduction

Automated map labeling attracts many researchers in computer science due to its numerous applications, *e.g.*, in cartography, geographic information systems, point pattern analysis, spatial statistics, and graphical interfaces.

The growing amount of data for which informational graphics have to be produced leads to an increasing need for automatic labeling procedures. Due to the complexity of the underlying problems, manual map labeling is a tedious and time-consuming task. In addition to the classical problem of labeling a two-dimensional cartographic map, labeling problems arise in geographic information systems, navigation systems, and fully automatically generated technical maps where manual labeling is impossible.

1.1 Point-Feature Map Labeling

A major problem in map labeling is the *point-feature label placement* where the task is to place labels adjacent to point features so that only a few or no labels overlap. These features may be cities, mountain peaks, or points in a plot that represent statistical data. Besides point features, also area-features, like countries and seas, or line-features, like streets and rivers, can be labeled. In this paper we focus on point-feature labeling and restrict the labels to be iso-oriented axis-parallel rectangles; the bibliography [16] maintained by Wolff is a good starting point for literature on other models.

Several criteria have been developed that characterize a high-quality labeling:

(C1) On a good map the placement of labels is as unambiguous as possible. It is intuitively apparent to the reader of the map which label belongs to which point feature. This implies that labels are close to the point features they belong to.
(C2) The information of the labels is legible.
(C3) No or only a few labels overlap. Obviously, overlaps decrease the legibility of a map.
(C4) The number of omitted labels is low.

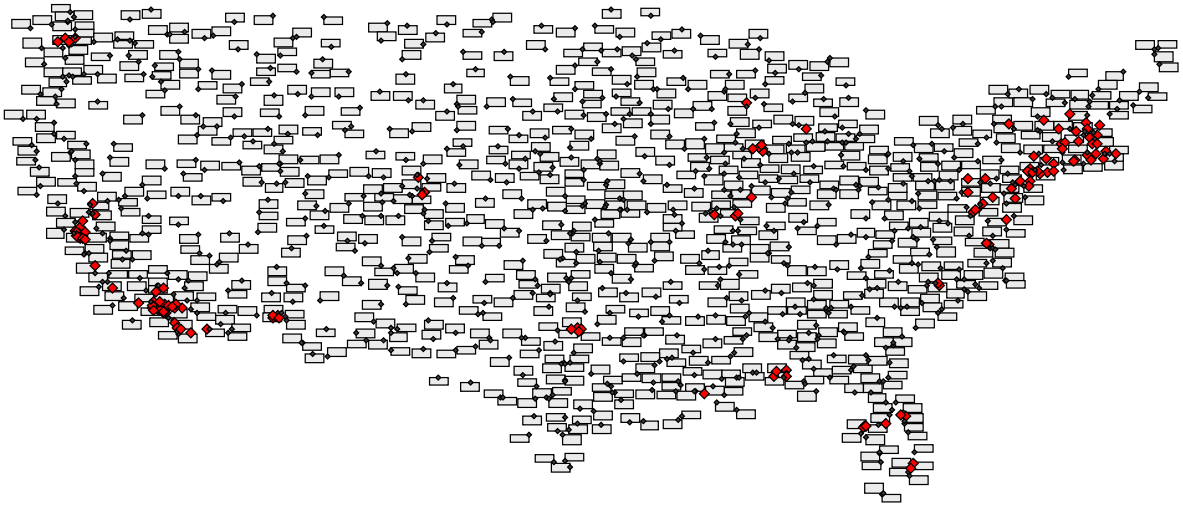


Fig. 1. Sample labeling of 910/1041 point features produced with our algorithm in the slider model. Larger dark points represent cities whose labels could not be placed.

The cartographic literature contains more rules, see, *e.g.*, the papers by Imhof [8] and Yoeli [17]. Yet, the overall aim in automatic map labeling is to devise algorithms that produce labelings of maximum legibility.

An instance of a labeling problem consists of a set of point features, information about the label sizes, and a mapping from labels to point features. In general it is not possible to place all the given labels in their original size without any overlap. The literature suggests several possibilities to deal with this problem; among these are decreasing the size of the labels to allow a placement of all labels without any overlap, and keeping the sizes of the labels fix while looking for the maximum number of labels that can be placed. The first possibility is referred to as the *label size maximization problem*, the second one as the *label number maximization problem*.

Research in automated map labeling has mainly focused on the six *labeling models* shown in Figure 2, the most popular of which are the four-position and the four-slider model. The dots in the figure represent the point feature to be labeled.

