Sociotropic Behavior in Voting

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Abstract

This paper proposes a formal model of sociotropic voting, where voters' ballots are based on the preferences of their peers in a group rather than their individual preferences. This behavior, prevalent in social settings and elections, can paradoxically lead to inferior outcomes for the group. We assume that voters have perfect knowledge of others' preferences and use an internal aggregation rule to construct their ballots, which may be distinct from the external voting rule. We find that sociotropic voting can alter outcomes in ordinal single-winner and in approval-based multi-winner elections, but not in single-winner elections under the classical approval voting rule. Our theoretical and experimental results show that sociotropic voting can be harmful when external voting rules are designed for consensus and fairness, such as Condorcet consistent rules. However, it can enhance equity in settings using simple voting rules like plurality.

1 Introduction

Suppose that a group of friends tries to pick a movie, and everyone—being polite and socially aware—votes for what they think the others would prefer. This ends with them watching a movie that no one actually wants to see. This behavior, where individuals aim to align with the group's perceived preferences, is known as *sociotropic* [5]. Note a crucial difference from *altruism* [23]: a sociotropic actor completely disregards her own preferences, while an altruist does not. Such a tendency is not limited to friendly gatherings; strong evidence shows that sociotropic behavior occurs in political elections, affecting choices on a national level [19, 22], a phenomenon that has intrigued political scientists for more than 40 years [18].

We propose the first formal model of sociotropic voting and explore its paradoxical effects. Example 1 illustrates a simple case where sociotropic voting changes the outcome. We will later see that the outcome may worsen, even with voters who aim to benefit the group, have perfect knowledge of others' preferences, and use a sophisticated rule to form their ballots.

Example 1. Consider four voters deciding between four candidates (a, b, c, d) in an election employing the famous Borda rule as follows: each voter reports a ranking of the candidates, each candidate receives 3 (respectively 2, 1, and 0) points when appearing in the first (respectively second, third, and last) position of a voter's ranking, and the candidate with the most total points wins. Suppose voters 1 and 2 individually prefer a to c to d to b, while voters 3 and 4 individually prefer b to c to d to a. The Borda rule here would choose c as the unique winner. Suppose, however, that all voters engage in sociotropic behavior—instead of reporting their individual preferences, they try to please their peers. To do so, they compute the popularity of each candidate via simple plurality. Excluding herself from the group, voter 1 (and similarly voter 2) sees candidate b as the most popular (being at the top for $2/3$ of the others' preferences), followed by a, c, and d. Analogously, voters 3 and 4 end up ranking a first, followed by b, c, and d. The Borda rule applied to these sociotropic ballots would now select a and b as tied winners. △

We base our model of sociotropic voting on a question that has been studied extensively in both economics and computer science: the impact that the voters' misrepresentation of preferences has on the outcome of an election [3, 11]. In particular, we follow the standard model of strategic manipulation and assume that voters have full knowledge of the other voters' preferences. However, a sociotropic voter does not aim at a better outcome for herself—instead, her ballot captures what is best for the rest of the group. Thus to form a ballot, sociotropic voters need to *aggregate* the others' preferences. To do so, we assume that they emlpoy an *internal* aggregation rule that might be different from the rule used to make the final group decision.

We study sociotropic voting within the two most prominent election frameworks in the (computational) social choice literature, namely ordinal elections [28] and approval voting [20]. In the former we examine only single-winner elections, while in the latter we consider both single- and multi-winner elections. For all models we first study whether sociotropic voting can change the outcome of an election at all. We find that this holds for ordinal single-winner voting and approval-based multi-winner voting but not for single-winner approval voting, at least under the standard voting rule AV and certain behavioral assumptions on sociotropic voters. Then, we investigate the situations in which sociotropic voting can lead to inferior or superior outcomes. We provide both general theoretical results and simulation experiments. Broadly speaking, we observe that sociotropic voting can be harmful if the *external* voting rule used to make the collective decision is already designed to provide a consensus outcome and satisfies desirable properties like Condorcet consistency or justified representation; but if the external voting rule is a simple majoritarian rule like plurality or AV, sociotropic voting can, under some conditions, help deliver a more equitable outcome. Omitted proofs can be found in the appendix.

Related work. *Strategic behavior* in elections has long been central to the social choice literature [16, 24]. The topic gained interest among computer scientists after it was shown that computational complexity theory can be used to gain new insights into the issue [4]. However, to the best of our knowledge, this literature so far has only focused on voters trying to improve the outcome according to their own preferences.

Public-spirited voting is also closely related to our model. The main difference is that voters in public-spirited voting have cardinal utilities, which are only translated into ordinal ballots after the public-spirited voters aggregate them. While this is very natural in the study of distortion [15, 2], the assumption of full knowledge of cardinal utility functions is much stronger than our assumption that voters know each others ordinal or approval preferences. Moreover we do not focus on distortion but study outcome changes more generally.

Finally, in the model of *decision-making in social networks*, some authors have considered scenarios where voters are influenced by the preferences of their neighbors [27, 10, 12, 26, 17]. While related in spirit, this framework is technically distinct from ours. Moreover, in this setting, most work has focused on the problem of electoral control, which we do not consider.

2 Model

This section formally introduces sociotropic voting in elections with approval and ordinal preferences.

2.1 Elections with Approval Preferences

In this model we are interested in electing a committee of (at most) k candidates, and voters express their preferences by approving a subset of the candidates.[1] Let N and X be the (finite) sets of voters and candidates, respectively. Voters are numbered from 1 to n and candidates are denoted by letters ($a, b, \ldots$ or $x, y, \ldots$). Each voter $i \in N$ is endowed with a nonempty set $A_i \subseteq X$, called her *individual preference*, and casts a nonempty *ballot* $B_i \subseteq X$, which might deviate from her individual preference. We indicate as $\boldsymbol{A} = (A_1, \ldots, A_n)$ and $\boldsymbol{B} = (B_1, \ldots, B_n)$ the *profiles* of individual preferences and ballots, respectively. Let $\boldsymbol{A}_{-i} = (A_1, \ldots, A_{i-1}, A_{i+1}, \ldots, A_n)$ be the profile obtained by excluding A_i from the profile $\boldsymbol{A}$. To compute the outcome of an election, we use an *external aggregation rule* F^{ext}

[1] Output committees can contain less than k candidates for technical reasons but, ideally, we want committees to be of size k.

that maps each profile $\boldsymbol{B}$ to a nonempty set of (tied) committees of size at most k (for some $k \in \mathbb{N}$), i.e., $F^{\text{ext}}(\boldsymbol{B}) \subseteq \{C \subseteq X : |C| \leq k\}$.

We designate a subset $\text{Soc} \subseteq N$ as the *sociotropic voters*, while voters in the set $N \setminus \text{Soc}$ are called *individualistic*. Each individualistic voter reports a ballot identical to her individual preference, i.e., $B_i = A_i$. Each sociotropic voter forms her ballot based on the individual preferences of all other voters. Formally, each $i \in \text{Soc}$ first applies a (shared) *internal* aggregation rule F^{int} to $\boldsymbol{A}_{-i}$ such that $F^{\text{int}}(\boldsymbol{A}_{-i}) \subseteq \{C \subseteq X : |C| \leq k\}$ and $F^{\text{int}}(\boldsymbol{A}_{-i}) \neq \varnothing$. Observe that the amount of representation any group of voters deserves is dependent on k. It is thus natural to assume that sociotropic voters use the same k for their internal aggregation as is used for the external aggregation.

Note that, in the case of ties, the output of the internal aggregation can be a set of committees, while a ballot is a set of candidates. We propose two ways in which sociotropic voters can handle ties, which we call *ballot functions* $\mathcal{B}$:[2]

- *lexicographic*: $\mathcal{B}(\mathcal{C})$ is the lexicographically minimal committee in $\mathcal{C}$ (assuming some linear ordering over X).
- *union*: $\mathcal{B}(\mathcal{C}) = \bigcup_{C \in \mathcal{C}} C$, i.e., B_i is the set of all the co-winning candidates in $\mathcal{C}$.

For each sociotropic voter, we set $B_i = \mathcal{B}(F^{\text{int}}(\boldsymbol{A}_{-i}))$. See Example 2 for an illustration of this. Finally, unless stated otherwise, we focus on the extreme case where every voter is sociotropic—we assume $\text{Soc} = N$.

We now introduce axioms and related notions. In multi-winner voting, one of the main requirements is *proportionality*, i.e., the idea that, if a group is large enough to deserve a certain amount of candidates in the committee, it should receive them. Many formalizations of this have been proposed [20, Chapter 4]. We present here a relatively weak axiom, which we use in our theoretical results. In the sequel, we write F to indicate either F^{ext} or F^{int}.

Definition 1 (Cohesive Groups). A set of voters $N' \subseteq N$ is *ℓ-cohesive* if $|N'| \geq \ell \cdot n/k$ and $|\bigcap_{i \in N'} A_i| \geq \ell$.

