Reallocating Wasted Votes in Proportional Parliamentary Elections with Thresholds

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Abstract

In many proportional parliamentary elections, electoral thresholds (typically 3-5%) are used to promote stability and governability by preventing the election of parties with very small representation. However, these thresholds often result in a significant number of "wasted votes" cast for parties that fail to meet the threshold, which reduces representativeness. One proposal is to allow voters to specify replacement votes, by either indicating a second choice party or by ranking a subset of the parties, but there are several ways of deciding on the scores of the parties (and thus the composition of the parliament) given those votes. We introduce a formal model of party voting with thresholds, and compare a variety of party selection rules axiomatically, and experimentally using a dataset we collected during the 2024 European election in France. We identify three particularly attractive rules, called Direct Winners Only (DO), Single Transferable Vote (STV) and Greedy Plurality (GP).

1 Introduction

Many democracies elect their parliaments using proportional representation, typically implemented using party lists of candidates, with each voter voting for one party. Most countries impose an *electoral threshold*, a minimum percentage of votes (usually between 0 and 6%) that is necessary for a party to enter parliament [14, 27]. Lists that do not gather the required votes get no seats, and the votes for those lists are "lost" or "wasted" and not used to distribute seats in parliament. In this paper, we will explore ways to prevent this phenomenon of wasted votes.

Not all jurisdictions that use proportional representation impose a threshold. While no votes are lost in such a system, it comes with the risk of having a fragmented parliament with many parties, which makes forming and maintaining a governing coalition difficult. Infamously, the Weimar Republic (1918–1933) did not use a threshold and saw the number of parties present in the *Reichstag* steadily grow to up to 15 in 1930. This led to political chaos: over 13 years there were 16 governments (only five of which had a majority) and 8 elections. The lack of a threshold and the resulting popular dissatisfaction with the political system are widely seen as one contributing factor to the rise of National Socialism [13], though the magnitude of its influence is disputed [1]. Citing this experience, post-war Germany instituted a 5% threshold in 1953.

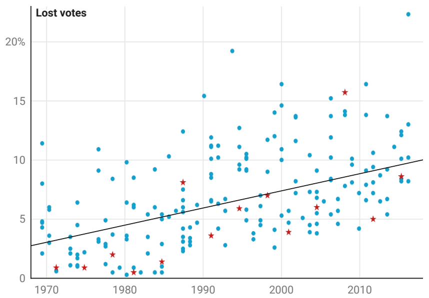

Figure 1: The fraction of lost votes in German federal and state elections (shown as red stars and blue circles, respectively) between 1970 and 2022. Reproduced from Benken [3].

While thresholds limit the number of parties in parliament and improve governability, they also have drawbacks. They can lead to a significant number of wasted votes, which violates the principles of proportionality and equality of votes. Benken [3] has cataloged the fraction of votes that were lost in German elections since 1970 and finds a steady upward trend (Figure 1). His dataset includes the 2022 election in the state of Saarland, where a record of 22% of votes were lost. Another example is the 2019 election for the French members of the European Parliament, where the 5% threshold led to 19.8% of wasted votes. An extreme example is Turkey's 2002 election that used a 10% threshold (since reduced to

7%) and where 46% of votes were lost – resulting in a party with 34% of the votes obtaining almost a two-thirds majority in parliament [26]. Besides wasting votes and thereby reducing representativeness or even creating false majorities, thresholds also discourage the formation of new parties, hinder the growth of small parties, and require voters to vote strategically [9]. In particular, voters who prefer a party that is unlikely to reach the threshold often strategically vote for a safer party (but it is not the only kind of strategic voting that can occur in this context, as we will see in Section 4.4).

Thus, naïvely, there appears to be tradeoff between the problems of a threshold and the risk of political fragmentation. However, there are promising proposals that could alleviate the problems of the threshold without taking away its advantages. In particular, we could elicit additional information from voters regarding their preferences over parties. For example, we could ask voters for a second choice of party. If their first choice of party misses the threshold, their vote is instead counted for the second choice. More generally, we could allow voters to provide a (partial) *ranking* of the parties, and keep redistributing the vote until we reach a party that meets the threshold.

This idea has been extensively discussed in Germany under the name "replacement vote" (*Ersatzstimme*, sometimes translated into English as "spare vote"). It appears in the election program of one party for the 2025 German parliament election [32, page 17], and laws implementing it have been proposed (but not adopted) in three German states in 2013–15. It is also the main subject of a recent academic edited volume in German language [4]. Elsewhere, the idea has been discussed by the Independent Electoral Review in New Zealand, a committee established by the Justice Minister, which noted the strong support that the proposal received during their consultation, though they recommended to instead lower the threshold to not complicate the voting process [23, numbers 4.34 and 4.58].

Despite this broad attention to these proposals, they have never been studied from a social choice perspective (to the best of our knowledge). As we will see, there are many interesting voting-theoretic questions not answered by the high-level description of how to process the voters' second choices or rankings. To study these questions, we introduce a new framework of *party selection rules*, which take as input a profile of (possibly truncated) rankings over parties and a threshold τ (an absolute number of votes). They output a subset of parties: those that will be included in the parliament. For a given selection of parties, a voter is *represented* by a party c if it is their most-preferred party in the selection. We require that a selection should be *feasible*, in the sense that each selected party represents at least τ voters. We view such a feasible selection as fully specifying the make-up of a parliament, though in practice we will need to apply an apportionment rule (such as D'Hondt) to determine exactly how many seats each party obtains, as a function of the number of voters it represents. Because our model allows truncated rankings, it in particular allows for applications where voters can rank at most two parties, which is the most commonly discussed variant in the threshold context.