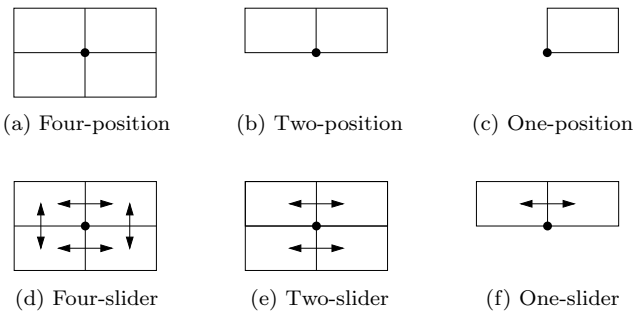


Fig. 2. Axis-parallel rectangular labeling models. A label can be placed in any of the positions indicated by the rectangles and can slide in the directions of the arrows

Definition 1 (Label Number Maximization problem). *Given a set $\Lambda = \{\lambda_1, \ldots, \lambda_k\}$ (the labels), two functions $w, h : L \to \mathbb{R}$ (the widths and heights of the labels), and a function $a : \Lambda \to \mathbb{R}^2$ (the points to be labeled), find a subset $\Lambda' \subseteq \Lambda$ of largest cardinality and a function $\rho : \Lambda' \to \mathbf{R}$, where $\mathbf{R}$ is the set of axis-parallel rectangles in the plane, so that the following conditions hold:*

(L1) Rectangle $\rho(\lambda)$ has width $w(\lambda)$ and height $h(\lambda)$ for every $\lambda \in \Lambda'$.
(L2) Point $a(\lambda)$ lies on the boundary of $\rho(\lambda)$ for all $\lambda \in \Lambda'$
(L3) The open intersection $\rho(\lambda) \cap \rho(\mu)$ is empty for all $\lambda, \mu \in \Lambda'$, $\lambda \neq \mu$.

An assignment of labels to rectangles that satisfies the three properties (L1)–(L3) is called a *labeling*. Properties (L1) and (L2) make sure that each label λ is drawn with the given size and attached correctly to its point feature $a(\lambda)$. Property (L3) forbids overlaps between the labels.

1.2 Previous Work

Force-based methods in graph drawing. Force-directed methods have originally been developed for drawing graphs. In practice, these techniques often perform remarkably well on medium-sized instances and are easy to implement. Further, the resulting drawings typically capture symmetries while avoiding the expensive computations to look for them explicitly.

These algorithms, going back to Eades [3] and Kruskal and Seery [12], view the input graph as a system of objects with forces acting between them. Configurations of the objects with low energy correspond to aesthetically pleasing layouts of the graph. Algorithms for this task are mostly variations of iterative gradient-based methods such as the Newton-Raphson method.

Davidson and Harel consider in [2] the number of edge crossings in a drawing as an additional, discrete term in the objective function and can therefore not apply gradient methods to find an equilibrium. The authors propose the *simulated annealing* approach introduced in [9] by Kirkpatrick, Gelatt Jr., and Vecchi. This approach defines for each configuration a finite set of neighboring configurations and tries one of them at random. New configurations are always accepted if they decrease the energy of the system but even if they increase the energy, they are accepted with a probability that decreases with time. As we will point out in Section 2, we will use simulated annealing for similar reasons as Davidson and Harel.

Point-feature map labeling. Most previous work on automated point-feature map labeling concentrates on the discrete models, where a label can only allocate a finite number of positions with respect to its point feature. See the bibliography [16] for an overview.

Van Kreveld, Strijk, and Wolff [15] define several variations of slider models, namely, the one-, two-, and four-slider model (see Figure 2) and show *NP*-completeness of the decision problem in the four-slider model (independently, Marks and Shieber have shown this in [13]). The main result in [15] is a $\frac{1}{2}$-approximation algorithm that is able to find a solution of LNM in any of the slider-models with unit height rectangles. The algorithm is a $\Theta(n \log n)$-time greedy sweep-line algorithm. For the same models, the authors develop a polynomial time approximation scheme. The respective algorithms label at least $(1 - \varepsilon)$ times the optimum number in overall running time $O(n^{4/\varepsilon^2})$. Strijk and van Kreveld extend the above mentioned $\frac{1}{2}$-approximation algorithm for the slider models in [14] to labels with different heights. If r denotes the number of different label heights, the running time of the algorithm is $O(rn \log n)$. The algorithm is based on the simple greedy strategy of iteratively placing the *leftmost label* until no more points can be labeled without intersections. The *leftmost label* is defined to be the label, whose right edge is leftmost among all label candidates, which are those labels that have not been placed yet minus a set of labels that are already known to be unplaceable in the current configuration.

Klau and Mutzel present in [11] an exact algorithm for the label number maximization problem that works in any of the labeling models. The method is based on a pair of so-called constraint graphs that code horizontal and vertical positioning relations. The key idea is to link the two graphs by a set of additional constraints, thus characterizing all feasible solutions of LNM. This combinatorial description enables the formulation of a zero-one integer linear program whose solution leads to an optimal labeling.

The paper [1] by Christensen, Marks, and Shieber contains an extensive computational study of labeling methods in the four-position model. The authors also present a simulated annealing method for this problem

that is the clear winner of the study in terms of labeling quality while still being reasonably fast. Furthermore, they propose a procedure for randomly creating labeling instances. We use this benchmark generator, which has become a widely used tool in map labeling research, for our computational experiments in Section 3.