Definition 2 (Justified Representation). A rule F satisfies *Justified Representation* (JR) if, for every profile $\boldsymbol{A}$, committee $C \in F(\boldsymbol{A})$ and each 1-cohesive group N', there is some $i \in N'$ with $|C \cap A_i| \geq 1$.

In the experiments, we also use a stronger notion.

Definition 3 (EJR+). A rule F satisfies EJR+ if, for every profile $\boldsymbol{A}$ and committee $C \in F(\boldsymbol{A})$, there is no candidate $x \in X \setminus C$, group $N' \subseteq N$, and $\ell \in \mathbb{N}$ with $|N'| \geq \ell \cdot n/k$ such that $x \in \bigcap_{i \in N'} A_i$ and, for all $i \in N'$, $|A_i \cap C| < \ell$.

Observe that EJR+ [9] implies JR [1]. We also consider the following mild conditions, where $\boldsymbol{A}$ is quantified universally over profiles:

- Rule F is *neutral* if, for every permutation $\pi : X \to X$ and committee $C \subseteq X$, we have $C \in F(\boldsymbol{A})$ iff $\pi(C) \in F(\pi(\boldsymbol{A}))$.[3]
- Rule F is *unanimous* if $|\bigcap_{i \in N} A_i| \leq k$ (resp. $>$) implies $\left(\bigcap_{i \in N} A_i\right) \subseteq C$ (resp. $\supseteq$) for all $C \in F(\boldsymbol{A})$.

[2] Because F^{int} might return an empty committee, for both ballot functions we assume $\mathcal{B}(\{\varnothing\}) = X$. This does not affect our results.

[3] Here, $\pi(\boldsymbol{A})$ is the profile obtained by replacing every occurrence of every candiate x in $\boldsymbol{A}$ with $\pi(x)$. The set $\pi(C)$ (with $C \subseteq X$) is defined analogously.

- Rule F is *exhaustive* if $|C| = k$ for all $C \in F(\boldsymbol{A})$.

For some of our results, we will focus on the well-behaved special case of *party-list profiles*, which resemble parliamentary elections in countries with proportional representation systems. Here, for each voters $i,\ j \in N$, we have either $A_i \cap A_j = \varnothing$ or $A_i = A_j$.

Finally, in this setting, one of the most natural rules is *approval voting* (AV), where we select all sets of k candidates that maximize the total number of approvals. Besides AV, we consider the following rules: sequential proportional approval voting (seqPAV), sequential Chamberlin-Courant (seqCC), and the Method of Equal Shares (MES) without completion [20, Chapter 2]. In the interest of space, we defer their definition to the appendix. Observe that, of these rules, only seqCC and MES satisfy JR [20, Chapter 4], and only the latter satisfies EJR+ [9].

Example 2. Consider a profile $\boldsymbol{A}$ with 3 voters with individual preferences $A_1 = \{a, b\}$, $A_2 = \{a, c\}$, and $A_3 = \{c, d\}$. Let $k = 2$ and suppose F^{int} is AV. We have $F^{\text{int}}(\boldsymbol{A}_{-1}) = \{\{a, c\}, \{c, d\}\}$. Therefore, if $\mathcal{B}$ is *lexicographic*, $B_1 = \{a, c\}$; if $\mathcal{B}$ is *union*, $B_1 = \{a, c, d\}$. Moreover, if $\text{Soc} = N$ and $\mathcal{B}$ is *lexicographic*, then $\boldsymbol{B} = (\{a, c\}, \{a, b\}, \{a, b\})$ and the final outcome for F^{ext} being AV is $\text{AV}(\boldsymbol{B}) = \{a, b\}$. △

2.2 Elections with Ordinal Preferences

As a second model we consider the case that voters express their preferences via strict rankings. For this setting we only consider single-winner elections.

Similarly to before, we have a set of voters N and of candidates X. Preferences and ballots are now in the form of strict linear orders over X. The set of all such orders is indicated as $\mathcal{L}(X)$. To distinguish from the previous model, we write the individual preference of a voter $i \in N$ as $R_i \in \mathcal{L}(X)$ and the profile of individual preferences as $\boldsymbol{R}$. Again, we indicate as $B_i \in \mathcal{L}(X)$ and $\boldsymbol{B}$ ballots and ballot profiles, respectively. If voter i prefers candidate x over candidate y, we write xR_iy (similarly for B_i); when clear from the context, we may simply write $x \succ_i y$ or $x \succ y$.

Here we have an external aggregation rule F^{ext} that maps each profile $\boldsymbol{B}$ of ordinal ballots to a nonempty subset of X, and an internal aggregation rule F^{int} that maps each $\boldsymbol{R}_{-i}$ to a nonempty subset of $\mathcal{L}(X)$. To form her ballot from the linear orders in $F^{\text{int}}(\boldsymbol{R}_{-i})$, each sociotropic voter $i \in \text{Soc}$ uses a different ballot function $\mathcal{B}_i$ that maximizes agreement with her individual preference R_i. Formally:

$$B_i \in \operatorname*{arg\,max}_{B \in F^{\text{int}}(\boldsymbol{R}_{-i})} |\{(x, y) \in X^2 : xBy \;\text{ and }\; xR_iy\}|$$

Note that for all specific rules studied in this paper, B_i is uniquely determined[4] and the above is well defined. Again, unless otherwise stated, we assume $\text{Soc} = N$.

We now introduce more notation. For a profile $\boldsymbol{R}$ and candidates $x,\ y \in X$, let $n^{\boldsymbol{R}}_{xy}$ be the number of voters preferring x over y. When clear from the context, we omit the profile in the superscript. If $n_{xy} > n_{yx}$ (resp., $n_{xy} = n_{yx}$) we say that x wins (resp., ties) in the pairwise contest against y. We say that x is the *Condorcet winner* of $\boldsymbol{R}$ if x wins the pairwise majority contest against all other candidates. When it exists, the Condorcet winner is unique. Moreover, we define the set of *weak Condorcet winners* of $\boldsymbol{R}$, called $\text{CW}(\boldsymbol{R})$, as the set of candidates that never lose a pairwise majority contest. If x is the Condorcet winner, then $\text{CW}(\boldsymbol{R}) = \{x\}$ (but the converse might not hold). The set $\text{CW}(\boldsymbol{R})$ might also be empty.

An external rule F^{ext} (resp. an internal rule F^{int}) is *Condorcet consistent* if, whenever $\boldsymbol{R}$ has a Condorcet winner x, then $F^{\text{ext}}(\boldsymbol{R}) = \{x\}$ (resp., x is ranked first in every order in $F^{\text{int}}(\boldsymbol{R})$). Furthermore, F^{ext} is

[4]As we explain later, all rules we consider work by first computing some weak order and then returning all its refining linear orders. Among these, there is always exactly one order that maximizes agreement with R_i.

weak-Condorcet consistent if, whenever $\mathrm{CW}(\boldsymbol{R}) \neq \varnothing$, then $F^{\text{ext}}(\boldsymbol{R}) = \mathrm{CW}(\boldsymbol{R})$. Similarly, F^{int} is weak-Condorcet consistent if, whenever $\mathrm{CW}(\boldsymbol{R}) \neq \varnothing$, for any order $\succ \in F^{\text{int}}(\boldsymbol{R})$, all candidates in $\mathrm{CW}(\boldsymbol{R})$ are ranked above all other candidates and $\pi(\succ) \in F^{\text{int}}(\boldsymbol{R})$ for all permutations $\pi : \mathrm{CW}(\boldsymbol{R}) \to \mathrm{CW}(\boldsymbol{R})$.[5]

We also say that x is the *unanimous winner* of $\boldsymbol{R}$ if all voters rank x first. The rule F^{ext} is *unanimous* if, whenever x is a unanimous winner of $\boldsymbol{R}$, then $F^{\text{ext}}(\boldsymbol{R}) = \{x\}$.

A popular family of rules in single-winner elections with ordinal preferences consists of *positional scoring rules*, where every rule is defined by a non-increasing vector

$$\boldsymbol{\alpha} = (\alpha(1),\ \ldots,\ \alpha(|X|)) \in \mathbb{R}^{|X|}_{\geq 0} \quad \text{with} \quad \alpha(1) > \alpha(|X|).$$

In words, every voter assigns to each candidate x she ranks at position ℓ from the top of her linear order exactly $\alpha(\ell)$ points, and the total score of x is the sum of points received by all voters. Then, based on $\boldsymbol{\alpha}$,

- the corresponding external rule F^{ext} outputs as winners the candidate(s) with maximal score.
- the corresponding internal rule F^{int} weakly ranks the candidates according to their score, and outputs all linear orders that are refinements of that weak ranking.

Notable positional scoring rules include the Borda rule, plurality, and veto (or anti-plurality). These are defined by the vectors $(|X|-1,\ |X|-2,\ \ldots,\ 0)$, $(1,\ 0,\ \ldots,\ 0)$, and $(1,\ \ldots,\ 1,\ 0)$, respectively. No positional scoring rule is Condorcet consistent [28], but plurality and Borda are unanimous.

