

On voting rules satisfying false-name-proofness and participation

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Abstract

We consider voting rules in settings where voters' identities are difficult to verify. Voters can manipulate the process by casting multiple votes under different identities or abstaining from voting. Immunities to such manipulations are called *false-name-proofness* and *participation*, respectively. For the universal domain of (strict) preferences, these properties together imply *anonymity* and are incompatible with *neutrality*. For the domain of preferences defined over all subsets of a given set of objects, both of these properties cannot be met by *onto* and *object neutral* rules that also satisfy the *tops-only* criterion. However, when preferences over subsets of objects are restricted to be separable, all these properties can be satisfied. Furthermore, the domain of separable preferences is maximal for these properties.

1 Introduction

Societies make decisions by means of voting rules, mapping profiles of voters' preferences into social alternatives. In highly anonymous settings, such as the Internet, there are various ways a voter can manipulate the voting mechanism. When participants' identities cannot be easily verified, or when the number of participants is unpredictable, opportunities for manipulation arise. One such manipulation involves a voter using multiple identities to cast several votes. We say that a rule immune to voters casting duplicate votes is "false-name-proof". More generally, a voting rule is "strongly false-name-proof" if it prevents voters from submitting multiple (and possibly different) votes.¹ A voter can also benefit by abstaining from voting, leading to what is known in the literature as the no-show paradox [10, 14]. We say that a rule that does not allow such behavior satisfies "participation". Since defining these properties requires a changing active set of voters, we consider societies with a variable set of voters.

We are interested in studying voting rules that satisfy false-name-proofness and participation in two different social choice problems. In the first, social alternatives do not have any specific structure. In the second, social alternatives consist of subsets from a given set of objects (candidates, binary issues, or alike).

When social alternatives are unstructured and all preferences over those alternatives are admissible, i.e., when we consider the universal domain of preferences, results on voting rules satisfying some form of false-name-proofness are rather negative. Bu [6] shows that strong false-name-proofness implies both "strategy-proofness" (no voter ever gains by untruthful voting) and "anonymity" (changing voters' identities does not affect the choice made by the rule). As it is well-known from Gibbard [11] and Satterthwaite [15] celebrated result, there are no non-constant strategy-proof and anonymous rules defined in the universal domain. Therefore, no non-constant strongly false-name-proof rule defined in the universal domain exists either. Nevertheless, the weakening of this requirement may allow for some possibility results.²

¹Our strong false-name-proofness property is typically called false-name-proofness in the literature [see, for example, 19, 8, 6].

²The property of strategy-proofness is central to the literature on mechanism design and has been extensively

Our first interest is to analyze the existence of voting rules that satisfy false-name-proofness and participation in the universal domain of preferences. In most voting settings, it is common to also assume that there are no alternatives that deserve special treatment. The requirement of “neutrality” formalizes this by demanding that the changing of alternatives’ names does not affect the choice made by the rule. We demonstrate that voting rules satisfying false-name-proofness and participation are inherently anonymous (Proposition 2) and, as a consequence, that there are no neutral rules that also satisfy our two requirements of immunity to manipulation (Proposition 3).

When the set of social alternatives consists of all the subsets of a given set of objects, an important restricted domain of preferences is that of “separable” preferences: adding an object to a set leads to a better set if and only if the object is “good” (as a singleton set, the object is better than the empty set). On that restricted domain, Fioravanti and Massó [9] characterize all voting rules that satisfy false-name-proofness, strategy-proofness, and “ontoness” (every subset of objects is a possible outcome) as the class of voting by quota [3], where to be chosen, each object needs either at least one vote or a unanimous vote.

Our second interest is to analyze what happens when separability is relaxed, i.e. when all preferences over subsets of objects are admissible.³ Besides false-name-proofness and participation, we would like to impose three other desirable properties. The first one is ontoness. As we previously said, it implies that no subset of objects should be discarded from consideration *a priori*. Second, the internal structure of this restricted domain allows us to define the weaker neutrality axiom of “object neutrality”, by which changing objects’ names does not affect the choice made by the rule. Third, as voters may not be willing to submit full preferences (this seems particularly important, for example, in online voting settings), we also require the informational simplicity property of “tops-onliness”, by which only the top choices of the voters are relevant for the rule. We demonstrate that there are no rules satisfying false-name-proofness and participation that also fulfill these three additional desiderata (Theorem 1). Even though it might seem to be over-demanding to require voting rules to satisfy so many properties, the impossibility result is far from being straightforward since (i) we show that the five axioms are independent in the domain of all preferences over subsets of objects, and (ii) the rules characterized by Fioravanti and Massó [9] satisfy all these axioms in the domain of separable preferences.

Finally, we ask to what extent the domain of separable preferences can be enlarged while maintaining the compatibility of all five properties. It turns out that such a restricted domain is maximal for those properties: adding a non-separable preference to the domain entails losing at least one of the properties involved (Theorem 2).

The property of (strong) false-name-proofness was introduced by Yokoo et al. [19], for the problem of assigning objects with transfers where agents have quasi-linear preferences. In voting environments, for the case when the preferences are single-peaked [4], Todo et al. [17] characterize the class of all strongly false-name-proof, anonymous, and “efficient” (no voter can be made better off without making some voter worse off) voting rules. Todo et al. [17] and Todo et al. [18] extend the analysis to the case where the set of alternatives has a tree structure. Moreover, Conitzer [8] characterizes all anonymous and neutral probabilistic voting rules over a finite set of alternatives that satisfy strong false-name-proofness and participation. Each element in the class identified by Conitzer [8] is characterized by a probability $p \in [0, 1]$. With

studied [see 2, for a comprehensive survey]. We deliberately depart from this line of inquiry and instead focus on rules that satisfy false-name-proofness and participation.