Force-based methods in map labeling. Already in 1982, Hirsch introduced a model that is similar to the four-slider model and proposed an algorithmic labeling method that can be interpreted as a force-directed approach. The algorithm starts with an initial label placement and tests for overlaps. Based on the amount of intersecting area, overlap vectors are computed for labels involved in an overlap conflict. For each label, the summation of these vectors helps in heuristically deciding where to move the label. Successive movements of all labels in conflict, which Hirsch calls a *map sweep*, is done by using one of the following two methods: (a) Moving labels in the direction of the computed vectors with sequential stops at preferred positions. This method allows the label to be placed at any possible position. (b) Performing a discrete jump to a position indicated by the vector angle. Here, the primary aim is to solve an overlap situation where the first method fails.

Hirsch does not consider the number maximization problem explicitly, but rather proposes interactive conflict resolution with the help of an experienced cartographer. Also, although his overlap vectors resemble the intersection-proportional component within our force system, he does not consider distance-related forces and suggests a different method for finding an equilibrium of minimum energy. His approach can be seen as a gradient-driven heuristic.

In a way, Hirsch's ideas are also applied in [5], a patent specification authored by Feigenbaum. The problem considered in the patent is related to, but significantly different from, the label number maximization problem: Here, labels should be placed as close as possible to the corresponding point features. The author defines forces that attract each label to its associated point feature and repulsive forces that reject labels from other features on the map. In contrast to our labeling model, labels in the referred patent should be placed as near as possible to their features, which allows a significantly higher number of labels to be placed (if the system cannot find a position without overlaps after a constant number of movements, overlapping labels are marked for the "intervention of an external operator"). The described system searches its final state by applying incremental moves according to the calculated forces. Initially, every label has size zero and grows slowly to its original dimensions to facilitate movements in dense areas.

1.3 Contribution and Overview

We present a new and very efficient heuristical approach in the most general of the labeling models, the four-slider model. Our new method is hybrid in the sense that it is based both on an underlying system of forces and a higher-ranking simulated annealing method. The abstraction of the problem to the virtual force system allows us to implement additional aesthetic criteria and to compute placements with good label distribution in a short amount of time. Furthermore, our algorithm can be applied as a postprocessing step to improve existing labelings. We find that the results often look similar to those of a human cartographer. Figure 1 on the first page of this paper shows a typical output produced by our algorithm.

The rest of this paper is organized as follows: Section 2 presents the new approach. We start by explaining the underlying force model and point out that it is not sufficient to use a classical approach to find a state of low energy within this system. We propose to combine the force directed method with simulated annealing to overcome this drawback and conclude the section by presenting our hybrid approach. Our experimental results in Section 3 show that our algorithm produces very good results very efficiently. To test the quality of the solutions computed by our new method, we have compared its results to optimal solutions of a large set of benchmark labeling instances. Finally, Section 4 concludes the paper and presents possible extensions and future lines of research.

2 Force-Directed Map Labeling

In this section we describe our force-based simulated annealing algorithm for the label number maximization problem. Our approach uses repulsive forces between labels, which are used to compute a force vector for

each label. The length and direction of these vectors gives us an idea of where to place individual labels and how to solve potential conflicts between two or more labels.

As a side-effect we achieve another important benefit, which makes the method usable for practical applications: Our forces are defined to grow super linearly with decreasing distance between two labels. Therefore, labels are not placed close to each other if possible and the method achieves a good distribution of the labels in the available space. This improves the readability of the labels and results in an aesthetically pleasing arrangement.

To avoid being trapped in local minima of the energy function, we combine the purely force directed method with the simulated annealing approach. A short overview of this method is given in Section 2.3.

2.1 Designing the Force Model

Every force-directed algorithm consists of two major parts: (a) a force-system between the objects and (b) a method that seeks an equilibrium of minimum energy.

In our case a low energy equilibrium configuration should correspond to a pleasing labeling. In contrast to applications in graph drawing our labels are bound to their point feature and may not be positioned freely in the available space. Each label must be placed in such a way that it touches its point feature and does not overlap other labels. Since we only allow (intermediate) positions that satisfy at least the first condition, we do not need any attractive forces between a point and its associated rectangle. Furthermore we restrict the computation of forces to pairs of labels that might intersect. We call the set of those labels for each label λ the *neighborhood* $N(\lambda)$.

Definition 2. *The* neighborhood *of a label $\lambda \in \Lambda$ with width $w(\lambda)$, height $h(\lambda)$ and associated point $a(\lambda) = (x_\lambda, y_\lambda)$ is*

$$N(\lambda) = \{\mu \in \Lambda \mid w(\lambda) + w(\mu) \geq |x_\lambda - x_\mu| \wedge h(\lambda) + h(\mu) \geq |y_\lambda - y_\mu|\} \ .$$

Our main goal is to place as many labels as possible in the available space without any intersections. Therefore the decisive factor in our force system is the amount of intersection between two labels. We call this force the *intersection-proportional* component. The amount of the second force acting in our model, the so-called *distance-related* part, depends on the distance between two labels and grows, if two rectangles are placed close to each other. If labels overlap a very small area ϵ the intersection-proportional component can become arbitrary small. Thus we add a constant value to the force function if and only if two labels overlap to punish this overlaps stronger. The distance-related part is not the significant value in our model, its only purpose is to guide the algorithm to a well distributed labeling.