Example 3. Suppose that both the external aggregation rule F^{ext} and the internal aggregation rule F^{int} are plurality. Consider the profile of individual preferences $\boldsymbol{R} = (a \succ b \succ c,\ b \succ c \succ a,\ c \succ b \succ a)$. Assume all voters are sociotropic. Then, the ballot profile is $\boldsymbol{B} = (b \succ c \succ a,\ c \succ a \succ b,\ b \succ a \succ c)$. For example, voter 1 looks at the preferences of the other voters, $b \succ c \succ a$ and $c \succ b \succ a$. Then, she ranks the candidates (according to their plurality score in $\boldsymbol{R}_{-i}$) as follows: b and c tie for the highest score, whereas a has the lowest score. Since she prefers b over c, she computes her ballot as $b \succ c \succ a$. Thus, we have $F^{\text{ext}}(\boldsymbol{B}) = \{b\} \subsetneq F^{\text{ext}}(\boldsymbol{R}) = \{a,\ b,\ c\}$. △

We then consider the next two internal aggregation rules.

Definition 4 (Copeland)**.** The aggregation rule F^{int} called *Copeland* works as follows. Every candidate $x \in X$ gets $+1$ point for each pairwise majority contest it wins and -1 point for each pairwise majority contest it loses. The candidates are weakly ranked according to their points, and the rule outputs all linear orders that refine that weak ranking.

Definition 5 (Simple Condorcet)**.** The aggregation rule F^{int} called *simple Condorcet* works as follows. If $\mathrm{CW}(\boldsymbol{R}) = \varnothing$, then $F^{\text{int}}(\boldsymbol{R}) = \mathcal{L}(X)$. Otherwise, $F^{\text{int}}(\boldsymbol{R}) = \{\succ \in \mathcal{L}(X) : x \succ y \text{ for all } x \in \mathrm{CW}(\boldsymbol{R}) \text{ and } y \in X \setminus \mathrm{CW}(\boldsymbol{R})\}$.

Both are Condorcet consistent, and simple Condorcet is also weak-Condorcet consistent. When used as an internal aggregation rule by sociotropic voters, simple Condorcet has the following effect in B_i for each $i \in \mathrm{Soc}$: the relative ordering of the candidates in $\mathrm{CW}(\boldsymbol{R}_{-i})$ and in $X \setminus \mathrm{CW}(\boldsymbol{R}_{-i})$ is the same as in R_i, but all candidates in $\mathrm{CW}(\boldsymbol{R}_{-i})$ are ranked above the candidates in $X \setminus \mathrm{CW}(\boldsymbol{R}_{-i})$.

After defining sociotropic behavior in elections, our next goal is to scrutinize its effects.

[5] Here, $\pi(\succ)$ is the order obtained by replacing in $\succ$ all occurrences of every candidate x (included in the domain of π) by $\pi(x)$.

3 Does Sociotropic Voting Change the Outcome?

We have already seen in Example 1 that sociotropic behavior may change the outcome of an election. In this section, we study more closely under which conditions this is possible.

3.1 Approval Preferences

First of all, we show that there is at least one important case in which sociotropic behavior can never change the outcome of an election. Namely, single-winner approval voting (when $k = 1$) where both the external rule and internal aggregation rule are AV and the ballot function is *union*.

Theorem 1. *Let both F^{ext} and F^{int} be AV, and let the ballot function be* union. *Moreover, assume $k = 1$. Then, for every profile $\boldsymbol{A}$ of approval preferences, $F^{ext}(\boldsymbol{A}) = F^{ext}(\boldsymbol{B})$.*

Importantly, Theorem 1 requires that all voters are sociotropic and fails otherwise. Moreover, observe that, if $k = 1$, most reasonable voting rules are equivalent to AV, so this result is rather general; still, it fails for approval-based rules that are distinct from AV for $k = 1$, e.g., SAV [8]. Finally, this result also fails if the ballot function is *lexicographic*, and not *union*. We present all the relevant examples in the appendix.

Let us now look at the general case where $k \geq 1$. Here, we do not know of any reasonable rules F^{ext} and F^{int} guaranteeing that $F^{\text{ext}}(\boldsymbol{A}) = F^{\text{ext}}(\boldsymbol{B})$ for every profile $\boldsymbol{A}$. Specifically, contrary to Theorem 1, if both F^{ext} and F^{int} are AV, the outcome can change even for the *union* ballot function and if voters approve of at least k candidates. See Example 4.

Example 4. Let $k = 2$ and consider profile $\boldsymbol{A} = (\{a,\ b,\ e\},\ \{c,\ d\},\ \{a,\ b,\ c\})$. Here, the AV winners are $\{a,\ b\}$, $\{a,\ c\}$, and $\{c,\ b\}$. If the voters use AV internally and the *union* ballot function, then $\boldsymbol{B} = (\{a,\ b,\ c,\ d\},\ \{a,\ b\},\ \{a,\ b,\ c,\ d,\ e\})$. Here, the AV winner is only $\{a,\ b\}$. △

3.2 Ordinal Preferences

In this setting, *contra* Theorem 1, Proposition 2 shows that, if the external aggregation rule is a positional scoring rule, sociotropic behavior may change the outcome whenever voters also use a positional scoring rule to form their ballots.

Proposition 2. *Let F^{ext} and F^{int} be positional scoring rules. Then, there exists a profile $\boldsymbol{R}$ such that $F^{ext}(\boldsymbol{R}) \neq F^{ext}(\boldsymbol{B})$.*

Notice that this does not show how $F^{\text{ext}}(\boldsymbol{R})$ and $F^{\text{ext}}(\boldsymbol{B})$ relate to each other, only that they might differ; they could be either fully disjoint or overlap significantly. For plurality, we prove that a new winner is never introduced; moreover, this holds independently of the number of sociotropic voters.

Proposition 3. *Let F^{ext} and F^{int} be plurality. Then, for every $Soc \subseteq N$ and profile $\boldsymbol{R}$ we have that $F^{ext}(\boldsymbol{B}) \subseteq F^{ext}(\boldsymbol{R})$.*

Proof (Sketch). Let $p_x(\boldsymbol{R})$ indicate the plurality score of candidate x in profile $\boldsymbol{R}$ and $top(R)$ the top-ranked candidate in R. In appendix, we prove the following useful fact.

Fact 1. If F^{ext} and F^{int} are plurality and $i \in \text{Soc}$ then $top(B_i) \in F^{\text{ext}}(\boldsymbol{R})$.

Next, for a candidate $x \in X$ and profile $\boldsymbol{R}$, let

$$s_x(\boldsymbol{R}) = |\{i \in \text{Soc}\colon x = top(R_i)\}| \quad \text{and} \quad e_x(\boldsymbol{R}) = |\{i \in N \setminus \text{Soc}\colon x = top(R_i)\}|.$$

Clearly $p_x(\boldsymbol{R}) = s_x(\boldsymbol{R}) + e_x(\boldsymbol{R})$ and $e_x(\boldsymbol{R}) = e_x(\boldsymbol{B})$ for all x. By Fact 1,

$$\sum_{x \in F^{\text{ext}}(\boldsymbol{R})} s_x(\boldsymbol{B}) = |\text{Soc}| \geq \sum_{x \in F^{\text{ext}}(\boldsymbol{R})} s_x(\boldsymbol{R}).$$

Hence, there is some $x^\star \in F^{\text{ext}}(\boldsymbol{R})$ such that $s_{x^\star}(\boldsymbol{B}) \geq s_{x^\star}(\boldsymbol{R})$. Crucially, this implies $p_{x^\star}(\boldsymbol{B}) \geq p_{x^\star}(\boldsymbol{R})$. For any $y \in X \setminus F^{\text{ext}}(\boldsymbol{R})$, we get $p_{x^\star}(\boldsymbol{B}) \geq p_{x^\star}(\boldsymbol{R}) > p_y(\boldsymbol{R}) \geq e_y(\boldsymbol{R}) = p_y(\boldsymbol{B})$, where the strict inequality follows because $x^\star$ is in $F^{\text{ext}}(\boldsymbol{R})$ while y is not, and the equality by Fact 1. Lastly, the above implies $y \notin F^{\text{ext}}(\boldsymbol{B})$ and, thus, $F^{\text{ext}}(\boldsymbol{B}) \subseteq F^{\text{ext}}(\boldsymbol{R})$. □

Proposition 3 does not hold for arbitrary positional scoring rules. For example, if F^{ext} and F^{int} are the Borda rule, then for $\boldsymbol{R} = (a \succ b \succ c \succ d,\ d \succ b \succ a \succ c,\ d \succ c \succ a \succ b)$ we have that $F^{\text{ext}}(\boldsymbol{R}) = \{d\} \not\subseteq \{a\} = F^{\text{ext}}(\boldsymbol{B})$.

Observe that plurality essentially corresponds to (single-winner) AV with singleton preferences; hence, one might expect a result analogous to Theorem 1 for plurality, rather than Proposition 3. The intuition behind this discrepancy lies in the ballot functions: Theorem 1 hinges on the *union* ballot function, which is not well-defined in the context of plurality (as it would allow voters to vote for multiple candidates).

4 The Disadvantages of Sociotropic Voting

In this section we focus on cases where sociotropic voting can alter the results of the election and study the disadvantages this may have for the group.