³A typical example of when this domain can be deemed relevant is inspired by Barberà et al. [3]. Suppose you are on a university professor hiring committee. You might think that Borda and Condorcet are outstanding professors, and would love to have any of them employed, but believe that the department will be chaos with the two of them in it (probably because of some dissidence on how they like to vote).

probability p , an alternative is chosen uniformly at random. With probability $1 - p$, a pair of alternatives is chosen uniformly at random. If all voters unanimously prefer one alternative over the other in the pair, the preferred alternative is chosen; otherwise, a fair coin is used to decide between the two. Although Conitzer's [8] result implies the impossibility of neutral and deterministic voting rules that satisfy strong false-name-proofness and participation, our Proposition 3 is not a direct corollary of his, as we use a weaker version of the axiom.

The plan of the paper is as follows. Section 2 presents the basic notions and axioms that we use, while we present the results in Section 3. Finally, Section 4 contains some concluding remarks.

2 Model

Let $\mathcal{N}$ be the family of all finite and non-empty subsets of the set of positive integers $\mathbb{Z}_+$. An element $N \in \mathcal{N}$ is interpreted as a society. We denote the cardinality of N by n and refer to an element $i \in N$ as a *voter*. Each set of voters $N \in \mathcal{N}$ has to collectively choose an *alternative* from a set $\mathcal{A}$. Let $\mathcal{U}_{\mathcal{A}}$ denote the set of all strict linear orders over $\mathcal{A}$. Therefore, each voter i is endowed with a *preference* $P_i \in \mathcal{U}_{\mathcal{A}}$, where $A P_i B$ means that for voter i , alternative A is preferred to alternative B . We denote the weak counterpart of P_i by R_i . When a preference order is not attached to a particular voter, we write it as P_0 . For each $N \in \mathcal{N}$, a *profile* is an ordered list of preferences $P_N = (P_i)_{i \in N} \in \mathcal{U}_{\mathcal{A}}^N$. Given a preference $P_i \in \mathcal{U}_{\mathcal{A}}$, denote with $t(P_i) \in \mathcal{A}$ to the *top* alternative for voter i and denote with $b(P_i) \in \mathcal{A}$ to the *bottom* alternative for voter i .

When the set of alternatives $\mathcal{A}$ is unstructured, we call $\mathcal{U}_{\mathcal{A}}$ the *universal domain* of preferences. Besides studying this domain, we will be interested in the domain arising from considering as alternatives all the subsets of a given set of *objects* $\mathcal{O} = \{1, \dots, O\}$ with $O \geq 2$, i.e., the case where $\mathcal{A} = 2^{\mathcal{O}}$. Call $\mathcal{U}_{\mathcal{O}}$ the *domain (of preferences) over subsets of objects*. Notice that, after some renaming of the alternatives involved, any domain over subsets of objects can be considered a universal domain but, in general, there are universal domains that cannot be considered as domains over subsets of objects.⁴

Given a domain $\mathcal{D} \subseteq \mathcal{U}_{\mathcal{A}}$, let $\mathcal{D}^N = \bigcup_{N \in \mathcal{N}} \mathcal{D}^N$. A *voting rule* on $\mathcal{D}$ is a mapping $f : \mathcal{D}^N \rightarrow \mathcal{A}$ that assigns, for each $N \in \mathcal{N}$ and each $P_N \in \mathcal{D}^N$, an element $f(P_N) \in \mathcal{A}$. Next, we define desirable properties for voting rules. To do this, fix a domain $\mathcal{D}$ and a rule $f : \mathcal{D}^N \rightarrow \mathcal{A}$.

The first property states that all alternatives should be feasible to be selected in all societies.

Onteness: For each $N \in \mathcal{N}$ and each $A \in \mathcal{A}$, there is a profile $P_N \in \mathcal{D}^N$ such that $f(P_N) = A$.

The next axiom asserts that all essential information for the voting rule is found in the top alternatives of the voters. Thus, the rule requires minimal information from the voters.

Tops-onliness: For each $N \in \mathcal{N}$ and each pair of profiles $P_N, P'_N \in \mathcal{D}^N$ such that $t(P_i) = t(P'_i)$ for all $i \in N$, it is the case that $f(P_N) = f(P'_N)$.

The following three properties are particularly relevant in contexts such as online voting, where a social planner cannot easily verify voters' identities or determine the total number of participants. The first property asserts that a voter should never have an incentive to cast repeated votes.

False-name-proofness: For each $N, N' \in \mathcal{N}$ with $N \cap N' = \emptyset$, each $i \in N$, each $P_N \in \mathcal{D}^N$, and each $P_{N'} \in \mathcal{D}^{N'}$ such that $P_j = P_i$ for each $j \in N'$, we have $f(P_N) R_i f(P_{N \cup N'})$.

⁴For example, if $|\mathcal{A}| = 3$ then there is no set of objects $\mathcal{O}$ such that $\mathcal{A} = 2^{\mathcal{O}}$.

Conitzer's [8] related condition imposes stronger restrictions on the voting rule by not requiring that the additional preferences submitted by voter $i \in N$ coincide with voter i 's original preference P_i .

Strong false-name-proofness: For each $N, N' \in \mathcal{N}$ with $N \cap N' = \emptyset$, each $i \in N$, each $P_N \in \mathcal{D}^N$, and each $P_{N'} \in \mathcal{D}^{N'}$, we have $f(P_N) R_i f(P_{N \cup N'})$.

The next axiom states that voters should be induced to vote.

Participation: For each $N \in \mathcal{N}$ with $|N| \geq 2$, each $i \in N$, and each $P_N \in \mathcal{D}^N$, we have $f(P_N) R_i f(P_{N \setminus \{i\}})$.

The following property states that no voter should receive a differential treatment.

Anonymity: For each permutation $\sigma : \mathbb{Z}_+ \rightarrow \mathbb{Z}_+$, each $N \in \mathcal{N}$, and each $P_N \in \mathcal{D}^N$, $f(\sigma(P_N)) = f(P_N)$, where $\sigma(P_N) = (P_{\sigma(i)})_{i \in N}$.

Conitzer [8] merges the properties of *strong false-name-proofness*, *participation* and *anonymity* under the name of *anonymity-proofness*. A principle similar to *anonymity*, but applied to alternatives, is provided next. Given a permutation $\gamma : \mathcal{A} \rightarrow \mathcal{A}$, and a profile $P_N \in \mathcal{D}^N$, let P_N^γ be the profile such that, for each $i \in N$ and each pair $A, A' \in \mathcal{A}$, $\gamma(A) P_i^\gamma \gamma(A')$ if and only if $A P_i A'$.