Definition 3. *For every two labels $\lambda, \mu \in \Lambda$, we define $d_{\min} : \Lambda \times \Lambda \to \mathbb{R}$ as*

$$d_{\min}(\lambda, \mu) = \begin{cases} 0 & \text{if } \lambda \text{ and } \mu \text{ overlap} \\ \min\{\|p, q\| \mid p \in \lambda, q \in \mu\} & \text{otherwise} \ . \end{cases}$$

The function $\|p, q\| : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ denotes the Euclidean distance between the two points p and q in the Euclidean plane $\mathbb{R}^2$.

Further we define $i_x(\lambda, \mu) : \Lambda \times \Lambda \to \mathbb{R}$ and $i_y(\lambda, \mu) : \Lambda \times \Lambda \to \mathbb{R}$ as the amount of intersection between labels λ and μ in horizontal and vertical direction, respectively.

We can now define the force function $f = (f_x, f_y)$ for each label in the following way:

Definition 4. *For each label $l \in \Lambda$ with center point $c_l = (x_l, y_l)$, the x-component of the force function $f_x : \Lambda \to \mathbb{R}$ is defined as*

$$f_x(l) = \sum_{r \in N(l)} \frac{(f_i(l, r) + f_a(l, r) + f_d(l, r))\ (x_l - x_r)}{\|c_l, c_r\|} \ . \tag{1}$$

The functions $f_i : \Lambda \times \Lambda \to \mathbb{R}$, $f_a : \Lambda \times \Lambda \to \mathbb{R}$ and $f_d : \Lambda \times \Lambda \to \mathbb{R}$ are defined as follows:

$$f_i(l, r) = \delta_1\, i_x(l, r)\, i_y(l, r) \ , \tag{2}$$

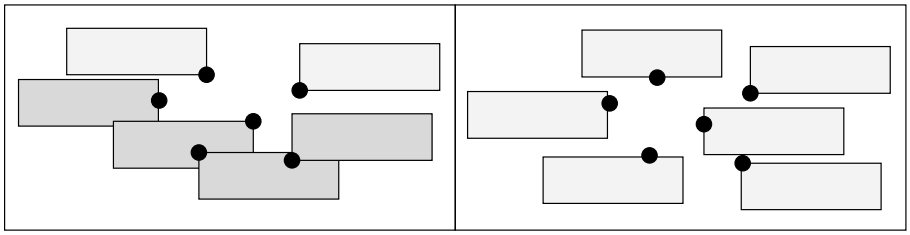


Fig. 3. A bad local minimum and an optimal labeling of the same instance

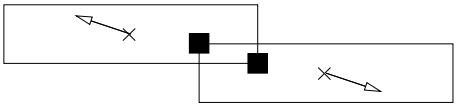


Fig. 4. Example configuration where the forces cannot resolve an overlap

$$f_d(l,r) = \frac{\delta_2}{\max(\varepsilon, d_{\min}(l,r))^2} \quad , \tag{3}$$

$$f_a(l,r) = \begin{cases} \delta_3 & \textit{if } \lambda \textit{ and } \mu \textit{ overlap} \\ 0 & \textit{otherwise} \ . \end{cases} \tag{4}$$

The y-component f_y *is defined analogously.*

The constants $\delta_1, \delta_2, \delta_3 \in \mathbb{R}$ control the influence of the particular term on the force function f. Note that the direction of the force between two labels is defined by the location of their center points and that ε limits the amount of f_d to a value of δ_2/ε^2.

2.2 Drawbacks of a Purely Force Directed Method

"Force has no place where there is need of skill." [6]

A purely force directed method performs poorly if the labels take a significant fraction of the available drawing area. There is only little space for manoeuvre when seeking an equilibrium, especially if incremental methods are used. Often, real-world labeling instances contain dense areas that do not leave much space for moving labels around without producing new intersections. The problem is aggravated by the fact that we only allow horizontal and vertical moves around the label's border. The same observation holds for the algorithm proposed by Hirsch in [7] and is well described by Christensen, Marks and Shieber in [1].

Figure 3 shows an example of a bad local minimum that is difficult to escape from by using incremental moves. It is not possible to transform the bad labeling on the left continuously into the good labeling on the right without a temporary increase of overlaps and thus of the overall energy of the system.