4.1 Approval Preferences

Let us begin with approval-based multi-winner voting, where we get the following result.

Proposition 4. *Let F^{ext} and F^{int} be unanimous rules, F^{ext} be neutral, and F^{int} be exhaustive. Moreover, let $\mathcal{B}$ be either* lexicographic *or* union. *Then, there exists a party-list profile $\boldsymbol{A}$ and a committee size k such that $F^{ext}(\boldsymbol{B})$ does not satisfy JR with respect to the individual preferences in $\boldsymbol{A}$.*

Proof. Consider profile $\boldsymbol{A} = (\{a, b\}, \{c\})$ and set $k = 2$. By unanimity of F^{int}, we have that $F^{\text{int}}(\boldsymbol{A}_{-2}) = \{\{a, b\}\}$ and that c must be in every $C \in F^{\text{int}}(\boldsymbol{A}_{-1})$. Thus, by exhaustiveness of F^{int}, we have three cases:

1. $F^{\text{int}}(\boldsymbol{A}_{-1}) = \{\{a, c\}\}$. Independently of the ballot function, $\boldsymbol{B} = (\{a, c\}, \{a, b\})$. By unanimity of F^{ext}, a must be in any $C \in F^{\text{ext}}(\boldsymbol{B})$. If $\{a, c\} \notin F^{\text{ext}}(\boldsymbol{B})$, we violate JR w.r.t. $\boldsymbol{A}$. Thus, assume $\{a, c\} \in F^{\text{ext}}(\boldsymbol{B})$. By neutrality of F^{ext}, this implies $\{a, b\} \in F^{\text{ext}}(\{a, b\}, \{a, c\})$. But then we can consider $\boldsymbol{A}' = (\{c\}, \{a, b\})$ as our initial profile and obtain $\{a, b\} \in F^{\text{ext}}(\boldsymbol{B}')$, a JR violation w.r.t. $\boldsymbol{A}'$.

2. $F^{\text{int}}(\boldsymbol{A}_{-1}) = \{\{c, b\}\}$. Independently of the ballot function, $\boldsymbol{B} = (\{c, b\}, \{a, b\})$. The analysis of this case is analogous to the previous one.

3. $F^{\text{int}}(\boldsymbol{A}_{-1}) = \{\{a, c\}, \{b, c\}\}$. Now, if $\mathcal{B}$ is *lexicographic*, we have $\boldsymbol{B} = (\{a, c\}, \{a, b\})$, and we are in the first case. If $\mathcal{B}$ is *union*, we have $\boldsymbol{B} = (\{a, b, c\}, \{a, b\})$. By unanimity of F^{ext}, we have $F^{\text{ext}}(\boldsymbol{B}) = \{\{a, b\}\}$, which violates JR w.r.t $\boldsymbol{A}$. □

Note that, disturbingly, this implies that sociotropic behavior can make the outcome unproportional (w.r.t. the true individual preferences) even if F^{ext} satisfies JR. Still, the proof of this quite general result hinges on the fact that F^{int} is exhaustive, and on some voter approving less than k candidates. We show next that, under additional assumptions, these two aspects are not required, at least for the union ballot function.

Proposition 5. *Let F^{ext} and F^{int} be neutral rules and let F^{int} also satisfy JR. Let $\mathcal{B}$ be* union. *Then, there exists a party-list profile $\boldsymbol{A}$ and a committee size k where all voters approve of at least k candidates such that $F^{ext}(\boldsymbol{B})$ does not satisfy JR with respect to the individual preferences in $\boldsymbol{A}$.*

4.2 Ordinal Preferences

Proposition 6 generalizes Examples 1 and 3.

Proposition 6. *Let F^{int} be a positional scoring rule. Then, there exists a profile $\boldsymbol{R}$ such that the Condorcet winner of $\boldsymbol{B}$ differs from the Condorcet winner of $\boldsymbol{R}$.*

We immediately get Corollary 7, showing that sociotropic voting might cause the removal of a Condorcet winner from the outcome. If one values Condorcet consistency, this can be seen as a negative consequence of sociotropic behavior.

Corollary 7. *Let F^{ext} be a Condorcet consistent rule and F^{int} a positional scoring rule. Then, there exists a profile $\boldsymbol{R}$ such that $\boldsymbol{R}$ has a Condorcet winner but $F^{ext}(\boldsymbol{R}) \cap F^{ext}(\boldsymbol{B}) = \varnothing$.*

5 The Advantages of Sociotropic Voting

We have seen that not only can sociotropic behavior change the voting outcome, but it can also lead to outcomes that are arguably less desirable. However, improvements are also possible, as the following example shows.

Example 5. Consider a profile $\boldsymbol{R}$ where two voters have preference $a \succ b \succ c$, two voters $b \succ a \succ c$, and three voters $c \succ b \succ a$. Here, plurality elects c, which is the Condorcet loser (i.e., c loses all majority contests). However, suppose that F^{int} is Borda. One can verify that all voters rank b first in $\boldsymbol{B}$. Hence, in $\boldsymbol{B}$, plurality elects b, which is the Condorcet winner w.r.t. the individual preferences. $\triangle$

In light of this, we investigate how sociotropic behavior can improve the outcome for the group.

5.1 Approval Preferences

Proposition 4 has demonstrated instances where a proportional rule might return an unproportional outcome due to sociotropic behavior. Despite this worst-case result, we will now argue that sociotropic behavior can be beneficial.

Theorem 8. *Let F^{ext} be AV, F^{int} be MES without any completion method, and the ballot function be* lexicographic. *If $\boldsymbol{R}$ is a party-list profile and $n \geq k+1$, then $F^{ext}(\boldsymbol{B})$ satisfies JR.*

Observe that the condition $n \geq k+1$ rules out the example used to prove Proposition 4. Moreover, a similar result is ruled out for the *union* ballot function by Proposition 5. Intuitively, the advantage of the *lexicographic* ballot function here is that it allows the sociotropic voters to coordinate on which candidates supported by a cohesive group to approve. Thus, from a theoretical perspective, successful sociotropic voting requires some coordination between the voters. We complement our theoretical findings experimentally.

Internal	Ballot func.	Improvement	Damage
AV	*lex.*	15% / 16%	0% / 0.1%
AV	*union*	0.1% / 0.2%	0.2% / 0%
seqPAV	*lex.*	100% / 100%	0% / 0%
seqPAV	*union*	88% / 94%	2% / 0.1%
seqCC	*lex.*	100% / 100%	0% / 0%
seqCC	*union*	52% / 66%	9% / 9%
MES	*lex.*	100% / 100%	0% / 0%
MES	*union*	88% / 94%	2% / 0.1%

Table 1: EJR+ experiments. Sampling: 1D / 2D-Euclidean.

Experimental questions. The objective is to quantify how often sociotropic behavior can be beneficial. To that end we consider the external rule to be AV paired with lexicographic tiebreaking, which fails basic proportionality requirements. Then, we investigate whether certain kinds of internal aggregation rules employed by the sociotropic voters can improve the proportionality of the outcome. By "improvement" we mean that the AV outcome over the individual preferences $\boldsymbol{R}$ does not satisfy some proportionality axiom, whereas the sociotropic outcome, i.e., the AV outcome over the ballot profile $\boldsymbol{B}$, does (and the other way around for "damage").

Experimental setup. To capture the **desired property** of the collective outcome, we encode proportionality via the EJR+ axiom. This is a strong notion but also poly-time verifiable [9], contrary to other strong proportionality axioms such as EJR [1].

For the **internal rule** F^{int}, we consider AV, seqPAV, seqCC, and MES. We employ AV as a baseline, and compare it to rules that aim to achieve proportional results. Focusing on sequential rules is computationally convenient, but also seems like a reasonable restriction: otherwise, we would require sociotropic voters to solve a hard computational problem to construct their ballots. We additionally consider the *lexicographic* and *union* **ballot functions**. For robustness and to minimize the assumptions on the behavior of sociotropic voters, we also ran the experiments pairing F^{int} with a randomized ballot function, i.e., where B_i is selected uniformly at random from $F^{\text{int}}(\boldsymbol{A}_{-i})$. This did not have a significant impact on our conclusions. See appendix for details.

Our **sample** is generated with the Euclidean model [25], where each voter and candidate is a point sampled uniformly at random from $[0,\ 1]^D$, and where each voter has a radius r. A voter approves of any candidate at a Euclidean distance smaller or equal than r. In particular, we use the 1D-Euclidean model with $r = 0.05$ and the 2D-Euclidean model with $r = 0.2$, we set 50 voters and 50 candidates, and k equal to 5. Our setup is deliberate: According to recent work on the "map of elections" [13], these parameters provide a substantial chance of EJR (and thus EJR+) being violated by AV, which we need for our results to be meaningful.[6] For each of the two models, we generate with rejection sampling 1000 profiles where the AV outcome satisfies EJR+ and 1000 where it does not.[7]

Experimental observations. Our results are presented in Table 6. Here 15% improvement means that among the 1000 profiles in which the AV outcome over the individual preferences violates EJR+, there are 150 profiles in which the AV outcome over the sociotropic ballots satisfies EJR+. Overall, it seems that sociotropic behavior can be beneficial regarding proportionality. Indeed, almost no damage was registered, meaning that the risk involved with sociotropic behavior is low. One slight exception is seqCC with the union ballot function. Moreover, when the internal aggregation rule is

[6]This work [13] considers profiles with 100 voters and 100 candidates, and sets $k = 10$; we focus on a smaller setting for computational limitations, but keep the size of the elections sufficiently large to ensure enough profiles failing EJR+.