As has been recognized in the discussion in Germany, there are at least two possible party selection rules [4, pp. 52–62]. The simplest is what we call the *direct winners only* (DO) rule, which selects exactly those parties who are ranked in first place by at least τ voters, and assigns voters who do not rank any of those parties in first place to their most-preferred selected party. Thus, under DO, there is just a single round of reassignments. Another option reassigns votes in multiple rounds. We call the resulting rule the *single transferable vote* (STV) due to its close similarity to the rule of the same name used in systems that let voters rank candidates instead of parties [30]. STV works by repeatedly identifying the party with the fewest first-place votes, and eliminates it from the profile. It repeats this until the set of remaining parties is feasible. We add a third party selection rule to the collection: the *greedy plurality* (GP) rule sorts parties by the number of voters placing it in first position, then iteratively adds parties to the selection starting from those with largest score, as long as the addition keeps the selection feasible. We will compare these three rules (DO, STV, GP) using the axiomatic method and the experimental method, focusing on the wasted votes and strategic behavior issues. In particular, we will show axiomatic characterizations of DO and STV.

Given the incentives for voters to misrepresent their preferences in the current (uninominal) systems, we would expect significant changes in voting behavior if any of our ranking-based methods were to be adopted. Since there is little empirical research about strategic voter behavior in threshold elections [22],[1] we decided to conduct our own experiment in the context of the 2024 election of the French representatives to the European parliament, motivated by the 2019 election having had a high percentage of lost votes. Our experiment was based on a survey inviting participants to report how they would vote if they could rank several parties in the election. By comparing the ranking data to the voting intentions of our survey participants, we are able to perform a counterfactual analysis. In particular, we can study the strategic behavior of the voters, and we can simulate the behavior of the party selection rules that we study. The results confirm the theoretical predictions: our party selection rules would significantly reduce the number of wasted votes without fragmenting the parliament.

Other Applications of the Model

Although our main motivation is to design better parliamentary election systems, our formal model of party selection rules applies more generally to any multi-item selection context where each item is required to have enough voter support, where 'voter support' means being the voter's preferred item within the selection. This applies whenever we have to find a clustering of voters, each cluster being associated with some item. For example, consider a university program that needs to select which optional courses to open in a particular year, given that each student will choose their preferred course from the selection, and that a course should be opened only if is taken by a minimal number of students. Other examples are selecting a set of activities to be organized for a group on a given day, such that every participant will choose one [the *group activity selection problem*, 7, 8]. Two major differences between parliamentary elections and clustering-based settings are that the latter do not involve an apportionment step, and that voters' preferences bear only on the item they are assigned to, while in parliamentary elections they usually depend on the final composition of the parliament.

2 Preliminaries

Let $C = \{c_1, \ldots, c_m\}$ be a set of *parties* and $N = \{1, \ldots, n\}$ be a set of *voters*. A *truncated ranking* $\succ$ is a ranking of a subset of parties. The non-ranked parties are considered to be less preferred than the ranked ones, and to be incomparable to each other. For sets of candidates $S, T \subseteq C$, we write $S \succ T$ if for all $a \in S$ and $b \in T$, we have $a \succ b$.

A preference *profile* is a collection of truncated rankings $P = (\succ_1, \ldots, \succ_n)$. A *full* profile is a profile in which every voter ranks all the parties (so the rankings are not truncated). A *uninominal* profile is a profile in which every voter ranks exactly one party.

An *outcome* is a (possibly empty) subset of parties $S \subseteq C$. Together, an outcome and a profile jointly define a unique mapping $\text{best}_{S,P} : N \to S \cup \{\emptyset\}$ that assigns every voter i to her most-preferred party $\text{best}_{S,P}(i)$ among those in S, and to the empty set if she does not rank any party in S. We say that $c = \text{best}_{S,P}(i)$ is the *representative* of voter i in S, and that i is *unrepresented* in S if $\text{best}_{S,P}(i) = \emptyset$.[2] For a party $c \in S$, we define the *supporters* of c as the set of voters of which it is the representative in S, $\text{supp}_{S,P}(c) = \{i \in N : \text{best}_{S,P}(i) = c\} \subseteq N$, and the *score* of c is $\text{score}_{S,P}(c) = |\text{supp}_{S,P}(c)|$. Finally, we define as $\text{share}_{S,P}(c) = \text{score}_{S,P}(c)/(\sum_{x \in S} \text{score}_{S,P}(x))$ the share of representation of each party in the outcome.

[1]A notable exception is a study by Graeb and Vetter [20], which asked $n = 828$ participants in Germany in 2017 how they would vote if they could indicate a second-choice replacement vote. However, their study does not compare different party selection rules, and it introduces two different changes to the voting system simultaneously: addition of a replacement vote but also a combination of the person-bound and party-bound votes (*Erststimme* and *Zweitstimme*) into a single vote, making it more difficult to estimate the impact of the replacement vote alone.