Neutrality: For each permutation $\gamma : \mathcal{A} \rightarrow \mathcal{A}$ and each $P_N \in \mathcal{D}^N$, $\gamma(f(P_N)) = f(P_N^\gamma)$.

A weaker notion of neutrality is available for voting rules defined on the domain of subsets of objects, $\mathcal{U}_\mathcal{O}$. Given a permutation $\mu : \mathcal{O} \rightarrow \mathcal{O}$, a subset of objects $S \in 2^\mathcal{O}$, and a profile $P_N \in \mathcal{U}_\mathcal{O}^N$, let $\mu(S) = \{\mu(x) : x \in S\}$ and let P_N^μ be the profile such that, for each $i \in N$ and each pair $S, T \in 2^\mathcal{O}$, $\mu(S) P_i^\mu \mu(T)$ if and only if $S P_i T$.

Object neutrality: For each permutation $\mu : \mathcal{O} \rightarrow \mathcal{O}$, each $N \in \mathcal{N}$, and each $P_N \in \mathcal{U}_\mathcal{O}^N$, $\mu(f(P_N)) = f(P_N^\mu)$.

3 Results

3.1 Universal domain

Our first result shows that if the identity of a voter changes while the ballot remains the same, the outcome of the rule remains unchanged. This result is instrumental in proving one of our main findings: that any false-name-proof voting rule satisfying participation must also be anonymous. Let $\mathcal{D} \subseteq \mathcal{U}_\mathcal{A}$, that is, a generic subset of the universal domain.

Proposition 1 *Let $f : \mathcal{D}^\mathcal{N} \rightarrow \mathcal{A}$ be a voting rule that satisfies false-name-proofness and participation. Let $N \in \mathcal{N}$, $i \in N$, and $P_N \in \mathcal{D}^N$. Then, if $i^* \notin N$ and $P_{i^*} \in \mathcal{D}$ is such that $P_{i^*} = P_i$, it is the case that $f(P_{(N \cup \{i^*\}) \setminus \{i\}}) = f(P_N)$.*

To show that Proposition 1 holds, we use the following result, which helps us get rid of repeated votes.

Lemma 1 *Let $f : \mathcal{D}^\mathcal{N} \rightarrow \mathcal{A}$ be a voting rule that satisfies false-name-proofness and participation. Let $N \in \mathcal{N}$, $i \in N$, and $P_N \in \mathcal{D}^N$. Then, if $i^* \notin N$ and $P_{i^*} \in \mathcal{D}$ is such that $P_{i^*} = P_i$, it is the case that $f(P_N) = f(P_{N \cup \{i^*\}})$.*

Proof. Let f , N , P_N , i , i^* , and P_{i^*} be as stated in the lemma. By *false-name-proofness*, $f(P_N) R_i f(P_{N \cup \{i^*\}})$. By *participation*, $f(P_{N \cup \{i^*\}}) R_{i^*} f(P_N)$. As $P_{i^*} = P_i$, we have that $f(P_{N \cup \{i^*\}}) R_i f(P_N)$. Thus, $f(P_N) R_i f(P_{N \cup \{i^*\}}) R_{i^*} f(P_N)$ and $f(P_N) = f(P_{N \cup \{i^*\}})$. $\square$

Proof of Proposition 1. Let f , N , P_N , i , i^* , and P_{i^*} be as stated in the proposition. By Lemma 1 and *participation*, $f(P_N) = f(P_{N \cup \{i^*\}}) R_i f(P_{N \setminus \{i\} \cup \{i^*\}})$. By Lemma 1 and *participation* again, $f(P_{N \setminus \{i\} \cup \{i^*\}}) = f(P_{N \cup \{i^*\}}) R_{i^*} f(P_N)$. As $P_i = P_{i^*}$, $f(P_N) R_i f(P_{N \setminus \{i\} \cup \{i^*\}}) R_{i^*} f(P_N)$ and, therefore, $f(P_N) = f(P_{N \setminus \{i\} \cup \{i^*\}})$. $\square$

Bu [6] and Fioravanti and Massó [9] explore the connection between *false-name-proofness* and *anonymity*. Our next result follows that line and shows that the names of the voters are not important for a rule that satisfies *false-name-proofness* and *participation*.

Proposition 2 *A voting rule $f : \mathcal{D}^N \rightarrow \mathcal{A}$ that satisfies false-name-proofness and participation, also satisfies anonymity.*

Proof. Let f satisfy *false-name-proofness* and *participation*. Consider $N \in \mathcal{N}$, a profile $P_N \in \mathcal{D}^N$, and a permutation $\sigma : \mathbb{Z}_+ \rightarrow \mathbb{Z}_+$. We need to show that $f(\sigma(P_N)) = f(P_N)$, where $\sigma(P_N) = (P_{\sigma(i)})_{i \in N}$. There are two cases to consider:

1. $N \cap \sigma(N) = \emptyset$. By iterating the result of Proposition 1, we obtain that $f(\sigma(P_N)) = f(P_N)$.
2. $N \cap \sigma(N) \neq \emptyset$. Let $N' = \sigma(N)$, and consider $N'' \in \mathcal{N}$ such that $N'' \cap (N \cup N') = \emptyset$ and $|N''| = |N|$. Then, there are two permutations $\tilde{\sigma}, \hat{\sigma} : \mathbb{Z}_+ \rightarrow \mathbb{Z}_+$ such that $\tilde{\sigma}(N) = N''$, $\hat{\sigma}(N'') = N'$, and $\sigma = \hat{\sigma} \circ \tilde{\sigma}$. By the previous case, $f(\sigma(P_N)) = f(P_{N'}) = f(\hat{\sigma}(P_{N''})) = f(P_{N''}) = f(\tilde{\sigma}(P_N)) = f(P_N)$.