[7]We are grateful to the `abcvoting` Python package [21] that we used in our experiments.

p_{Soc}	**Improvement**	**Damage**
0.5	100% / 100%	0% / 0%
0.25	99.9% / 99.9%	0% / 0%
0.1	97% / 97%	0% / 0%
0.05	80% / 81%	0% / 0%
0.01	26% / 26%	0% / 0%

Table 2: Probabilistic EJR+ experiments, where F^{int} is seqPAV and $\mathcal{B}$ is *lexicographic*. Sampling: 1D / 2D-Euclidean.

proportional (seqPAV, seqCC, MES) the outcome can often become more proportional. If the internal rule is unproportional (AV), then we see only slight improvements. Specifically, it seems like seqPAV and MES help improve the outcome independently of the ballot function used, and with low risk. SeqCC seems to drop significantly in performance if paired with the union ballot function. Generally, the union ballot function performs worse than the lexicographic function. This is reasonable, because the lexicographic function (as mentioned) can be seen as a form of coordination among voters.

Clearly, it is a simplification to assume that $\text{Soc} = N$. Thus, to test the robustness of our results, we performed the following experiment. We assume that each voter has probability p_{Soc} of being sociotropic. If they are, they compute their ballot as described above. Otherwise, they just submit their individual preference. We ran this experiment for the case where F^{int} is seqPAV with *lexicographic* ballot function, as in this case sociotropic behavior seems quite beneficial.

The results are shown in Table 2. From this, we can draw two conclusions. First, it seems like, our previous observations about the low risk of sociotropic behavior are quite robust. Secondly, it seems that sociotropic behavior can be beneficial even if a large portion of the population is individualistic, assuming that the sociotropic voters construct their ballot in a proportional way. In the most extreme case, even if on average one voter out of 100 is sociotropic, we still get an improvement rate of 26%.

This last result is remarkable; to explain it, we manually inspected some profiles we sampled. We noted that, often, there is a candidate $x \in X$ such that (1) x is not in the outcome, (2) the inclusion of x in the outcome would satisfy EJR+, and (3) x has quite high approval score (e.g., it could have maximal score but be excluded from the outcome by tiebreaking). Hence, a small change in the input in favor of x can make x win, and thus satisfy EJR+. It seems that, if sociotropic voters construct their ballots using a proportional rule, they often include an alternative such as x in their ballots.

These experiments suggest that if the external rule is not fair but sociotropic voters use a proportional rule to form their ballots, then this can often positively impact the outcome (in terms of proportionality). Given our negative theoretical results (Section 4.1), one might wonder if the opposite also holds: If the external rule is proportional but sociotropic voters use AV internally, do we just as often lose proportional guarantees (e.g., EJR+)? We ran further experiments to check this, but found it untrue: we rarely lose proportional guarantees due to sociotropic behavior even if sociotropic voters use a simple rule like AV. We show this experiment in appendix.

5.2 Ordinal Preferences

Let us now turn to ordinal preferences. We again focus on Condorcet consistency and start with internal aggregation rules that are weak-Condorcet consistent.

Proposition 9. *Let F^{int} be a weak-Condorcet consistent rule. If $\boldsymbol{R}$ has a strict Condorcet winner x, then x is the unanimous winner of $\boldsymbol{B}$.*

Proof. Let x be strict Condorcet winner in $\boldsymbol{R}$ and fix $i \in N$. First, observe that x must be a weak Condorcet winner in $\boldsymbol{R}_{-i}$, because for any $y \in X \setminus \{x\}$ we have that $n_{xy}^{\boldsymbol{R}_{-i}} \geq n_{xy}^{\boldsymbol{R}} - 1 \geq n_{yx}^{\boldsymbol{R}} \geq n_{yx}^{\boldsymbol{R}_{-i}}$, where the second inequality follows from the fact that x is the Condorcet winner in $\boldsymbol{R}$. Hence, by weak-Condorcet consistency, candidate x is ranked above any $y \notin \mathrm{CW}(\boldsymbol{R}_{-i})$ in all orders in $F^{\text{int}}(\boldsymbol{R}_{-i})$.

Next, observe that, if $\boldsymbol{R}_{-i}$ has some weak Condorcet winner y distinct from x, then we must have xR_iy. To see this, note that $n_{xy}^{\boldsymbol{R}} > n_{yx}^{\boldsymbol{R}}$ but $n_{xy}^{\boldsymbol{R}_{-i}} = n_{yx}^{\boldsymbol{R}_{-i}}$; this is possible only if i prefers x over y. Thus, by weak-Condorcet consistency of F^{int} and the definition of $\mathcal{B}_i$, x is ranked first in B_i.

Since this holds for an arbitrary $i \in N$, we have that x is ranked first by all voters, completing the proof. □

A straightforward consequence of this is the following. Suppose that F^{ext} satisfies the quite innocuous property of unanimity. Then, a certain kind of sociotropic behavior (corresponding to F^{int} being weak-Condorcet consistent) guarantees that, if a Condorcet winner exists, it will be elected. If one values Condorcet consistency, this can be seen as a positive effect of (a certain kind of) sociotropic behavior. The following result follows immediately from Proposition 9.

Corollary 10. *Let F^{ext} be a unanimous and F^{int} a weak-Condorcet consistent rule. Then, for every profile $\boldsymbol{R}$ with a strict Condorcet winner x, we have that $F(\boldsymbol{B}) = \{x\}$.*

Note that the last two results do not hold if we consider x to be a weak Condorcet winner instead of a strict one, even if it is the unique such winner (see appendix for details). Next, we study the case where the internal rule used for sociotropic voting is a specific weak-Condorcet consistent rule, namely simple Condorcet. Here, we show that the weak Condorcet winners under sociotropic voting are among the weak Condorcet winners of the individual preference profile.

Proposition 11. *Let F^{int} be simple Condorcet. Then, for every Soc $\subseteq N$ and profile $\boldsymbol{R}$ with CW$(\boldsymbol{R}) \neq \varnothing$, we have that $\varnothing \subsetneq$ CW$(\boldsymbol{B}) \subseteq$ CW$(\boldsymbol{R})$.*

For other weak-Condorcet consistent rules beyond simple Condorcet, Proposition 11 does not necessarily hold. Proposition 11 also implies the following result.

Corollary 12. *Let F^{int} be simple Condorcet and F^{ext} be weak-Condorcet consistent. Then, for any Soc $\subseteq N$ and profile $\boldsymbol{R}$ with CW$(\boldsymbol{R}) \neq \varnothing$, we have $F^{ext}(\boldsymbol{B}) \subseteq F^{ext}(\boldsymbol{R})$.*

As for multi-winner voting, we complement the theoretical analysis with some brief experimental observations.

Experimental questions. To quantify the benefits of sociotropic behavior in ordinal elections, we focus on Condorcet consistency. Here, by "improvement" we mean that the outcome of an external rule F^{ext} over the individual preferences $\boldsymbol{R}$ does not elect an existing strict Condorcet winner x, whereas the outcome of F^{ext} over the sociotropic ballots $\boldsymbol{B}$ does elect x (and the other way around for "damage").

Experimental setup. For the **external rule** F^{ext}, we consider two basic positional scoring rules, plurality and Borda, since no positional scoring rule is Condorcet consistent. For the **internal rule** F^{int}, we consider two Condorcet consistent rules, Copeland and simple Condorcet.

Our **sample** is generated with two complementary models. First, the impartial culture (IC) distribution that samples profiles uniformly at random is taken as the baseline. Second, Mallow's model with a rel-ϕ parameter (sampled uniformly at random), is found to be the most appropriate distribution for capturing real-world elections [7].[8] We sample profiles with 30 candidates and 31 voters, having a Condorcet winner (we do not consider larger profiles because such instances with a Condorcet winner are rare). We generate with rejection sampling 1000 profiles where F^{ext} elects the Condorcet winner (possibly together with other candidates), and 1000 profiles where F^{ext} does not.

[8]We are grateful to the `pref_voting` Python package (`pref-voting.readthedocs.io`) that we used in our experiments.

External	Internal	Improvement	Damage
Plurality	Copeland	98% / 99%	2% / 0%
Plurality	Simple Condorcet	88% / 78%	12% / 5%
Borda	Copeland	89% / 98%	0% / 0%
Borda	Simple Condorcet	90% / 92%	0% / 0%

Table 3: Condorcet experiments. Sampling: IC / rel-ϕ Mallows.

p_{Soc}	Improvement	Damage
0.5	60% / 74%	6% / 0%
0.25	53% / 62%	8% / 2%
0.1	35% / 43%	8% / 1%
0.05	27% / 26%	6% / 0%
0.01	8% / 8%	2% / 0%

Table 4: Probabilistic Condorcet experiments, where F^{ext} is Borda and F^{int} is simple Condorcet. Sampling: IC / rel-ϕ Mallows.