[2]Thus, the 'representative' of a voter is a party, not an individual candidate.

Given a profile P and a *threshold* $\tau \in \mathbb{N}$ with $0 \leq \tau \leq |N|$, an outcome S is *feasible* (for P and τ) if every party $c \in S$ has at least τ supporters, that is, $\text{score}_{S,P}(c) \geq \tau$.[3] Clearly, the empty set is feasible, and a subset of a feasible set is feasible. If P is a full profile, then every singleton set $S = \{c\}$ is feasible, because we will then have $\text{score}_{S,P}(c) = |N| \geq \tau$. When $\tau = 0$, every subset of parties is feasible. When $\tau = 1$ and we have a full profile, then S is feasible if and only if S does not contain two parties c and c' such that c Pareto-dominates c'. When $\tau = n$ (or more generally $\tau > \frac{n}{2}$) and we have a full profile, then S is feasible if and only if it is singleton or the empty set. Hence, single-winner voting on full profiles is a special case of our model with $\tau = n$ if we force rules to return a non-empty outcome.

Once the party selection S has been determined, the parliament is made up using an apportionment method, each party $c \in S$ being represented proportionally to $\text{score}_{S,P}(c)$. Our setup abstracts away issues arising due to apportionment: while in practice an outcome will need to be reduced to a fixed number of available parliament seats, we will not study this "second step," trusting that it won't affect our conclusions in interesting ways.

3 Party Selection Rules

A *party selection rule* is a function f that takes as input a profile P and a threshold τ, and returns a feasible outcome $f(P, \tau)$. In this paper, we will focus on three specific rules:

- *Direct winners only (DO):* This rule selects the outcome whose support consists of all parties who are ranked in top position by at least τ voters.
- *Single transferable vote (STV):* This rule starts with the set $S_0 = C$. Then, at each step $k \geq 0$, if S_k is feasible, it returns this set. Otherwise, the rule identifies the party $c \in S_k$ who is ranked first (among parties of S_k) by the fewest voters and sets $S_{k+1} = S_k \setminus \{c\}$.
- *Greedy plurality (GP):* This rule starts with the empty set and goes over each party in decreasing order of plurality score (the number of voters ranking it first), adding it to the outcome set if the outcome remains feasible; otherwise it is skipped.

Except for DO, there might be ties in the rules when two parties have the same plurality scores at some point during the execution of STV or GP. In this case, we use some fixed tie-breaking order on the parties to decide how to proceed. This assumption makes it easier to analyze the rules. However, all the results we present in this paper can be extended to the case of irresolute rules with parallel-universe tiebreaking [5, 18], where all possible ways to break ties are considered to obtain the set of outcomes.

DO, STV, and GP are polynomial-time computable, as well as easy to understand. Many other interesting party selection rules can be defined. For example, one can consider rules that optimize some objective function over the set of feasible outcomes, such as maximizing the number of represented voters or the number of voters whose most-preferred party is in the outcome. However, these rules are hard to compute, as well as hard to verify and understand by voters, thus not particularly suitable to be used in parliamentary elections. However, they are interesting for some other contexts covered by our model, such as group activity selection or facility location. We define these rules in the full version [11].

On uninominal profiles, DO, STV, and GP are equivalent. However, in the general case, they can give different outcomes, as shown in the following example.

Example 3.1. *Consider the profile* $P = \{4 : a \succ b \succ c, 3 : b \succ c, 2 : c \succ b \succ a, 2 : d, 4 : d \succ b\}$*, with the threshold* $\tau = 5$*. In this profile, the only party with a plurality score of at least* 5 *is* d*, thus DO returns* $\{d\}$*. If we run the STV rule, we eliminate c first, as it has the lowest plurality score, and then* a *because the* c *voters are now supporting* b*, and we obtain the outcome* $\{d, b\}$*. Finally, if we run the GP rule, we*

[3] Note that in our model, the threshold is an absolute number of voters rather than a fraction. This choice makes the notation clearer. Also, we do not consider issues of abstention, and the total number of voters is fixed.

	DO	STV	GP
Set-maximality	✗	✗	✓
Inclusion of direct winners	✓*	✓*	✓
Representation of solid coalitions	✗	✓	✗
Threshold monotonicity	✓	✓	✗
Independence of definitely losing parties	✗	✓*	✗
Independence of clones	✗	✓	✗
Reinforcement for winning parties	✓*	✗	✗
Monotonicity	✓	✗	✗
Representative-strategyproofness (one risky party)	✗	✗	✓
Share-strategyproofness (safe first or second)	✓	✗	✗
Share-strategyproofness (representative ranked first)	✓	✗	✓

Table 1: Properties satisfied by the rules. The "*" indicates characterization results.

add d first, then we add a since it is the party with the second highest plurality score and $\{d, a\}$ is feasible. However, we do not add b since $\{d, a, b\}$ is not feasible (a has only 4 supporters), and same for c. Thus, the outcome of GP is $\{d, a\}$.

Note that the uninominal rule used in actual parliamentary elections with thresholds selects exactly those parties whose plurality score exceeds the threshold. Therefore, as a party selection rule, it coincides with DO. However, because it forces every voter to cast a *uninominal* ballot instead of a ranking, this rule cannot be defined directly within our model (one can also think of it as first transforming the profile by removing everything below the first party ranked by each voter, and then running DO on the resulting uninominal profile). Thus, the uninominal system *ignores* all votes that did not rank one of the selected parties on top, whereas DO reassigns such ballots to the highest-ranked selected party.