Therefore, f satisfies *anonymity*. $\square$

Remark 1 Both requirements in Proposition 2 are necessary to obtain *anonymity*, i.e., there are non-anonymous voting rules that satisfy either *false-name-proofness* or *participation*. The rule that selects voter 1's top alternative whenever 1 is present, and otherwise assigns a status-quo alternative, satisfies *participation* and is not *anonymous*. Furthermore, the rule that selects voter 1's bottom alternative whenever 1 is present, and otherwise assigns a status-quo alternative, satisfies *false-name-proofness* and is not *anonymous*.

It is well known that, for most choices of $|N|$ and $|\mathcal{A}|$, there is no way to make *anonymity* and *neutrality* compatible on the universal domain.⁵ This implies that both requirements cannot be met for voting rules defined in a variable population environment.

Remark 2 There is no voting rule $f : \mathcal{U}_{\mathcal{A}}^N \rightarrow \mathcal{A}$ that satisfies *anonymity* and *neutrality*.

Our first impossibility result says that for rules defined in the universal domain, *false-name-proofness* together with *participation* are incompatible with *neutrality*.

⁵In fact, such compatibility exists if and only if $|\mathcal{A}|$ cannot be written as the sum of divisors of $|N|$ different than 1 [see, for example, Exercise 9.9 in 13, for more details].

Proposition 3 *There is no voting rule $f : \mathcal{U}_A^N \rightarrow \mathcal{A}$ that satisfies false-name-proofness, participation, and neutrality.*

Proof. Let f satisfy false-name-proofness and participation. Then, by Proposition 2, f satisfies anonymity. By Remark 2, f cannot be neutral. $\square$

Remark 3 Proposition 3 is not directly implied by Theorem 1 of Conitzer [8], which states there is no neutral and deterministic voting rule that satisfies these properties, as he uses *strong false-name-proofness* for his negative result. Still, using our weaker *false-name-proofness*, an impossibility result is obtained.

3.2 Domain over subsets of objects

Now, we turn our attention to rules defined in the domain over subsets of objects, $\mathcal{U}_O$. In this new environment, it makes sense to relax *neutrality* to *object neutrality*, in order to look for positive results. In the following example we show the existence of *false-name-proof* rules that also satisfy *participation* and *object neutrality*. Alas, neither *onteness* nor *tops-onliness* are guaranteed.

Example 1 First, define rule $f^O : \mathcal{U}_O^N \rightarrow 2^O$ as follows. For each $N \in \mathcal{N}$ and each $P_N \in \mathcal{U}_O^N$, $f^O(P_N) = O$. This constant rule always selects the whole set of objects O and clearly satisfies all properties but *onteness*.

Next, given $N \in \mathcal{N}$ and $P_N \in \mathcal{U}_O^N$, let $\tilde{O}(P_N) = \{i \in N : t(P_i) \neq O \text{ and } O \text{ } P_i \text{ } S \text{ for each } S \in 2^O \setminus \{t(P_i), O\}\}$. Define rule $\tilde{f} : \mathcal{U}_O^N \rightarrow 2^O$ as follows. For each $N \in \mathcal{N}$ and each $P_N \in \mathcal{U}_O^N$,

$$\tilde{f}(P_N) = \begin{cases} t(P_i) & \text{if } i \in \tilde{O}(P_N) \text{ and } t(P_i) = t(P_j) \text{ for each } j \in \tilde{O}(P_N) \\ O & \text{otherwise} \end{cases}$$

Rule $\tilde{f}$ selects the set of all objects, O , unless all voters who consider O as their second choice share their top choice, in which case the rule recommends such top choice. This rule satisfies all properties but *tops-onliness*. To see this, let $O = \{x, y\}$, $N \in \mathcal{N}$, and consider $P_N, P'_N \in \mathcal{U}_O^N$ such that $t(P_i) = t(P'_i) = \{x\}$ for each $i \in N$, $b(P_i) = O$ for each $i \in N$, and $\tilde{O}(P'_N) = N$. Then, $\tilde{f}(P_N) = O \neq \{x\} = \tilde{f}(P'_N)$. $\diamond$

Our second impossibility result says that, in the domain over subsets of objects, our five desired properties are not compatible.

Theorem 1 *There is no voting rule $f : \mathcal{U}_O^N \rightarrow 2^O$ that satisfies ontteness, tops-onliness, false-name-proofness, participation, and object neutrality.*

Proof. Assume there is a voting rule $f : \mathcal{U}_O^N \rightarrow 2^O$ that satisfies the five axioms. First, we claim that there are $N \in \mathcal{N}$, a profile $P_N \in \mathcal{U}_O^N$, and a voter $i \in N$, such that

$$f(P_N) \neq f(P_{N \setminus \{i\}}). \quad (1)$$

If this is not the case, for each $\{j, k\} \in \mathcal{N}$ and each $(P_j, P_k) \in \mathcal{U}_O^{\{j, k\}}$, we have $f(P_j) = f(P_j, P_k) = f(P_k)$ and thus $f(P_j) = f(P_k)$, implying that all one-voter societies are assigned the same alternative. This violates *onteness*. So (1) holds.

Next, let $P'_i \in \mathcal{U}_O$ be such that $t(P'_i) = t(P_i)$ and $t(P'_i) R'_i f(P_{N \setminus \{i\}}) P'_i T$ for each $T \in 2^O \setminus \{t(P_i), f(P_{N \setminus \{i\}})\}$. By *participation*, $f(P'_i, P_{N \setminus \{i\}}) R'_i f(P_{N \setminus \{i\}})$. By *tops-only*, $f(P'_i, P_{N \setminus \{i\}}) = f(P_N)$. Thus, (1) implies $f(P'_i, P_{N \setminus \{i\}}) \neq f(P_{N \setminus \{i\}})$ and, therefore, we have $f(P'_i, P_{N \setminus \{i\}}) = t(P'_i)$. Hence, using *tops-only* again, we get

$$f(P_N) = t(P_i). \quad (2)$$

By Lemma 1, we can safely assume that all the tops in P_N are different. Let $j \in N \setminus \{i\}$ and consider $P'_j \in \mathcal{U}_O$ such that $t(P'_j) = t(P_j)$ and $b(P'_j) = t(P_i)$. By *tops-only* and (2), $f(P'_j, P_{N \setminus \{j\}}) = t(P_i)$. By *participation*, $f(P'_j, P_{N \setminus \{j\}}) R'_j f(P_{N \setminus \{j\}})$ and, since $b(P'_j) = t(P_i)$, we have $f(P_{N \setminus \{j\}}) = t(P_i)$. Removing in the same way each remaining voter, one at a time, we obtain $f(P_i) = t(P_i)$. Thus, by *object neutrality*, $f(P'_i) = \mu(f(P_i))$ for any permutation $\mu : O \rightarrow O$. Together with *anonymity*, granted by Proposition 2, this implies that

$$f(P'_\ell) = t(P'_\ell) \text{ for each } \{\ell\} \in \mathcal{N} \text{ and } P'_\ell \in \mathcal{U}_O \text{ with } |t(P'_\ell)| = |t(P_i)|. \quad (3)$$