Another problem arises from the direction of our forces. We designed them in Section 2.1 to act between the center points of two labels. But since labels have non uniform size and they are bound to their point features, the direction of our resulting force vector does not always indicate a solution for the conflict. Figure 4 shows a very simple example consisting of just two point features. Any algorithm that strictly follows the direction of the force vector is not able to resolve the shown configuration, even though the optimal solution is self-evident.

Therefore, we need to accept worse intermediate configurations to be able to escape local minima and we propose to use the simulated annealing method for this purpose. Details of the adaptation of the general simulated annealing algorithm to our needs are described in the next section.

2.3 The Hybrid Algorithm

Simulated annealing is a very flexible optimization method and can be used in a wide range of combinatorial optimization problems. It has been proposed in [9] and is derived from the following observation: When cooling down a liquid rapidly to a crystal form, the system results in amorphous structures with a high energy while slow cooling results in a crystal structure with lower energy.

The general simulated annealing procedure applies a series of sequential moves while simultaneously decreasing the temperature. The main idea is that the probability with which the change from a state with energy E_1 to a state with energy E_2 will be accepted is $e^{-\frac{E_2-E_1}{k\,T}}$, where k is a positive constant. Thus the probability for moves that increase the energy decreases with a falling temperature.

We now present the hybrid force-based simulated annealing algorithm for the label number maximization problem and describe some of its implementation details. The necessary steps are outlined in pseudocode in Algorithm 1.

Algorithm 1: Force-directed point feature labeling

compute random initial labeling σ in the eight-pos. model
initialize temperature T and cooling rate α
compute forces for the current configuration and initialize the set of overlapping labels Φ.
$M_{\max} \leftarrow 30|\Lambda|$; taken $\leftarrow$ rejected $\leftarrow 0$;
repeat
  $\hat{\sigma} \leftarrow \sigma$; {*save current configuration* σ}
  choose random candidate $\lambda \in \Phi$
  if $|f_x(\lambda)| > F_{\min} \vee |f_y(\lambda)| > F_{\min}$ **then**
    change σ by moving λ in the direction indicated by its force vector
    if $|f_x(\lambda)| > F_{\min} \vee |f_y(\lambda)| > F_{\min}$ **then**
      change σ by moving λ to a random position
  else
    change σ by moving λ to a random position
  if force(σ) < force($\hat{\sigma}$) $\vee$ random $r \in [0 \dots 1] < e^{\frac{\text{force}(\sigma)-\text{force}(\hat{\sigma})}{T}}$ **then**
    taken $\leftarrow$ taken + 1
    update set of obstructed labels Φ
  else
    rejected $\leftarrow$ rejected + 1
    $\sigma \leftarrow \hat{\sigma}$;
  if taken + rejected $\geq M_{max}$ **then**
    if taken = 0 **then**
      if point selection is disabled $\vee \neg\exists$ overlapping label $\hat{\lambda} \in \Phi$ **then**
        return current labeling σ
      else
        $\sigma \leftarrow \sigma \setminus \hat{\lambda}$ {*remove label* $\hat{\lambda}$ *from the current solution* σ}
        update set of obstructed labels Φ
    $T = \alpha T$ {*decrease temperature* T}
    $M_{\max} = \max(|\Lambda|, \min(10|\Lambda|, 50|\Phi|))$; taken $\leftarrow$ rejected $\leftarrow 0$
until $|\Phi| = 0$
return current labeling σ

The algorithm performs a series of temperature stages. After each stage the temperature is decreased by a constant precomputed factor, which decreases the probability of accepting moves that lead to a higher energy state. To speed up convergence we compute a set of obstructed labels Φ, which either intersect at least one other label or their associated force vector indicates movement to a new position with lower energy. The algorithm returns the current solution if $|\Phi| = 0$ and chooses the label with the most overlaps for removal if no move has been accepted for a full temperature stage.

In each iteration we randomly choose a label $\lambda \in \Phi$ and try to move it according to its force vector. If this move does not lead to an equilibrium or the force vector does not indicate movement even though the

label is involved in an overlap, we move the label to a random position in the eight-position model instead. The new position is always accepted if it decreases the energy and may be accepted if it does not increase the energy by more than the current temperature allows.

At each temperature stage we perform $M_{\max}$ moves. We initialize this value with $30|\Lambda|$ and perform $\max(|\Lambda|, \min(10|\Lambda|, 50|\Phi|))$ moves in all subsequent stages. The initial temperature is chosen such that we accept an increase in the overall force of f_{avg} with a probability of 30%, where f_{avg} represents the amount of force for an overlap of 50% of two average sized labels. The cooling rate α is chosen such that the temperature T becomes less than 1 after 15 stages. The parameter α should be changed to adjust the trade-off between quality and speed. The above settings yield high-quality labelings in reasonable computation time.

Whenever we move a label λ to a different position or remove it from the labeling in line 25, the forces on all labels $\lambda' \in N(\lambda)$ change. Since a simple approach takes time $O(n^2)$ in the worst case we store the forces between each pair of labels in a quadratic matrix. This enables us to update the forces in linear time by recalculating only the change of the particular addend for each neighbor $N(\lambda)$. Furthermore we have to update the set of obstructed labels Φ, since some labels $\lambda' \in N(\lambda)$ may have to be added to or removed from this set.