4 Axiomatic Analysis

In this section, we define a set of axioms that we believe are desirable for a party selection rule in the context of proportional representation with thresholds. We then analyze the different rules with respect to these axioms. We will go over different kinds of axioms. Table 1 summarizes the results of the axiomatic analysis. Some of the axioms (namely set-maximality, independence of clones and monotonicity) are discussed only in the full version [11], where all proofs can also be found.

4.1 Representation Axioms

We first consider representation axioms, which ensure in different ways that groups of voters of size at least τ must be represented. The most basic axiom requires that all parties that receive enough first-place votes must be winners. This axiom seems essential for political applications.

Definition 4.1. *A party selection rule satisfies* inclusion of direct winners *if for every profile P and threshold τ, whenever c is a party such that at least τ voters rank c in top position, then $c \in f(P, \tau)$.*

It is easy to see that DO, STV, and GP satisfy this axiom. Moreover, since DO returns *only* the direct winners, we always have $\text{DO}(P, \tau) \subseteq \text{STV}(P, \tau)$ and $\text{DO}(P, \tau) \subseteq \text{GP}(P, \tau)$. Thus, there are at least as many unrepresented voters under DO as under STV or GP.

One can strengthen inclusion of direct winners to apply to cases where enough voters support a *set* of parties. For example, suppose that there are three "green" parties and that at least τ voters rank them in the top three ranks, though they may disagree on their relative ordering. The following axiom requires that at least one of the green parties is included in the outcome. It is inspired by the "proportionality for solid coalitions" (PSC) axiom in multi-winner voting [12].

Definition 4.2. *A party selection rule satisfies* representation of solid coalitions *if for every profile P and threshold τ, if $T \subseteq C$ is a set of parties that has at least τ supporters in the sense that $|\{i \in N : T \succ_i C \setminus T\}| \geq \tau$, then $T \cap f(P, \tau) \neq \emptyset$.*

This property is satisfied by STV, but it is failed by DO, since it can happen that none of the parties supported by a solid coalition has enough first-place votes. It is also failed by GP.

Proposition 4.3. *STV satisfies representation of solid coalitions, but DO and GP do not.*

Note that representation of solid coalitions implies inclusion of direct winners (consider singleton T). One could strengthen this axiom further by forbidding that there is a party c outside the outcome S for which τ voters prefer c to all parties in S. Thus, the axiom requires that, for all $c \notin S$, $|\{i \in N : c \succ_i S\}| < \tau$. This is a version of the local stability axiom studied in multi-winner voting [2, 24]. Unfortunately, we can show that this axiom cannot be satisfied for any $\tau \notin \{1, n\}$.

4.2 Varying the Threshold

We now discuss what should happen to the outcome when the threshold changes. The first axiom says that a losing party should stay losing if the threshold increases, and conversely a winning party should stay winning if the threshold decreases. This is a natural requirement, as a higher threshold should make it harder for a party to be selected.

Definition 4.4. *A party selection rule satisfies* threshold monotonicity *if for all profiles P and all thresholds $\tau \leq \tau'$, we have $f(P, \tau) \supseteq f(P, \tau')$.*

It is easy to see that DO and STV satisfy this axiom, but GP does not.

Proposition 4.5. *DO and STV satisfy threshold monotonicity, but GP does not.*

Proponents of STV often argue in its favor using the following principle of procedural fairness: once we have decided that some candidates are losing, we should continue the procedure as if that party hadn't run in the first place [25]. We can formalize this principle using the following independence axiom, which says that once some parties are losing at some threshold, then for all larger thresholds, the rule should behave as if none of the losing parties had been available.

Definition 4.6. *A party selection rule satisfies* independence of definitely losing parties *if for every profile P and thresholds $\tau \leq \tau'$, we have $f(P, \tau') = f(P|_S, \tau')$, where $S = f(P, \tau)$.*

Here, $P|_S$ denotes the profile obtained from P by deleting all parties outside S. Independence of definitely losing parties implies threshold monotonicity, because it requires that $f(P, \tau') = f(P|_S, \tau') \subseteq S = f(P, \tau)$. The axiom is related to the "independence at the bottom" axiom of Freeman et al. [17], and it encodes a key intuition behind the functioning of the STV rule. In fact, in combination with inclusion of direct winners, this axiom characterizes STV. To state the result formally, we say that a profile P is *generic* if for every $S \subseteq C$, in the profile $P|_S$ there is a unique party with the lowest plurality score $\text{score}_{S,P}(c)$. On generic profiles, the STV rule never encounters a tie.

Theorem 4.7. *Let f be a party selection rule satisfying inclusion of direct winners and independence of definitely losing parties. Then f equals STV on all generic profiles.*

Note that this characterization cannot be used as a starting point for an axiomatic characterization of STV as a single-winner voting rule, because it crucially depends on a variable-threshold setup.