Now, let $\{j, k\} \in \mathcal{N}$ and $(P_j, P_k) \in \mathcal{U}_O^{\{j, k\}}$ be such that $t(P_j) \neq t(P_k)$ and $|t(P_j)| = |t(P_k)| = |t(P_i)|$. There are three cases to consider:

1. $f(P_j, P_k) \notin \{t(P_j), t(P_k)\}$. Let $P'_j \in \mathcal{U}_O$ be such that $t(P'_j) = t(P_j)$ and $b(P'_j) = f(P_j, P_k)$. By *tops-only*, $f(P'_j, P_k) = f(P_j, P_k)$. By *participation*, $f(P'_j, P_k) R'_j f(P_k)$. Thus, $f(P_k) = f(P_j, P_k) \neq t(P_k)$, contradicting (3).
2. $f(P_j, P_k) = t(P_j)$. Consider a permutation $\mu : O \rightarrow O$ such that $\mu(t(P_j)) = t(P_k)$ and $\mu(t(P_k)) = t(P_j)$. By *object neutrality* we obtain $f(P'_j, P'_k) = \mu(f(P_j, P_k)) = \mu(t(P_j)) = t(P_k)$, and thus $f(P'_j, P'_k) = t(P_k)$. By Theorem 2, f is *anonymous*. Therefore, by *anonymity* and *tops-only*,
$$f(P_j, P_k) = f(P_k, P_j) = f(P'_j, P'_k) = t(P_k),$$
and so $f(P_j, P_k) = t(P_k)$, contradicting this case's hypothesis.
3. $f(P_j, P_k) = t(P_k)$. A similar reasoning to the previous case allows us to deduce that $f(P_j, P_k) = t(P_j)$, contradicting this case's hypothesis.

Since in each case we reach a contradiction, we conclude that no such rule f exists. $\square$

It is important to notice that we are not demanding voting rules to satisfy an excessively high number of properties, i.e., there are no redundant axioms in Theorem 1. To see this, we consider several voting rules. Each one satisfies all the axioms but one.

- All but *ontoness*: The rule f^O in Example 1.
- All but *tops-onliness*: The rule $\tilde{f}$ in Example 1.
- All but *false-name-proofness*: For each $N \in \mathcal{N}$ and each $P_N \in \mathcal{U}_O^N$, $f^{\min}(P_N) = t(P_{\min\{i : i \in N\}})$. This rule selects the top of the voter with the minimum index in the society. This rule satisfies *participation* but is not *anonymous*, thus, by Proposition 2, $f^{\min}$ is not *false-name-proof*.
- All but *participation*: For each $N \in \mathcal{N}$ and each $P_N \in \mathcal{U}_O^N$, $x \in f^*(P_N)$ if and only if $|\{i \in N \mid x \in t(P_i)\}| = 1$. This rule selects those objects that belong to only one top-choice set of the preference profile. To see that f^* does not satisfy *participation*, let $O =$

$\{x, y, z\}$, $N = \{i, j\}$ and $(P_i, P_j) \in \mathcal{U}_O^{\{i,j\}}$ be such that $t(P_i) = \{x, y\}$, $\{x, y, z\} \succ P_i \{z\}$, and $t(P_j) = \{x, y, z\}$. Then, if voter i does not participate, she can manipulate f^* since $f^*(P_j) = \{x, y, z\} \succ P_i \{z\} = f^*(P_i, P_j)$.

- *All but object-neutrality*: Let $\succ$ be a linear order over 2^O . For each $N \in \mathcal{N}$ and each $P_N \in \mathcal{U}_O^N$, $f^\succ(P_N) = \max_\succ \{t(P_i) : i \in N\}$. This rule selects, for each profile, the best positioned top according to $\succ$. To see that $f^\succ$ does not satisfy *object-neutrality*, let $O = \{x, y, z\}$, $\{x\} \succ \{y\} \succ \{z\}$, $N = \{i, j\}$, and $P_N \in \mathcal{U}_O^N$ be such that $t(P_i) = \{x\}$ and $t(P_j) = \{z\}$. Thus $f(P_N) = \{x\}$. If we consider a permutation $\mu : O \rightarrow O$ such that $\mu(x) = z$, $\mu(y) = x$, and $\mu(z) = y$, then $\mu(f^\succ(P_N)) = \mu(\{x\}) = \{z\} \neq \{y\} = f^\succ(P_N^\mu)$.

3.3 Domain of separable preferences: maximality

We have seen that when we consider the domain of all preferences over subsets of objects, there are no voting rules that satisfy *ontoness*, *tops-onliness*, *false-name-proofness*, *participation*, and *object neutrality*. Nevertheless, we can find several rules that satisfy all of them when the preferences of the voters are separable. We can find two examples in Fioravanti and Massó [9], with voting by quota 1 and voting by unanimous quota. A natural question is whether there is a domain larger than the domain of separable preferences in which voting rules still satisfy all the axioms. Next, we answer the latter in the negative.