Experimental observations. See Table 3. Table 4 presents the results of the corresponding experiment focusing on Borda and simple Condorcet, where we consider varying probabilities of a voter being sociotropic (denoted by p_{Soc}). For example, a 98% improvement means that, among the 1000 profiles in which the F^{ext} outcome over the individual preferences does not elect the Condorcet winner (say, x), there are 980 profiles in which the F^{ext} outcome over the sociotropic ballots elects x. Overall, improvements are prevailing, especially when the internal rule F^{int} is Copeland. Damages are rather negligible (usually 0%), with the strongest ones (12%) being observed when the external and internal rules are plurality and simple Condorcet, respectively. Notably, for F^{ext} being Borda and F^{int} being simple Condorcet, we have substantial improvements even when only 10% of the voters are sociotropic (see the row with $p = 0.1$ in Table 4, where improvements are 43% for the Mallows distribution.)

6 Conclusion

We introduced a first formal model of sociotropic voting for three of the most common settings in computational social choice: single- and multi-winner voting with approval preferences and single-winner voting with ordinal preferences. We identified one important case where sociotropic behavior cannot affect the outcome, single-winner approval voting with AV, at least under the *union* ballot function that is natural for this setting. For the other cases, our results suggest that sociotropic behavior can have negative consequences, especially if the voting rule used in the election already embodies some form of fairness (i.e., Condorcet consistency, JR). However, if the rule is not geared towards fairness (e.g., AV, plurality), then both theory and simulations show that sociotropic behavior can lead to more equitable outcomes for the group, especially when sociotropic voters use proportional rules like MES or seqPAV for their internal aggregation.

Our paper is a first step in the formal study of sociotropic voting. An interesting future direction would be to investigate what happens when sociotropic voters have only limited or imprecise knowledge about the preferences of their peers. Next, one could investigate what happens when different sociotropic voters use different internal rules to compute their ballots.

Acknowledgements

We thank the anonymous reviewers for their valuable feedback, which will significantly improve future versions of this work. This research was funded in whole or in part by the Austrian Science Fund (FWF), Austria under grant 10.55776/PAT7221724 and 10.55776/COE12, by netidee Förderungen, Austria (https://www.netidee.at/), Vienna Science and Technology Fund (WWTF), Austria (Grant ID: 10.47379/ICT23025), and by the European Union's Horizon 2020 research and innovation programme (grant agreement 101034440).

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A Omitted Rule Definitions

In this section, we define some rules we employ in this paper for the multi-winner setting.

We only consider *sequential* rules, i.e., we select candidates sequentially in (at most) k rounds. For each round r, let C_r be the set of candidates selected before this round. Note that, at each step, one might have several candidates to pick from (i.e., ties might occur). We assume that each rule returns the set of all committees that result from the described sequential process given *some* tiebreaking rule.

SeqPAV. Here, there are exacly k rounds. For each round r, we select some candidate x maximizing the following score:

$$x \in \operatorname*{arg\,max}_{x \in X \setminus C_r} \sum_{i \in N} \sum_{\ell=1}^{\mathrm{sat}_r(i,\,x)} 1/\ell, \quad \text{where} \quad \mathrm{sat}_r(i,\,x) = |A_i \cap (\{x\} \cup C_r)|.$$

SeqCC. This rule is analogous to seqPAV; the only difference is the score to be maximized:

$$x \in \arg\max_{x \in X \setminus C_r} \sum_{i \in N} \mathbb{1}[\mathrm{sat}_r(i,\, x) \geq 1],$$

where $\mathbb{1}[\cdot]$ is the indicator function (i.e., $\mathbb{1}[p] = 1$ if p is true and $\mathbb{1}[p] = 0$ otherwise).

MES. Initially, all voters are assigned a budget of k/n. Let $b_r(i)$ be the budget of voter i before round r (hence, $b_1(i) = k/n$ for all $i \in N$). Let $N(x) \subseteq N$ be the set of voters approving of x. In round r, we consider the set $X_r \subseteq X$ of all candidates in $X \setminus C_r$ such that $\sum_{i \in N(x)} b_r(i) \geq 1$. If X_r is empty, the rule terminates and returns C_r. Otherwise, we compute $\alpha(x)$ for each $x \in X_r$ as

$$\alpha(x) = \min \left\{ \alpha \in \mathbb{R} \mid \sum_{i \in N(x)} \min(\alpha,\, b_r(i)) = 1 \right\}.$$

We select a candidate $x^\star$ with minimal $\alpha(x^\star)$ and set $b_{r+1}(i) = \max(0, b_r(i) - \alpha(x^\star))$ if $i \in N(x^\star)$ and $b_{r+1}(i) = b_r(i)$ otherwise.

Note that MES might terminate in less than k rounds, and thus return a committee with fewer than k candidates. To enforce output committees to have size exactly k, one need to extend MES with a so-called *completion method* [20, Chapter 2]. However, in this paper, we only consider MES *without* completion methods.

B Further Examples

B.1 Approval Preferences

We first show that Theorem 1 does not hold if $\mathrm{Soc} \subsetneq N$. Consider profile $\boldsymbol{A} = (\{a\},\, \{a\},\, \{b\},\, \{b\})$ and let $k = 1$. Here, the AV winners are a and b. Now, assume that only voter 2 is sociotropic. Then, for any ballot function we get $\boldsymbol{B} = (\{a\},\, \{b\},\, \{b\},\, \{b\})$ and lose the approval winner a.

Moving on, besides AV, a popular single-winner approval-based rule is Satisfaction Approval Voting (SAV) [8], defined as

$$\mathrm{SAV}(\boldsymbol{A}) = \arg\max_{x \in X} \sum_{i \in N} \frac{|\{x\} \cap A_i|}{|A_i|},$$

Not only does Theorem 1 cease to hold if at least one of F^{ext} and F^{int} is SAV, but it might also hold that $F^{\mathrm{ext}}(\boldsymbol{A}) \cap F^{\mathrm{ext}}(\boldsymbol{B}) = \varnothing$, even if $\mathrm{Soc} = N$.

Example 6. Let F^{ext} and F^{int} be SAV. Take a profile $\boldsymbol{A} = (\{a\},\, \{b,\, c\},\, \{a,\, d,\, e\},\, \{b,\, c,\, e\},\, \{b,\, c,\, e\},\, \{b,\, c,\, e\})$. Then, $F^{\mathrm{ext}}(\boldsymbol{A}) = \{b,\, c\}$ but $F^{\mathrm{ext}}(\boldsymbol{B}) = \{a\}$. The same holds if F^{ext} is SAV and F^{int} is AV, or vice versa. △

Finally, suppose that the ballot function $\mathcal{B}_i$ is *lexicographic*, and not *union*. Again, Theorem 1 ceases to hold. Indeed, consider a profile with three voters, tiebreaking order $a > b$, and approvals $\{a\}$, $\{b\}$ and $\{b\}$. Here, the approval winner is b. However, the ballot profile becomes $\{b\}$, $\{a\}$ and $\{a\}$, with approval winner a.

B.2 Ordinal Preferences

We show that Proposition 9 and Corollary 10 fail for the case of weak Condorcet winners (even if only one weak Condorcet winner exists). Consider the following weak-Condorcet consistent rule F^{int}: If there are weak Condorcet winners, rank them first together, and the other candidates tie in the second place. Otherwise, apply Borda.[9] (This is a weak variant of Black's rule [14].) Consider profile $\boldsymbol{R}$:

$$
\begin{aligned}
R_1 &: a \succ b \succ c, \\
R_2 &: c \succ a \succ b, \\
R_3 &: c \succ a \succ b, \\
R_4 &: b \succ c \succ a, \\
R_5 &: b \succ c \succ a, \\
R_6 &: b \succ c \succ a.
\end{aligned}
$$

One can verify the following. The (only) weak Condorcet winner here is b, but in $\boldsymbol{B}$, the weak Condorcet winners are both b and c.

Next, we show that Proposition 9 fails for general (i.e., non-weakly) Condorcet consistent rules. Consider the following rule F^{int}: If there is a strict Condorcet winner, rank it first, and all other candidates tie in the second place; otherwise, apply Borda. (This is Black's rule [6].) Consider profile $\boldsymbol{R} = (a \succ b \succ c,\ a \succ b \succ c,\ b \succ c \succ a)$. One can verify the following: Here, the Condorcet winner is a, but in $\boldsymbol{B}$, the Condorcet winner is b.

C Omitted Experiments

In Tables 5 and 6 we report the omitted experiments.

Internal	Ballot func.	Improvement	Damage
AV	*random*	26% / 29%	0% / 0.1%
seqPAV	*random*	99% / 99%	0.1% / 0%
seqCC	*random*	86% / 93%	1% / 0.2%
MES	*random*	98% / 99%	0% / 0%

Table 5: EJR+ experiments for the *random* ballot function. Sampling: 1D-Euclidean (left) and 2D-Euclidean (right).