Since labels have to be placed according to the four-slider model, moving a label alongside its force vector becomes more difficult than moving, *e.g.*, zero-sized nodes in a graph drawing application. A position that corresponds to an equilibrium of the forces is not always valid with respect to the point. Furthermore our forces depend on a combination of the overlapping area and the distance between two labels, which are both defined differently depending on the specific domain, and are thus not continuous[1]. Thus we can not apply numerical algorithms like the Newton-Raphson method or similar techniques, since they require at least the first derivation of the function. In place of this we start moving the label by 20% of the remaining width/height in the particular direction and halve the amount of movement if the indicated direction changes until we achieve an equilibrium or the maximum number of moves has been performed.

We perform at least $|\Lambda|$ moves before removing a heuristically chosen label. Thus the running time depends to a great extent on the number of labels that the algorithm cannot place. Most problem instances in our test suits of real world labeling problems do not contain many of these unplaceable labels. Therefore, our method performs well on these problems. However, if running time is a critical criterion, this step can be replaced through a faster cleanup heuristic.

3 Computational Study

In this section we report on the extensive computational experiments we have performed to evaluate quality and resource requirements of our new method in comparison to the best-known algorithms for label number maximization. Due to space limitations, we will only provide a summary, more detailed results can be found in [4]. Section 3.1 presents the algorithms under evaluation and contains details on their implementation. In Section 3 we discuss our observations. We want to emphasize that both the data we used and our implementation of the evaluated algorithms are publicly available under the Gnu General Public License at `http://www.ads.tuwien.ac.at/research/labeling`.

3.1 Evaluated Labeling Algorithms

We have implemented all major map labeling algorithms that we found in the literature on point feature map labeling in the slider model. All computations were done on a Pentium 4 with 2.8GHz and 2GB of RAM. For each run, we set a limit of 30 minutes computation time.

- The algorithm RANDOM, which places labels randomly, if possible, has been incorporated into the study only for comparative reasons.
- Christensen, Marks and Shieber present in [1] a simulated annealing approach that beats most other algorithms in both speed and quality. Since their implementation uses the four-position model, in general, the quality of their solutions cannot be as good as those of algorithms for the four-slider model.

[1] Remember that the distance between to labels remains constant while they overlap in respect to the horizontal or vertical position, but grows with $\sqrt{\delta_x^2, \delta_y^2}$ if not.

Nevertheless we decided to include this algorithm in our computational study to compare one of the best known labeling methods in the four-position model to the remaining algorithms.
To be able to compare our results to those published in [1] we used the same cooling scheme and used the number of pairwise overlaps plus the number of deleted labels as objective function in SA4POS, our implementation of this method. Furthermore we isolated configuration changes to either obstructed or deleted labels, since this causes the algorithm to converge much faster.

– As already described in Section 1.2 the model proposed by Hirsch in [7] is quite similar to the slider model and thus we decided to include this algorithm in our study (HIRSCH). As shown in Figure 5 preferred positions are defined around a circle, whose center point represents the particular point feature. Furthermore any intermediate position tangent to the circle in each of the four quadrants is allowed. In addition each label is allowed to slide along a line tangent to the circle at each of the quadrant boundaries (0, 90, 180 and 270 degrees). In order to compare the quality of Hirsch's algorithm against the remaining

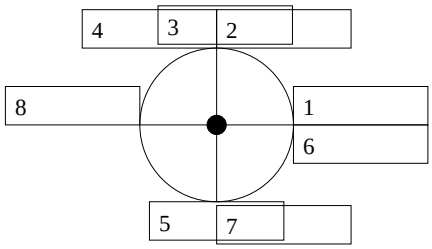


Fig. 5. Preferred positions on the circle in Hirsch's model. Positions 3 and 7 are slightly displaced for better legibility.

methods, we followed the suggestion of Christensen, Marks, and Shieber in [1] and reduced the radius of the circle to zero. Furthermore we neglect any cartographic preferences, since they are also ignored in the rest of the considered algorithms.

– APPROX is our implementation of the algorithm described in [15] and [14]. In order to achieve the $O(n \log n)$ time bound, the authors use three heaps, which contain horizontal and vertical line segments that either intersect the current frontier, lie completely right of it, or both. Their purpose is to determine the next label to be placed. The placement of a new label changes the frontier, and the elements in the heaps may move, cease to exist or new (vertical) segments may arise. Therefore the algorithm requires two priority search trees and red-black trees for each vertical segment.
Since this is quite complicated we decided to implement another variant that runs in $O(n^2)$, does not rely on unique label heights, and is quite fast in practice.
– We computed optimal labelings in the 4-slider model using OPT, an implementation of the algorithm presented in [11]. Note that, due to the running time limit, only instances up to maximally 850 labels could be computed.
– Finally, FDL is our JAVA implementation of the new force-directed method.