First, we remember the definition of separability. For a voter, an object is *good* if it is better to choose this object alone than choosing no object at all; otherwise, the object is *bad*. A preference is separable if the division between good and bad objects guides the ordering of subsets, in the sense that adding a good object leads to a better set while adding a bad object leads to a worse set. Formally, preference P_0 is *separable* if for each $S \in 2^O$ and each $x \in O \setminus S$,

$$S \cup \{x\} \succ P_0 S \text{ if and only if } \{x\} \succ P_0 \emptyset.$$

Let $\mathcal{S}$ be the domain of separable preferences. An important characterization of separability is presented in the following remark.

Remark 4 [3] *Preference $P_0 \in \mathcal{U}_O$ is separable if, for each $S \in 2^O$ and each $x \in O \setminus S$,*

$$S \cup \{x\} \succ P_0 S \text{ if and only if } x \in t(P_0).$$

Let $\mathcal{F}$ denote the class of all rules defined on the domain of separable preferences that are *onto*, *tops-only*, *false-name-proof*, satisfy *participation*, and are *object neutral*. The following definition, inspired by Bonifacio et al. [5] and Arribillaga and Bonifacio [1], formalizes the idea of maximal domain for a set of rules satisfying a list of properties.⁶

Definition 1 *Let $\mathcal{S}^*$ be such that $\mathcal{S} \subseteq \mathcal{S}^* \subseteq \mathcal{U}_O$ and let $\mathcal{F}^* \subseteq \mathcal{F}$. Domain $\mathcal{S}^*$ is **maximal for $\mathcal{F}^*$** if*

- (i) *for each $f \in \mathcal{F}^*$ the tops-only extension of f to $\mathcal{S}^*$ satisfies ontoness, false-name-proofness, participation and object neutrality,⁷ and*

⁶Previous studies on maximal domains, mostly focus on the property of *strategy-proofness* [see, for example, 16, 7, 12].

⁷Given a *tops-only* rule $f : \mathcal{S}^N \rightarrow 2^O$ and a domain $\mathcal{S}^*$ such that $\mathcal{S} \subseteq \mathcal{S}^*$, the **tops-only extension of f to $\mathcal{S}^*$** is such that, for each $N \in \mathcal{N}$ and each $P_N \in \mathcal{S}^*$, $f(P_N) = f(\bar{P}_N)$ for some $\bar{P}_N \in \mathcal{S}$ with $t(\bar{P}_i) = t(P_i)$ for each $i \in N$.

- (ii) for each $P_0 \in \mathcal{U}_O \setminus \mathcal{S}^*$ there is $f \in \mathcal{F}^*$ such that the tops-only extension of f to $\mathcal{S}^* \cup \{P_0\}$ violates (at least) one of the properties listed in (i).

An important fact about the maximality of a domain with respect to a list of properties thus defined is its monotonicity: the bigger the set of rules considered for maximality, the smaller the domain of preferences in which the properties hold. We highlight this observation in the following remark.

Remark 5 Assume that $\mathcal{S}^*$ is maximal for $\mathcal{F}^*$. If $\mathcal{F}^i \subseteq \mathcal{F}^*$ and $\mathcal{S}^i$ is maximal for $\mathcal{F}^i$ with $i \in \{1, 2\}$, then by Definition 1 it follows that $\mathcal{S}^* \subseteq \mathcal{S}^1 \cap \mathcal{S}^2$.

Next, we present our maximality result.

Theorem 2 *The domain of separable preferences is maximal for the set of all onto, tops-only, and false-name-proof rules that satisfy participation and object neutrality, i.e., $\mathcal{S}$ is maximal for $\mathcal{F}$.*

Proof. Let $\bar{P}_0 \in \mathcal{U}_O \setminus \mathcal{S}$. By Remark 4, there are $S \subseteq \mathcal{O}$ and $x \in \mathcal{O} \setminus S$ such that either

$$x \in t(\bar{P}_0) \text{ and } S \bar{P}_0 S \cup \{x\} \quad (4)$$

or

$$x \notin t(\bar{P}_0) \text{ and } S \cup \{x\} \bar{P}_0 S. \quad (5)$$

First, consider rule $f^> : \mathcal{S}^N \rightarrow 2^{\mathcal{O}}$ such that, for each $N \in \mathcal{N}$ and each profile $P_N \in \mathcal{S}^N$ satisfies⁸

$$x \in f^>(P_N) \text{ if and only if } |\{t(P_i) \in t(P_N) : x \in t(P_i)\}| > \frac{|t(P_N)|}{2}.$$

Clearly, $f^>$ is *tops-only* and *object neutral*. Since the rule only depends on the set of top subsets of options and not on how many times each subset appears, $f^>$ is *false-name-proof*. Moreover, as casting a vote can only add support to a good object for a voter, $f^>$ also satisfies *participation*. Therefore, $f^> \in \mathcal{F}$.

Assume further that $t(\bar{P}_0) \neq \emptyset$ and consider the *tops-only* extension of $f^>$ to $\mathcal{S} \cup \{\bar{P}_0\}$. There are two cases to consider. If (4) holds, let $(P_1, P_2) \in (\mathcal{S} \cup \{\bar{P}_0\})^{\{1,2\}}$ be such that $t(P_1) = S$ and $t(P_2) = S \cup \{x\}$. Let $i^* \notin \{1, 2\}$ and endow voter i^* with preference $\bar{P}_0$. Notice that, by (4), $t(\bar{P}_{i^*}) \neq S \cup \{x\}$. Then,

$$f^>(P_1, P_2) = S \bar{P}_{i^*} S \cup \{x\} = f^>(P_1, P_2, \bar{P}_{i^*}),$$

contradicting *participation*. If (5) holds and $t(\bar{P}_0) \not\subseteq S$, let $y \in t(\bar{P}_0) \setminus S$ and consider $(P_1, P_2, P_3) \in (\mathcal{S} \cup \{\bar{P}_0\})^{\{1,2,3\}}$ such that $t(P_1) = S$, $t(P_2) = S \cup \{x\}$, and $t(P_3) = S \cup \{x, y\}$. Let $i^* \notin \{1, 2, 3\}$ and endow voter i^* with preference $\bar{P}_0$. Then,

$$f^>(P_1, P_2, P_3) = S \cup \{x\} \bar{P}_{i^*} S = f^>(P_1, P_2, P_3, \bar{P}_{i^*}),$$

contradicting *participation*. If (5) holds and $t(\bar{P}_0) \subseteq S$, let $y \in t(\bar{P}_0)$ and consider $(P_1, P_2, P_3) \in (\mathcal{S} \cup \{\bar{P}_0\})^{\{1,2,3\}}$ such that $t(P_1) = S$, $t(P_2) = S \cup \{x\}$, and $t(P_3) = (S \setminus \{y\}) \cup \{x\}$. Let $i^* \notin \{1, 2, 3\}$ and endow voter i^* with preference $\bar{P}_0$. Notice that, by (5), $t(\bar{P}_{i^*}) \neq S$. Then,

$$f^>(P_1, P_2, P_3) = S \cup \{x\} \bar{P}_{i^*} S = f^>(P_1, P_2, P_3, \bar{P}_{i^*}),$$