Corollary 4.8. *DO and GP do not satisfy independence of definitely losing parties.*

4.3 Reinforcement for Winning Parties

The next axiom connects the outcomes of a party selection rule on profiles defined on different sets of voters, and imposes a consistency condition. The axiom is inspired by the reinforcement axiom introduced by Young [33]. Our axiom says that if a party is winning in a profile P_1 with a threshold τ_1 and in a profile P_2 with a threshold τ_2, then it should also be winning in the profile $P_1 + P_2$ with threshold $\tau_1 + \tau_2$, where $P_1 + P_2$ is the profile obtained by "concatenating" P_1 and P_2.

Definition 4.9. *A party selection rule satisfies* reinforcement for winning parties *if for all profiles P_1 and P_2 and all thresholds τ_1 and τ_2, if $c \in f(P_1, \tau_1)$ and $c \in f(P_2, \tau_2)$, then $c \in f(P_1 + P_2, \tau_1 + \tau_2)$.*

DO satisfies this. In fact, DO is the only party selection rule satisfying inclusion of direct winners and reinforcement for winning parties, thus providing an axiomatic characterization of this rule.

Theorem 4.10. *DO is the only party selection rule that satisfies inclusion of direct winners and reinforcement for winning parties.*

Corollary 4.11. *STV and GP do not satisfy reinforcement for winning parties.*

4.4 Incentive Issues

A major drawback of uninominal voting is that it incentivizes voters to strategically misreport their preferences. In particular, in standard uninominal elections, voters whose favorite party will not reach the threshold may instead vote for a large party (and thereby increase its share of the parliament) or vote for a party near the threshold (and thereby potentially move it above the threshold). We refer to this type of strategic voting as *tactical voting*. We will study the extent to which tactical voting can be avoided when using party selection rules.

A second distinct type of manipulation has been called *coalition insurance voting* [21, 15, 29]. Here, a voter whose preferred party is guaranteed to reach the threshold decides to instead vote for a less-preferred party that is in danger of missing the threshold. If that smaller party reaches the threshold, it may form a governing coalition with the voter's preferred party. Thus, while the voter is now contributing their support to a worse representative, the voter will be more satisfied with the parliament as a whole. This manipulation is specific to parliamentary elections with thresholds and has no analogue in single-winner voting. We will consider this second type of manipulation separately.

4.4.1 Tactical Voting

Consider the common tactical vote in a uninominal election: a voter that supports a sub-threshold party instead votes for a larger party in order to have her vote counted towards the parliament's final composition. By doing this, the voter increases the share of representation of the party she votes for without any cost to her true favorite party (which anyway was not going to meet the threshold). In the extreme case, a tactical vote can even cause a new party to enter the winning set. In our model, we can define tactical voting in two natural ways, depending on how we measure the satisfaction of the voter with an outcome S. The first possibility is to consider that the satisfaction of a voter corresponds *solely* to the highest position of a party in S in the voter's truthful ranking.[4] The second possibility is to consider that the satisfaction of a voter corresponds *both* to the highest position of a party in S in the voter's truthful ranking, *and* to the share of representation $\mathrm{share}_{S,P}(c)$ of that party.

Other notions of satisfaction are possible and could in principle depend on the entire vector of shares of representation and the voter's truthful truncated ranking. However, the two notions above will be

[4] We note that this notion is particularly natural in a clustering-based context, where an agent prefers an outcome to another whenever they are assigned to a better representative.

sufficient to capture typical manipulations while being permissive enough to allow for positive results. Note that obtaining full strategyproofness with respect to either of these notions is hopeless. Assume that $\tau = n$ and that all voters submit full rankings. Then this is almost exactly the single-winner case (since sets with more than one party are not feasible), and we know from the Gibbard–Satterthwaite theorem [19, 28] that any rule that is strategyproof is either a dictatorship (i.e., the outcome is always the favorite party of some voter $i \in N$) or imposing (i.e., some alternative is never selected).

We will therefore consider strategyproofness in restricted cases, considering a sequence of politically plausible situations where some of our rules turn out to be immune to manipulations. The notion of strategyproofness that we will first explore in such restricted cases is the following. It requires that no voter can improve her most-preferred party among those selected.

Definition 4.12. *A party selection rule is* representative-strategyproof *for a profile P and a threshold τ, if for any voter i and any misreport $\succ'_i$, if $P' = (\succ_1, \dots, \succ'_i, \dots, \succ_n)$, $S = f(P, \tau)$, and $S' = f(P', \tau)$, we have best$_{S',P'}(i) \not\succ_i$ best$_{S,P}(i)$.*

In search of positive results, we will make statements that distinguish three types of parties from the perspective of a voter i on a particular profile, assuming that all other votes are fixed. More precisely, we say that: a party is (1) *safe* if it is included in the outcome no matter how i votes, it is (2) *risky* if it might be included or not in the outcome depending on how i votes, and it is (3) *out* if it is not included in the outcome no matter how i votes. Note that this partition of the parties depends on the rule that is used. However, for the three rules we consider, parties that are ranked first by more than $\tau + 1$ voters are always safe. On the other hand, parties that are ranked by fewer than $\tau - 1$ voters among the voters who did not rank a safe party first are always out. The remaining parties can be either safe, risky or out. Intuitively, risky parties are the ones that are neither clear winners nor clear losers. In a real-world scenario, this corresponds to the parties that are close to the threshold according to the polls.