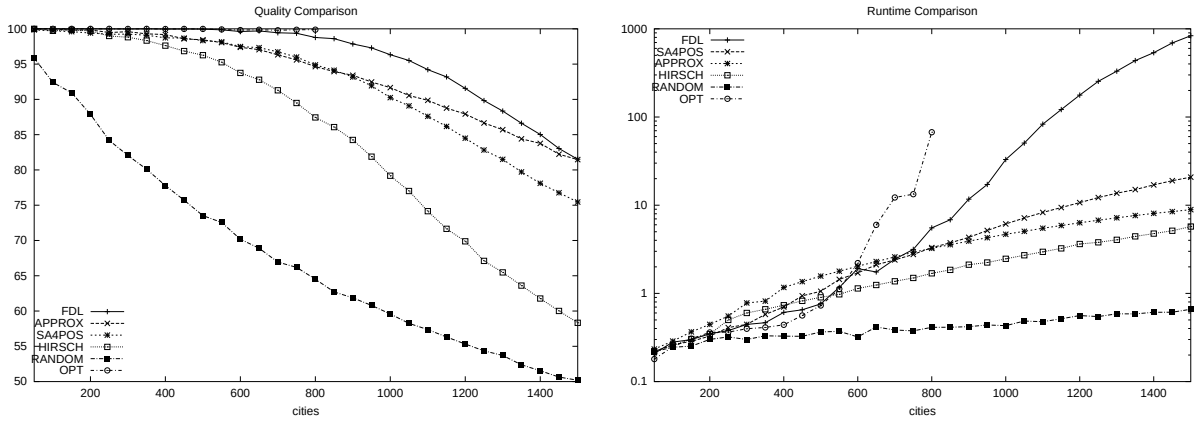
3.2 Results

We ran the implementations on different data sets. Among them are (a) instances generated with the widely used benchmark generator by Christensen, Marks, and Shieber and (b) instances derived from real world data giving the positions of ground water drill holes in Munich.

We generated 25 random problem instances of type (a) for each instance size in $\{100, 150, \ldots, 1450, 1500\}$ labels, resulting in 685 instances, as in the study on the four-position model [1]. The numbers of labels in the real-world problem set (b) are in the set $\{250, 500, 750, \ldots, 2750, 3000\}$ and there are 30 instances for each number of labels.

Figure 6(a) illustrates the performance of the evaluated algorithms in terms of quality, whereas Figure 6(b) displays their running time behavior. Of course, OPT performs best in terms of quality but also needs the

largest amount of resources. Among the heuristic methods, our new algorithm produces the best scores but also takes more time to compute them—especially for large instances. We want to remark, however, that the random instances larger than 1000 labels do not resemble real-world instances since they get very dense (see the discussion on the real-world Munich drill hole instances below). The plots also reveal that the approximation algorithm performs surprisingly well in terms of quality (for very large instances it becomes as good as FDL) with the advantage that its running time does not explode.



(a) Percentage of labeled point features

(b) Runtime of the algorithms in seconds

Fig. 6. Results for the random benchmark set

We then compared the heuristic methods on the easier real-world instances. Figures 7(a) and 7(b) show the results. It can be seen that all methods apart from RANDOM have quite good results with FDL being the winner. In fact, these instances have been generated so that always 100% of the labels can be placed—even in the four-position model. FDL is the only method that achieved the perfect score on all instances. As already mentioned at the end of Section 2.3, the running time of FDL depends heavily on the number of labels that cannot be placed. As this number is zero for these instances, the running time behavior is very good for FDL as for all other methods apart from the approximation algorithm.

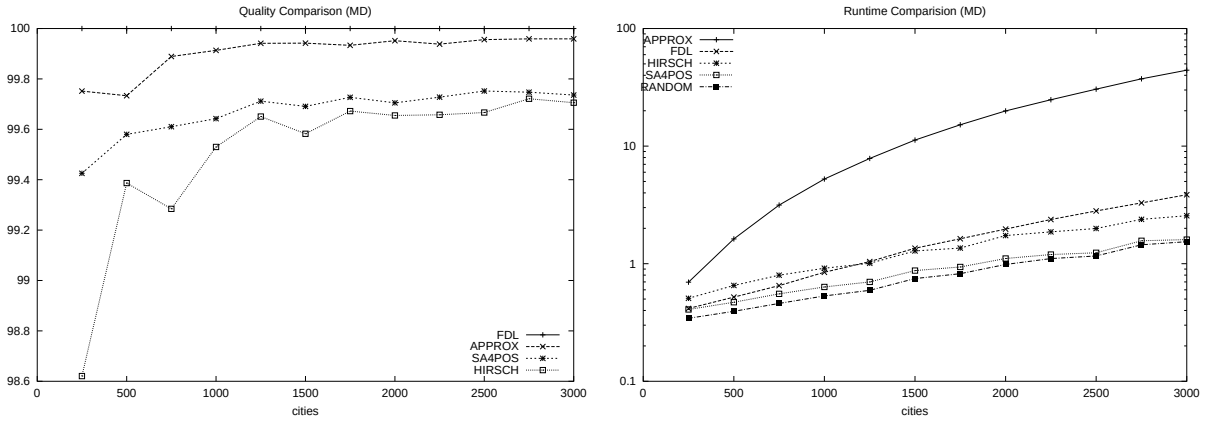
In Figure 8 we show the results for a typical random instance. Our computational results confirm the outcome of the 1995 study [1]: simulated annealing is very well-suited for labeling problems and outperforms other methods in terms of quality.