⁸**Notation:** given a society $N \in \mathcal{N}$ and profile $P_N \in \mathcal{U}_O^N$, let $t(P_N) = \{t(P_i) \mid i \in N\}$ be the collection of (different) tops of profile P_N .

contradicting *participation*. Since in both cases we reach a contradiction, it follows that $t(\bar{P}_0) = \emptyset$. Thus, this implies that

$$\text{if } \mathcal{S}^> \text{ is maximal for } \{f^>\}, \text{ then } \mathcal{S}^> \subseteq \mathcal{S} \cup \{\bar{P}_0 \in \mathcal{U}_{\mathcal{O}} \setminus \mathcal{S} : t(\bar{P}_0) = \emptyset\}. \quad (6)$$

Second, consider the rule $f^{\geq} : \mathcal{S}^{\mathcal{N}} \rightarrow 2^{\mathcal{O}}$ such that, for each $N \in \mathcal{N}$ and each profile $P_N \in \mathcal{S}^N$ satisfies that

$$x \in f^{\geq}(P_N) \text{ if and only if } |\{t(P_i) \in t(P_N) : x \in t(P_i)\}| \geq \frac{|t(P_N)|}{2}.$$

A reasoning similar to the one used for $f^>$ shows that $f^{\geq} \in \mathcal{F}$.

Assume now that $t(\bar{P}_0) \neq \emptyset$ and consider the *tops-only* extension of $f^{\geq}$ to $\mathcal{S} \cup \{\bar{P}_0\}$. There are two cases to consider. If (4) holds and $S \neq \emptyset$, let $(P_1, P_2, P_3) \in (\mathcal{S} \cup \{\bar{P}_0\})^{\{1,2,3\}}$ be such that $t(P_1) = S$, $t(P_2) = S \cup \{x\}$, and $t(P_3) = \emptyset$. Let $i^* \notin \{1,2,3\}$ and endow voter i^* with preference $\bar{P}_0$. Notice that, by (4), $t(\bar{P}_{i^*}) \neq S \cup \{x\}$. Then,

$$f^{\geq}(P_1, P_2, P_3) = S \cup \bar{P}_{i^*} \cup \{x\} = f^{\geq}(P_1, P_2, P_3, \bar{P}_{i^*}),$$

contradicting *participation*. If (4) holds and $S = \emptyset$, let $y \in \mathcal{O} \setminus t(\bar{P}_0)$ and consider $(P_1, P_2, P_3) \in (\mathcal{S} \cup \{\bar{P}_0\})^{\{1,2,3\}}$ be such that $t(P_1) = \emptyset$, $t(P_2) = \{x\}$, and $t(P_3) = \{y\}$. Let $i^* \notin \{1,2,3\}$ and endow voter i^* with preference $\bar{P}_0$. Notice that, by (4), $t(\bar{P}_{i^*}) \neq \{x\}$.⁹ Then,

$$f^{\geq}(P_1, P_2, P_3) = \emptyset \cup \bar{P}_{i^*} \cup \{x\} = f^{\geq}(P_1, P_2, P_3, \bar{P}_{i^*}),$$

contradicting *participation*. If (5) holds, let $(P_1, P_2) \in (\mathcal{S} \cup \{\bar{P}_0\})^{\{1,2\}}$ be such that $t(P_1) = S$ and $t(P_2) = S \cup \{x\}$. Let $i^* \notin \{1,2\}$ and endow voter i^* with preference $\bar{P}_0$. Notice that by (5), $t(\bar{P}_{i^*}) \neq S$. Then,

$$f^{\geq}(P_1, P_2) = S \cup \{x\} \cup \bar{P}_{i^*} = f^{\geq}(P_1, P_2, \bar{P}_{i^*}),$$

contradicting *participation*. Since in each case we reach a contradiction, it follows that $t(\bar{P}_0) = \emptyset$. Thus, this implies that

$$\text{if } \mathcal{S}^{\geq} \text{ is maximal for } \{f^{\geq}\}, \text{ then } \mathcal{S}^{\geq} \subseteq \mathcal{S} \cup \{\bar{P}_0 \in \mathcal{U}_{\mathcal{O}} \setminus \mathcal{S} : t(\bar{P}_0) = \emptyset\}. \quad (7)$$

Finally, let $\mathcal{S}^*$ be maximal for $\mathcal{F}$. By Definition 1, $\mathcal{S} \subseteq \mathcal{S}^*$. Since $f^>, f^{\geq} \in \mathcal{F}$, Remark 5 implies that $\mathcal{S}^* \subseteq \mathcal{S}^> \cap \mathcal{S}^{\geq}$. By (6) and (7), $\mathcal{S}^> \cap \mathcal{S}^{\geq} \subseteq \mathcal{S}$ and thus $\mathcal{S}^* \subseteq \mathcal{S}$. Hence, $\mathcal{S}^* = \mathcal{S}$. $\square$

Remark 6 Notice that to prove Theorem 2, the only property used, besides *tops-onliness*, is *participation*. Therefore, a more general maximality result is available considering only these two properties.

