Heuristic Approaches for the Probabilistic Traveling Salesman Problem

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1 Introduction

The probabilistic traveling salesman problem (PTSP) was introduced by Jaillet [6] as an extension of the classical traveling salesman problem (TSP). In the PTSP, the salesman does not necessarily have to visit every node, but a probability of requiring a visit is given for each node. The goal is to find an a-priori tour that includes every node and minimizes the expected length of an a-posteriori tour which contains the nodes with the given probabilities and skips the other ones. A real world application of the PTSP would be e.g. to plan an every-day tour for the postman. After each delivery he always proceeds to the next address on the pre-planned tour for which a delivery is required.

Formally we are given a complete graph $G = \langle V, E\rangle$ with V containing n nodes and E containing m edges. Each edge $(i, j) \in E$ is assigned a cost $d_{ij} > 0$ and each node $v \in V$ has a probability p_v of being visited. If the probabilities p_v are equal for every node, the problem is called homogeneous and otherwise it is heterogeneous [5]. A solution is an a-priori tour $T = \langle v_1, \ldots, v_n\rangle$ over all nodes with v_n connected to v_1 and its expected costs are defined as

$$c(T) = \sum_{i=1}^{2^n} p(R_i) \cdot L(R_i)$$

where R_i is a possible realization, i.e., one possible a-posteriori tour, and $p(R_i)$ its occurrence probability. Since there are exponentially many different realizations, it is in practice not convenient to obtain the objective value in such a way. Therefore Jaillet [6] showed that the expected length can be calculated in $O(n^2)$ time by explicitly enumerating all realizations using the following term:

$$c(T) = \sum_{i=1}^{n} \sum_{j=i+1}^{n} d_{v_i v_j} p_{v_i} p_{v_j} \prod_{k=i+1}^{j-1} (1 - p_{v_k}) + \sum_{j=1}^{n} \sum_{i=1}^{j-1} d_{v_j v_i} p_{v_i} p_{v_j} \prod_{k=j+1}^{n} (1 - p_{v_k}) \prod_{k=1}^{i-1} (1 - p_{v_k})$$

The PTSP is a well studied problem in the literature. Bianchi et al. [4, 3] proposed metaheuristic approaches based on ant colony optimization and local search. Balaprakash et al. [1] analysed sampling and estimation based approaches. Weyland et al. [8] considered new sampling and ad-hoc approximation methods for local search and ant colony system. Marinakis and Marinaki [7] proposed a hybrid swarm optimization approach.

In this work we consider different heuristic approaches for the PTSP. First we analyze various popular construction heuristics for the classical TSP applied on the PTSP: nearest neighbor, farthest insertion, nearest insertion, radial sorting, space filling curve. Then we investigate their adaptations to the PTSP and compare them to their classical counterparts. The performances differ greatly depending on the problem type, instance size and the visiting probabilities. For example it was shown that an optimal PTSP solution may intersect itself on Euclidean instances whereas this is not possible for the classical TSP [6]. Therefore this property may not be relied on by the algorithms. In order to further improve the solutions from the construction heuristics, we use two popular neighborhood structures in a variable neighborhood search (VNS) framework: the 2-opt neighborhood and 1-shift neighborhood. Since evaluating a solution for the PTSP is much more expensive than for the classical TSP, we use a delta evaluation proposed by Bertsimas et al. [2]. The idea is the same as in the case of the classical TSP: When comparing two solutions, the calculations on the part of the solution that is not relevant can be skipped. Therefore the complexity for evaluation can be reduced from $O(n^2)$ to $O(1)$. We will extend this concept to other neighborhood structures as well. We ran preliminary tests on TSPlib instances from the literature and the VNS turned out to be an efficient and powerful approach for the PTSP. Compared to other state-of-the-art approaches, it was able to exceed their results on most of the tested instances and identify new best solutions.

References

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