We begin by considering the case where there exists at most one risky party. This is often not an unrealistic assumption.[5] However, uninominal voting fails representative-strategyproofness even in this case since a voter who prefers a party that is out can instead cast their vote for a risky party that is their second choice, causing the risky party to be included. We can show the following result:

Proposition 4.13. *GP satisfies representative-strategyproofness when there is at most one risky party from the perspective of each voter. DO and STV do not.*

We now turn to the second, stronger notion of strategyproofness, where the satisfaction of a voter depends on both the position and the share of representation of their favorite selected party.

Definition 4.14. *A party selection rule is* share-strategyproof *for a profile P and a threshold τ if for any voter i and any misreport $\succ'_i$, if $P' = (\succ_1, \dots, \succ'_i, \dots, \succ_n)$, $S = f(P, \tau)$, $S' = f(P', \tau)$, and $c =$ best$_{S,P}(i)$, we have (1) best$_{S',P'}(i) \not\succ_i c$ and (2) share$_{S',P'}(c) \leq$ share$_{S,P}(c)$.*

First, we consider the case where every voter has a safe party among their top two preferences. Clearly, uninominal voting fails share-strategyproofness in this case, as a voter can elevate a safe party that is their second choice ahead of a party that is out but their true favorite, thus increasing the share of the vote allocated to the safe party. Notably, this strategy is ineffective under DO.

Proposition 4.15. *DO satisfies share-strategyproofness whenever the most-preferred or second-most-preferred party of every voter is safe from the perspective of that voter. GP and STV do not.*

Finally, we prove a result in which the set of possible misreports of a voter is restricted. In particular, we assume that voters will only misreport by promoting their most-preferred party that is selected

[5]For example, in the 2023 New Zealand general election, with a threshold of 5%, only one party's vote share fell in the interval $(3.08\%, 8.64\%)$. The same is true in 2020 for the interval $(2.60\%, 7.86\%)$ and in 2017 for $(2.44\%, 7.20\%)$.

under truthful voting into first position in their misreport. This restriction can be thought of as giving voters perfect knowledge of which parties are out (and therefore not worth voting for), but not enough sophistication to perform arbitrarily "complex" manipulations.

Proposition 4.16. *DO and GP are share-strategyproof under the restriction that $c = best_{S,P}(i)$ is ranked first in $\succ'_i$, but STV is not.*

It should be observed that Proposition 4.16 covers the typical manipulation that occurs with uninominal voting: voters put their favorite *safe* party first instead of their favorite party (otherwise, their ballot would not be considered). Consider for instance the profile $P = \{1 : a \succ b, 3 : b, 3 : c\}$ with $\tau = 3$. With our rules, the first voter can vote sincerely: her vote will support b since a is out. On the other hand, under uninominal voting, she has an incentive to vote for b.

Finally, note that STV is not covered by any of these positive results. However, in cases where STV is manipulable, the voter causes the elimination order to change, and thus some parties not to be added to the outcome anymore, possibly increasing the vote share of her preferred party. This is arguably a very unnatural manipulation for voters, who need almost full knowledge of the preferences of the other voters to be able to predict the correct manipulation.

4.4.2 Coalition Insurance Voting

A separate type of manipulation is *coalition insurance voting* [6], where a supporter of a safe party c instead votes for a risky party d that she also likes in order to push d over the threshold, thereby potentially allowing c and d to form a governing coalition (while in a parliament without d, there would be no majority for c). Indeed, while c loses one supporter, party d gains τ supporters by virtue of being included in the outcome. In many cases, the voter will be more satisfied with the new outcome as a whole. In many countries with proportional representation systems, parties announce intended coalitions in advance of the elections, and safe parties such as c might even encourage their supporters to vote for d instead. Coalition insurance voting has been observed in several countries including Germany and Sweden, and is well-studied using survey and lab experiments [e.g., 15, 16].

Can such voting behavior be avoided when using party selection rules? Unfortunately, a simple example suffices to show that no rule that satisfies the inclusion of direct winners axiom is immune to manipulations of this type. Consider the profile $P = \{3 : a, 3 : b, 4 : c, 2 : d, 1 : c \succ d\}$ with $\tau = 3$, and suppose that the last voter likes both parties c and d (which might form a coalition), but dislikes a and b (which might form another coalition). The outcome is $S = \{a, b, c\}$ as all these parties have at least τ first-place votes, and including d would violate feasibility. With this outcome, $\{a, b\}$ may form the governing coalition with $6/11 \approx 55\%$ of the votes. Now, if the last voter changes her vote to $d \succ c$, then the outcome is $S' = \{a, b, c, d\}$ as all parties now get at least τ first choices. After this manipulation, $\{c, d\}$ receives $7/13 \approx 54\%$ of the votes and may now be forming a governing coalition, thus including the most-preferred party of the manipulating voter.

While party selection rules cannot completely avoid this effect, one might expect that in practice there is less motivation for this kind of manipulation in the case that voters can submit a ranking. For instance, in the previous example, the d voters might have put c in second place if c and d were running on similar platforms or had announced an intention of forming a coalition. However, there is also a possibility that coalition insurance voting might *increase* under rankings, since this strategy is less risky in this situation. Indeed, if voters preferring c instead cast the ranking $d \succ c$, then either the manipulation is successful (and d is selected), or it is unsuccessful and the vote is transferred to c, which does not hurt the manipulator. In contrast, under uninominal voting, a vote for d carries a risk that the vote will be lost.