4 Conclusions

We have presented a new hybrid heuristical approach for the label number maximization problem. Our algorithm uses an underlying force system that serves two purposes. First, a minimum energy configuration of this system corresponds to placements with evenly distributed labels that is appealing to a human observer. The second task of the force system is to determine which labels should be left out to obtain a labeling without overlaps. We combine this with a simulated annealing algorithm to escape local minima.

Our extensive computational experiments on widely used benchmark data show that our algorithm finds labelings that are close to optimality in a short amount of computing time. We find that our results often look similar to those of a human cartographer.

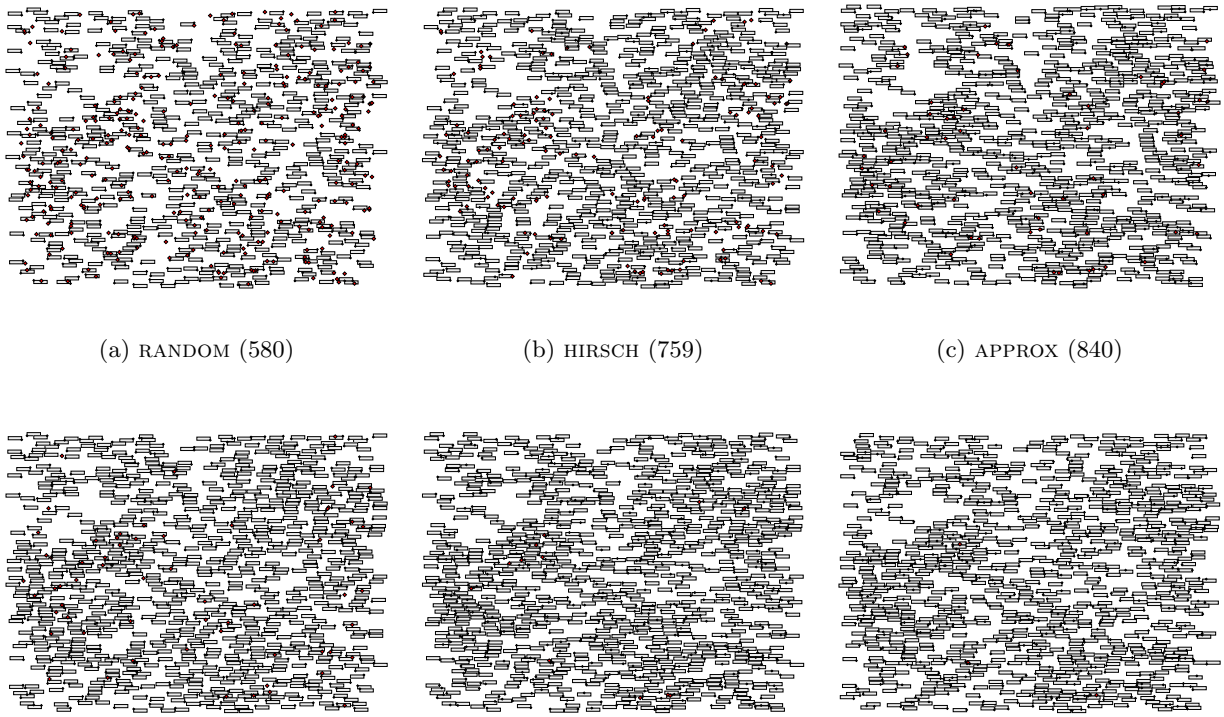
Future lines of research might include to adapt the approach to line and area labeling. We will also investigate how to combine force-based graph drawing with our approach to attack the combined drawing and labeling problem. Further, we want to integrate the approach into the Human-Guided Search (HuGS) system, see [10], to allow for human interaction.



(a) Percentage of labeled point features (RANDOM is far behind and thus not shown)

(b) Runtime of the algorithms in seconds

Fig. 7. Results for the real-world benchmark set



(a) RANDOM (580)

(b) HIRSCH (759)

(c) APPROX (840)

(d) SA4POS (840)

(e) FDL (888)

(f) OPT (897)

Fig. 8. Results for a typical randomly generated instance with 900 labels

Acknowledgments

We thank Tycho Strijk for useful discussions regarding the implementation APPROX and one of the anonymous referees of a previous version for his constructive feedback.

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