4 Conclusion

We analyze voting rules that satisfy *false-name-proofness* and the *participation* criterion. We show that these two axioms imply *anonymity* and that this holds for any domain of preferences. Moreover, we further extend previous negative results for the case of the universal domain, showing that there are no *neutral* voting rules consistent with these two axioms. For the domain where preferences are strict linear orders over subsets of objects, we show that compatibility of these two properties of immunity to manipulation with *onteness*, *tops-onliness* and *object neutrality* can only be achieved, in a strong sense, in the domain of separable preferences.

⁹Also note that, if $\mathcal{O} = \{x, y\}$, then (4) is trivially contradicted.

As a by-product of Proposition 2 we obtain a new characterization of voting rules over the domain of separable preferences. This follows from two observations. The first one is that the combination of *false-name-proofness*, *strategy-proofness*, and *ontoness* is equivalent, due to Propositions 6 and 8 of Fioravanti and Massó [9], to the combination of *strong false-name-proofness*, *participation*, *ontoness*, and *anonymity*. The second is due to our Proposition 2: *anonymity* is superfluous in the latter list. Therefore, by Theorem 1 of Fioravanti and Massó [9] we can characterize all voting rules that satisfy *strong false-name-proofness*, *participation*, and *ontoness* as the class of voting rules in which an object is chosen if it has either at least one vote in every society or a unanimous vote in every society.

References

- [1] R Pablo Arribillaga and Agustín G Bonifacio. Not obviously manipulable allotment rules. *Economic Theory*, page to appear, 2025.
- [2] Salvador Barberà. Strategyproof social choice. *Handbook of Social Choice and Welfare*, 2: 731–831, 2011.
- [3] Salvador Barberà, Hugo Sonnenschein, and Lin Zhou. Voting by committees. *Econometrica*, 59(3):595–609, 1991. ISSN 00129682, 14680262. URL <http://www.jstor.org/stable/2938220>.
- [4] Duncan Black. On the rationale of group decision-making. *Journal of Political Economy*, 56(1):23–34, 1948.
- [5] Agustín G. Bonifacio, Jordi Massó, and Pablo Neme. Preference restrictions for simple and strategy-proof rules: Local and weakly single-peaked domains. *Journal of Mathematical Economics*, 106:102845, 2023. ISSN 0304-4068. doi: <https://doi.org/10.1016/j.jmateco.2023.102845>. URL <https://www.sciencedirect.com/science/article/pii/S0304406823000381>.
- [6] Nanyang Bu. Unfolding the mystery of false-name-proofness. *Economics Letters*, 120(3): 559–561, 2013. ISSN 0165-1765. doi: <https://doi.org/10.1016/j.econlet.2013.06.011>. URL <https://www.sciencedirect.com/science/article/pii/S0165176513002930>.
- [7] Stephen Ching and Shigehiro Serizawa. A maximal domain for the existence of strategy-proof rules. *Journal of Economic Theory*, 78(1):157–166, 1998.
- [8] Vincent Conitzer. Anonymity-proof voting rules. In Christos Papadimitriou and Shuzhong Zhang, editors, *Internet and Network Economics*, pages 295–306, Berlin, Heidelberg, 2008. Springer Berlin Heidelberg. ISBN 978-3-540-92185-1.
- [9] Federico Fioravanti and Jordi Massó. False-name-proof and strategy-proof voting rules under separable preferences. *Theory and Decision*, 97:391–408, 2024.
- [10] Peter C. Fishburn and Steven J. Brams. Paradoxes of preferential voting. *Mathematics Magazine*, 56(4):207–214, 1983. ISSN 0025570X, 19300980. URL <http://www.jstor.org/stable/2689808>.
- [11] Allan Gibbard. Manipulation of voting schemes: A general result. *Econometrica*, 41(4): 587–601, 1973. URL <https://EconPapers.repec.org/RePEc:ecm:emetrp:v:41:y:1973:i:4:p:587-601>.
- [12] Jordi Massó and Alejandro Neme. Maximal domain of preferences in the division problem. *Games and Economic Behavior*, 37(2):367–387, 2001.
- [13] Hervé Moulin. *Axioms of cooperative decision making*. Cambridge University Press, 1988.
- [14] Hervé Moulin. Condorcet’s principle implies the no show paradox. *Journal of Economic Theory*, 45(1):53–64, 1988. ISSN 0022-0531. doi: [https://doi.org/10.1016/0022-0531\(88\)90253-0](https://doi.org/10.1016/0022-0531(88)90253-0). URL <https://www.sciencedirect.com/science/article/pii/0022053188902530>.
- [15] Mark Allen Satterthwaite. Strategy-proofness and Arrow’s conditions: Existence and correspondence theorems for voting procedures and social welfare functions. *Journal of Economic Theory*, 10(2):187–217, 1975. ISSN 0022-0531. doi: [https://doi.org/10.1016/0022-0531\(75\)90009-0](https://doi.org/10.1016/0022-0531(75)90009-0).

1016/0022-0531(75)90050-2. URL <https://www.sciencedirect.com/science/article/pii/0022053175900502>.

- [16] Shigehiro Serizawa. Power of voters and domain of preferences where voting by committees is strategy-proof. *Journal of Economic Theory*, 67(2):599–608, 1995.
- [17] Taiki Todo, Atsushi Iwasaki, and Makoto Yokoo. False-name-proof mechanism design without money. In *The 10th International Conference on Autonomous Agents and Multiagent Systems - Volume 2, AAMAS '11*, page 651–658, Richland, SC, 2011. International Foundation for Autonomous Agents and Multiagent Systems. ISBN 0982657161.
- [18] Taiki Todo, Nodoka Okada, and Makoto Yokoo. False-name-proof facility location on discrete structures. In *Proceedings of ECAI*, volume 325, pages 227–234. IOS Press, 2020.
- [19] Makoto Yokoo, Yuko Sakurai, and Shigeo Matsubara. The effect of false-name bids in combinatorial auctions: New fraud in internet auctions. *Games and Economic Behavior*, 46(1):174–188, 2004. ISSN 0899-8256. doi: [https://doi.org/10.1016/S0899-8256\(03\)00045-9](https://doi.org/10.1016/S0899-8256(03)00045-9). URL <https://www.sciencedirect.com/science/article/pii/S0899825603000459>.

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