External	Internal	Ballot func.	Damage
seqPAV	AV	*lex.*	0.1% / 0.9%
seqPAV	AV	*union*	0.3% / 1.1%
seqPAV	AV	*random*	0.1% / 0.9%
seqCC	AV	*lex.*	4.0% / 8.3%
seqCC	AV	*union*	5.9% / 11%
seqCC	AV	*random*	2.8% / 7.3%
MES	AV	*lex.*	0.1% / 0.9%
MES	AV	*union*	0.3% / 1.1%
MES	AV	*random*	0.1% / 0.9%

Table 6: Additional EJR+ experiments. Here, we sampled 1000 profiles (per statistical culture) where seqPAV, seqCC and MES all satisfied EJR+. The "damage" percentage refers to the percentage of profiles such that, if all voters are sociotropic and sociotropic voters form their ballot with AV, then the outcome does not satisfy EJR+ anymore. Sampling: 1D / 2D-Euclidean.

[9] Of course, the procedure we describe returns a weak order. One can unambiguously transform such weak order into a set of (tied) linear orders.

D Omitted Proofs

Theorem 1. *Let both F^{ext} and F^{int} be AV, and let the ballot function be* union. *Moreover, assume $k = 1$. Then, for every profile $\boldsymbol{A}$ of approval preferences, $F^{\text{ext}}(\boldsymbol{A}) = F^{\text{ext}}(\boldsymbol{B})$.*

Proof. In the following, given a profile $\boldsymbol{A}$ and two subsets of candidates $Y,\ Z \subseteq X$, let

$$A_{\boldsymbol{A}}(Y^+,\ Z^-) = |\{i \in N : Y \subseteq A_i \subseteq X \setminus Z\}|.$$

Moreover, given a set $\mathcal{S} \subseteq 2^X$, let

$$\sqcup\mathcal{S} = \bigcup_{S \in \mathcal{S}} S.$$

In particular, $\sqcup F^{\text{ext}}(\boldsymbol{A})$ are all the cowinners of $F^{\text{ext}}(\boldsymbol{A})$. Next, let $\{x\} \in F^{\text{ext}}(\boldsymbol{A})$. Note that $A_{\boldsymbol{A}}(\{x\}^+)$ and $A_{\boldsymbol{B}}(\{x\}^+)$ are the approval scores of x in $\boldsymbol{A}$ and $\boldsymbol{B}$, respectively. Moving from $\boldsymbol{A}$ to $\boldsymbol{B}$, the approval winner x will be additionally approved by every voter i with $A_i \cap (\sqcup F^{\text{ext}}(\boldsymbol{A})) = \varnothing$, and also by every voter i such that $x \notin A_i$ but $A_i \cap (\sqcup F^{\text{ext}}(\boldsymbol{A})) \neq \varnothing$. This holds because when the voter excludes herself from $\boldsymbol{A}$, $\{x\}$ will be an approval winner of $F^{\text{ext}}(\boldsymbol{A}_{-i})$. However, x will lose the approval of every voter i such that $x \in A_i$ but $y \notin A_i$ for some $\{y\} \in F^{\text{ext}}(\boldsymbol{A})$ (if any). This holds because when the voter excludes herself from profile $\boldsymbol{A}$, $\{x\}$ will not be an approval winner of $F^{\text{ext}}(\boldsymbol{A}_{-i})$, as it loses to y. So, we have the following:

$$\begin{aligned} A_{\boldsymbol{B}}(\{x\}^+) = A_{\boldsymbol{A}}(\{x\}^+) &+ A_{\boldsymbol{A}}((\sqcup F^{\text{ext}}(\boldsymbol{A}))^-) \\ &+ \sum_{\{y\} \in F^{\text{ext}}(\boldsymbol{A})} A_{\boldsymbol{A}}(\{x\}^-,\ \{y\}^+) \quad - \sum_{\{y\} \in F^{\text{ext}}(\boldsymbol{A})} A_{\boldsymbol{A}}(\{x\}^+,\ \{y\}^-). \end{aligned}$$

Note that $\{x\} \in F^{\text{ext}}(\boldsymbol{A})$ and $\{y\} \in F^{\text{ext}}(\boldsymbol{A})$ implies that $A_{\boldsymbol{A}}(\{x\}^-,\ \{y\}^+) = A_{\boldsymbol{A}}(\{x\}^+,\ \{y\}^-)$. Hence,

$$A_{\boldsymbol{B}}(\{x\}^+) = A_{\boldsymbol{A}}(\{x\}^+) + A_{\boldsymbol{A}}((\sqcup F^{\text{ext}}(\boldsymbol{A}))^-). \tag{1}$$

Now, consider an arbitrary $\{z\} \notin F^{\text{ext}}(\boldsymbol{A})$. We know that z may be approved in $\boldsymbol{B}$ only by a voter i such that $z \notin A_i$ but $\sqcup F^{\text{ext}}(\boldsymbol{A}) \subseteq A_i$. This is because when such a voter (and only such a voter) excludes herself from $\boldsymbol{A}$, all approval cowinners of $\boldsymbol{A}$ will have their score decreased by one point, while z, preserving its score, may become an approval cowinner of $F^{\text{ext}}(\boldsymbol{A}_{-i})$. Therefore,

$$A_{\boldsymbol{B}}(\{z\}^+) \leq A_{\boldsymbol{A}}((\sqcup F^{\text{ext}}(\boldsymbol{A}))^+,\ \{z\}^-). \tag{2}$$

Because we have assumed that $\{x\} \in F^{\text{ext}}(\boldsymbol{A})$, it also holds that $A_{\boldsymbol{A}}((\sqcup F^{\text{ext}}(\boldsymbol{A}))^+,\ \{z\}^-) \leq A_{\boldsymbol{A}}(\{x\}^+)$, and thus

$$A_{\boldsymbol{B}}(\{z\}^+) \leq A_{\boldsymbol{A}}((\sqcup F^{\text{ext}}(\boldsymbol{A}))^+,\ \{z\}^-) \leq A_{\boldsymbol{A}}(\{x\}^+) \leq A_{\boldsymbol{B}}(\{x\}^+), \tag{3}$$

with the first inequality following from Inequality (2) and the last from Equation (1). Thus $A_{\boldsymbol{B}}(\{z\}^+) \leq A_{\boldsymbol{B}}(\{x\}^+)$.

Suppose, aiming for a contradiction, that this is an equality *for all* $\{x\} \in F^{\text{ext}}(\boldsymbol{A})$. Then, it must hold that $A_{\boldsymbol{A}}((\sqcup F^{\text{ext}}(\boldsymbol{A}))^+,\ \{z\}^-) = A_{\boldsymbol{A}}(\{x\}^+)$ for all $\{x\} \in F^{\text{ext}}(\boldsymbol{A})$ and that $A_{\boldsymbol{A}}((\sqcup F^{\text{ext}}(\boldsymbol{A}))^-) = 0$. In detail:

- $A_{\boldsymbol{A}}((\sqcup F^{\text{ext}}(\boldsymbol{A}))^+,\ \{z\}^-) = A_{\boldsymbol{A}}(\{x\}^+)$ for all $\{x\} \in F^{\text{ext}}(\boldsymbol{A})$ means that every voter i with $A_i \cap (\sqcup F^{\text{ext}}(\boldsymbol{A})) \neq \varnothing$ will have $(\sqcup F^{\text{ext}}(\boldsymbol{A})) \subseteq A_i$ and $z \notin A_i$.
- $A_{\boldsymbol{A}}(\sqcup(F^{\text{ext}}(\boldsymbol{A}))^-) = 0$ means that for every voter i it holds that $A_i \cap (\sqcup F^{\text{ext}}(\boldsymbol{A})) \neq \varnothing$.

Combining the two, every voter must approve all cowinners of $F^{\text{ext}}(\boldsymbol{A})$ and must not approve z (where $\{z\} \notin F^{\text{ext}}(\boldsymbol{A})$). Because $n \geq 2$, we have that $A_{\boldsymbol{A}}(\{x\}^+) \geq 2$, but $A_{\boldsymbol{A}}(\{z\}^+) = 0$. This means that when a voter i who approves all cowinners in $F^{\text{ext}}(\boldsymbol{A})$ excludes herself from $\boldsymbol{A}$, the candidates in $\sqcup F^{\text{ext}}(\boldsymbol{A})$ will still have a higher approval score than z in $\boldsymbol{A}_{-i}$ and thus voter i will not include z in B_i. Thus, $A_{\boldsymbol{B}}(\{z\}^+) = 0$. From Equation (1), $A_{\boldsymbol{B}}(\{x\}^+) = A_{\boldsymbol{A}}(\{x\}^+) \geq 2 > 0 = A_{\boldsymbol{B}}(\{z\}^+)$, a contradiction.