5 Experiments

Inspired by our theoretical analysis, we now want to evaluate the extent to which ranking-based systems allow better representation by decreasing the number of unrepresented voters, without drastically increasing the number of parties included in the parliament. We are also interested in the strategic behavior of voters in these systems. We base our study on a survey we ran in the context of the election of the French representatives to the European Parliament held in June 2024. The French representatives are elected by a nationwide party-list proportional representation system, using the D'Hondt apportionment method with a 5% threshold. In the 2024 election, 38 lists took part in the election and 12.08% of votes were cast on lists that did not reach the threshold. Seven lists reached it, two of which were just above the threshold (with 5.47% and 5.5% of vote share), so the proportion of wasted votes could have been much worse (which happened in 2019, when it was around 20%).

5.1 The Datasets

Our datasets [10] were collected through a survey that invited participants to consider how they would vote in a system that allowed them either (1) to rank at most two lists, or (2) to rank an arbitrary number of lists. The survey was administered through a custom-built online platform in French language. Participants were led through several steps, that we detail in the full version (with screenshots) [11].

Our analysis in this section is based on the answers of 2 840 voluntary participants to the survey, recruited through social media and mailing lists. They answered the survey the week *before* the election. These participants are not representative of the French population (they are very left-leaning, young, and highly educated), so we used their voting intentions to assign them weights in order to reduce (but of course not eliminate) the representation bias. This weighting methodology is standard for surveys of alternative voting methods [31, 20]. Participants who did not provide any voting intention were assigned weight zero, leaving $n = 2\,838$ participants with non-zero weight for the self-selected sample. This gives us two preference profiles: one in which participants are allowed to rank at most two lists, and one in which they can rank an unlimited number of lists. In the full version [11], we also present the results for a second sample of 1 000 *paid* participants recruited through a survey research company, and which are representative of the French population.

All the collected data were anonymized. The participants were informed before participating that the data would be used for research purposes only. Participants had the possibility to skip any question they did not wish to answer. The study received ethics approval from the relevant board.

5.2 Analysis of the Results

For this analysis, we divide the parties into three groups according to their vote share in the actual election. Intuitively, we want this partition to reflect the safe/risky/out categorization that we introduced in Section 4.4.1. We have (1) 5 *safe* parties that received at least 7% of the votes at the actual election, (2) 2 *risky* parties that received between 5% and 6% of the votes and were thus in danger of not reaching the threshold, and (3) 31 *out* parties that received less than 3% of the votes. This classification based on the vote shares matches expectations from polls.

Strategic Voting Our first observation is that an important share of participants (between 22.6% and 28.6% in our sample) put first in their ranking a party that received fewer votes than the party they actually intended to vote for. This is the canonical example of "tactical voting": putting an *out* party first and voting at the election for a *safe* or *risky* party. A detailed analysis of this phenomenon and of voters' strategic behavior is provided in the full version [11].

Lost votes We now compare the share of voters that are unrepresented (i.e., that did not rank any party that is selected by the rule). Table 2 shows these shares obtained from (1) the actual election

(12.1%), (2) the uninominal rule which deletes everything below voters' first choice, selects all parties meeting the threshold, but leaves voters unrepresented if their first choice is not selected, and (3) the party selection rules DO, STV, and GP.

The discrepancy between (1) and (2) is due to voters' strategic behaviour in the actual election. Indeed, as we just mentioned, many voters put a party that is *out* in first position of their ranking, but voted for a *safe* or *risky* party at the election. Thus, applying the uninominal rule on the "sincere" rankings would lead these voters to be unrepresented in our dataset, while they were represented in the actual election. This phenomenon additionally caused one *risky* party to not meet the threshold anymore in our datasets, causing its supporters to be unrepresented. This explains why we observe a much higher percentage of wasted votes for the uninominal rule with our datasets.

	2 votes	Ranking
Actual	12.1%	12.1%
Uninominal	37.9%	34.4%
DO	11.7%	3.2%
STV	7.0%	2.3%
GP	7.0%	2.3%

Table 2: Percentage of unrepresented voters.

We are presenting the numbers in (2) as an extreme case of what the effect of the threshold would be if voters were to strategize less. But the numbers are not realistic, since participants decided on their rankings based on an understanding that a party selection rule would be used. Comparing the numbers in (1), (2), and (3), we see that while strategizing is crucial to make the current election work well, it is not necessary when using party selection rules. Indeed, even with the reduced need for strategic behavior, our rules decrease the share of unrepresented voters even compared to (1).

Let us next compare the performance of the different party selection rules. Recall that the outcome of DO is always a subset of the outcomes of STV and GP. For each of our datasets, it turns out that STV and GP return the same set of parties, and DO returns one fewer party (see Figure 2(b) in the full version [11]). This explains why there are fewer unrepresented voters with STV and GP than with DO. The representation gain is higher with unlimited-ranking ballots than with 2-truncated ballots. This is not surprising: when participants rank several parties, they are more likely to rank one that will be selected than when they rank only two. Moreover, for the unlimited-ranking dataset, one additional party (namely the *Pirate party*) is selected in the outcome for each rule when evaluated on the rankings, further decreasing the number of unrepresented voters. However, this is almost entirely due to a selection bias: the survey was shared among the supporters and people familiar with this party, leading to more people ranking it first than in a representative sample.[6]

Impact of ballot size Another interesting observation is that the representativity gain with ranked ballots is already quite high with short ballots. Indeed, a large majority of voters are represented by a party ranked very high in their ranking. For instance, we can see in Figure 3 that for the unlimited-ranking profile, if we use STV or GP, around 70% of voters are represented by their favorite party, and almost all voters have their representative in their top 3 choices.

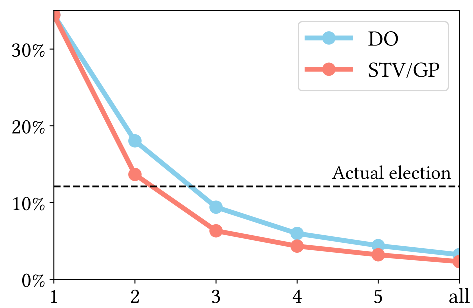

Figure 2: Percentage of unrepresented voters after truncating their rankings to a particular length, indicated on the horizontal axis.

To complete this analysis, Figure 2 shows the fraction of lost votes if all rankings of length greater than k are truncated to rank k. We observe that for $k \geq 5$, we reach an almost maximal representativity level. This is important for the practical implementation of such rules: limiting the number of parties to rank would arguably limit voters' cognitive load to an acceptable level (and would thus be more likely to be adopted in real-world political settings), while still ensuring good representativity.

[6]Note that this bias is only partially corrected by our weighting method, as many participants ranked this party first without indicating voting for it at the election, probably for strategic reasons.

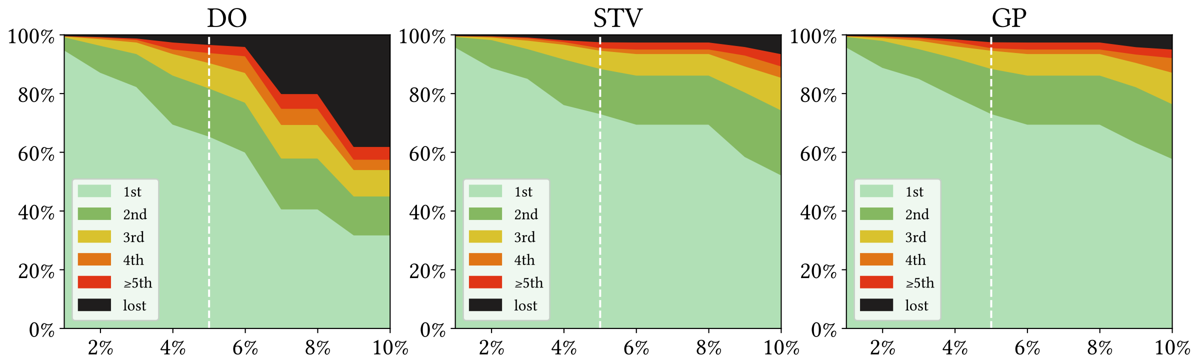

Figure 3: Distribution of the ranks of the representatives in the rankings of the voters with different thresholds. The threshold used is indicated on the horizontal axis. For each threshold, the vertical slice above it shows how the voters are divided into unrepresented voters (black area) and represented voters (colored areas, colored according to the rank of the party assigned to the voter). The results for the 2-truncated votes are provided in the full version [11].

Varying the threshold The results above were obtained with a threshold of 5%, the one used in this election. As a robustness check, we ran the analysis with other thresholds. In particular, we applied the rules to our datasets with thresholds varying between 1% and 10%, and computed the percentage of unrepresented voters, as well as the distribution of ranks of the representatives. The results are shown in Figure 3. They are compatible with our previous observations: STV and GP consistently give similar results and a better representation than DO, especially for high threshold values, for which DO ignores a significant part of the voters. Moreover, we observe that even for high thresholds, most voters are represented by one of their top 3 choices. We also show in the full version that these observations hold when we add random noise to the data [11].

6 Discussion

We studied whether allowing voters to rank party lists instead of voting for a single list could help obtain more representativeness in parliamentary elections by reducing the amount of unrepresented voters. Both our theoretical and our empirical results suggest that rankings can indeed be helpful, with results varying by rule. STV and GP allow more parties to be represented, and, relatedly, leave fewer voters unrepresented than DO. Axiomatically, the three rules are incomparable: DO and GP enjoy stronger strategyproofness guarantees than STV, while STV represents solid coalitions. All three rules are simple and easy to understand, with DO being closest to the current system and STV being closely related to election systems that are widely used, especially in English-speaking countries.

Our theoretical results operate within social choice theory and the axiomatic method. It would be interesting to study this setting from the perspective of strategic candidacy, evaluating the rules' impact on party formation and political innovation. Our experiment could also be productively repeated in other countries, to better understand the robustness of our conclusions, and to understand whether the electorate would accept or welcome the switch to one of these rules. In addition, studying effects on coalition insurance voting would be interesting.

We hope that our work can support the discussion about proportional representation and thresholds in several countries that are facing issues with their election systems. In our view, the results of our work might even provide reassurance for countries currently using majoritarian first-past-the-post systems to switch to proportional representation. Countries like France that place substantial weight on governability could in principle use methods like the ones we studied to combine proportional representation with very high thresholds (perhaps as high as 10%) without causing unacceptable amounts of wasted votes.

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