We conclude that one of the inequalities in (3) must be strict. So, z has strictly smaller approval score in $\boldsymbol{B}$ than some x with $\{x\} \in F^{\text{ext}}(\boldsymbol{A})$, and it cannot be an approval winner. Because $\{z\} \notin F^{\text{ext}}(\boldsymbol{A})$ was arbitrary, we have that $F^{\text{ext}}(\boldsymbol{B}) \subseteq F^{\text{ext}}(\boldsymbol{A})$. Moreover, from Equation (1) we know that all approval winners of $\boldsymbol{A}$ will have the same approval score in $\boldsymbol{B}$, implying that $F^{\text{ext}}(\boldsymbol{B}) = F^{\text{ext}}(\boldsymbol{A})$. $\square$

Proposition 2. *Let F^{ext} and F^{int} be positional scoring rules. Then, there exists a profile $\boldsymbol{R}$ such that $F^{ext}(\boldsymbol{R}) \neq F^{ext}(\boldsymbol{B})$.*

Proof. Take a set of three candidates $X = \{a, b, c\}$. W.l.o.g., we can assume F^{ext} to be defined by some score vector $(s_1 + s_2,\ s_1,\ 0)$ such that $s_1, s_2 \geq 0$ and $s_1 + s_2 > 0$. Likewise, let us assume that F^{int} is defined by a vector $(t_1 + t_2,\ t_1,\ 0)$ such that $t_1, t_2 \geq 0$ and $t_1 + t_2 > 0$.

First, let us assume that $s_2 > 0$, that is, F^{ext} is not the veto rule (we will analyse the case where F^{ext} is the veto rule after). Consider profile $\boldsymbol{R}$:

$$
\begin{aligned}
R_1 &: a \succ b \succ c,\\
R_2 &: b \succ c \succ a,\\
R_3 &: c \succ a \succ b,\\
R_4 &: b \succ c \succ a,\\
R_5 &: c \succ b \succ a.
\end{aligned}
$$

Let $f(x)$ indicate the score of candidate x under F^{ext} in $\boldsymbol{R}$. We have that $f(b) = f(c) = 4s_1 + 2s_2 > f(a) = 2s_1 + s_2$. Thus, $F^{\text{ext}}(\boldsymbol{R}) = \{b,\ c\}$. Next, let $g_i(x)$ be the score of candidate x under F^{int} in $\boldsymbol{R}_{-i}$. We have:

$$
\begin{aligned}
&g_1(a) = t_1, && g_1(b) = 3t_1 + 2t_2, && g_1(c) = 4t_1 + 2t_2,\\
&g_2(a) = 2t_1 + t_2, && g_2(b) = 3t_1 + t_2, && g_2(c) = 3t_1 + 2t_2,\\
&g_3(a) = t_1 + t_2, && g_4(b) = 4t_1 + 2t_2, && g_4(c) = 3t_1 + t_2,\\
&g_4(a) = 2t_1 + t_2, && g_3(b) = 3t_1 + t_2, && g_3(c) = 3t_1 + 2t_2,\\
&g_5(a) = 2t_1 + t_2, && g_5(b) = 3t_1 + 2t_2, && g_5(c) = 3t_1 + t_2.
\end{aligned}
$$

Using these scores, as well as the ballot function, we can determine the profile $\boldsymbol{B}$. There are three subcases.

1. $t_1,\ t_2 > 0$. Then, we have:

$$
\begin{aligned}
B_1 &: c \succ b \succ a,\\
B_2 &: c \succ b \succ a,\\
B_3 &: b \succ c \succ a,\\
B_4 &: c \succ b \succ a,\\
B_5 &: b \succ c \succ a.
\end{aligned}
$$

Since $s_2 > 0$, we have $F^{\text{ext}}(\boldsymbol{B}) = \{c\} \neq \{b,\ c\} = F^{\text{ext}}(\boldsymbol{R})$.

2. $t_1 > t_2 = 0$. Then (by the ballot function), we have:

$$\begin{aligned} B_1 &: c \succ b \succ a, \\ B_2 &: b \succ c \succ a, \\ B_3 &: b \succ c \succ a, \\ B_4 &: b \succ c \succ a, \\ B_5 &: c \succ b \succ a. \end{aligned}$$

   Since $s_2 > 0$, we have $F^{\text{ext}}(\boldsymbol{B}) = \{b\} \neq \{b,\ c\} = F^{\text{ext}}(\boldsymbol{R})$.

3. $t_2 > t_1 = 0$. Then (by the ballot function), we have:

$$\begin{aligned} B_1 &: b \succ c \succ a, \\ B_2 &: c \succ b \succ a, \\ B_3 &: b \succ c \succ a, \\ B_4 &: c \succ b \succ a, \\ B_5 &: b \succ c \succ a. \end{aligned}$$

   Since $s_2 > 0$, we have $F^{\text{ext}}(\boldsymbol{B}) = \{b\} \neq \{b,\ c\} = F^{\text{ext}}(\boldsymbol{P})$.

In all cases, we get that $F^{\text{ext}}(\boldsymbol{B}) \neq F^{\text{ext}}(\boldsymbol{R})$. This shows that if F^{ext} is not the veto rule, our statement holds. Now, let F^{ext} be the veto rule, and consider profile $\boldsymbol{R}$:

$$\begin{aligned} R_1 &: a \succ b \succ c, \\ R_2 &: a \succ b \succ c, \\ R_3 &: c \succ b \succ a. \end{aligned}$$

We have $F^{\text{ext}}(\boldsymbol{R}) = \{b\}$. Again, let $g_i(x)$ be the score of candidate x under F^{int} in $\boldsymbol{R}_{-i}$. We have:

$$\begin{aligned} g_1(a) &= t_1 + t_2, & g_1(b) &= 2t_1, & g_1(c) &= t_1 + t_2, \\ g_2(a) &= t_1 + t_2, & g_2(b) &= 2t_1, & g_2(c) &= t_1 + t_2, \\ g_3(a) &= 2t_1 + 2t_2, & g_4(b) &= 2t_1, & g_4(c) &= 0. \end{aligned}$$

Now, there are five subcases.

1. $t_1 > t_2 > 0$. Then, we have (by the ballot function):

$$\begin{aligned} B_1 &: b \succ a \succ c, \\ B_2 &: b \succ a \succ c, \\ B_3 &: a \succ b \succ c. \end{aligned}$$

   We have $F^{\text{ext}}(\boldsymbol{B}) = \{b,\ a\} \neq \{b\} = F^{\text{ext}}(\boldsymbol{R})$.

2. $t_2 > t_1 > 0$. Then, we have (by the ballot function):

$$\begin{aligned} B_1 &: a \succ c \succ b, \\ B_2 &: a \succ c \succ b, \\ B_3 &: a \succ b \succ c. \end{aligned}$$

   We have $F^{\text{ext}}(\boldsymbol{B}) = \{a\} \neq \{b\} = F^{\text{ext}}(\boldsymbol{R})$.

3. $t_1 = t_2 > 0$. Then, we have (by the ballot function):

$$B_1 \colon a \succ b \succ c,$$
$$B_2 \colon a \succ b \succ c,$$
$$B_3 \colon a \succ b \succ c.$$

   We have $F^{\text{ext}}(\boldsymbol{B}) = \{a,\ b\} \neq \{b\} = F^{\text{ext}}(\boldsymbol{R})$.

4. $t_1 > t_2 = 0$. Then, we have (by the ballot function):

$$B_1 \colon b \succ a \succ c,$$
$$B_2 \colon b \succ a \succ c,$$
$$B_3 \colon b \succ a \succ c.$$

   We have $F^{\text{ext}}(\boldsymbol{B}) = \{a,\ b\} \neq \{b\} = F^{\text{ext}}(\boldsymbol{R})$.

5. $t_2 > t_1 = 0$. Then, we have (by the ballot function):

$$B_1 \colon a \succ c \succ b,$$
$$B_2 \colon a \succ c \succ b,$$
$$B_3 \colon a \succ c \succ b.$$

   We have $F^{\text{ext}}(\boldsymbol{B}) = \{a,\ b\} \neq \{b\} = F^{\text{ext}}(\boldsymbol{R})$.

In no case, $F^{\text{ext}}(\boldsymbol{B}) = \{b\}$. This concludes the proof. □

Proposition 3. *Let F^{ext} and F^{int} be plurality. Then, for every Soc $\subseteq N$ and profile $\boldsymbol{R}$ we have that $F^{ext}(\boldsymbol{B}) \subseteq F^{ext}(\boldsymbol{R})$.*

Proof. Let $p_x(\boldsymbol{R})$ indicate the plurality score of candidate x in profile $\boldsymbol{R}$ and $top(R)$ the top-ranked candidate in R. We first show the following useful fact.

Fact 1. If F^{ext} and F^{int} are plurality and $i \in \text{Soc}$ then $top(B_i) \in F^{\text{ext}}(\boldsymbol{R})$.

To show this, fix a profile $\boldsymbol{R}$, a sociotropic voter $i \in \text{Soc}$, and let $t = top(R_i)$. There are two cases.

1. $F^{\text{ext}}(\boldsymbol{R}) \neq \{t\}$. Here, for all $x \in F^{\text{ext}}(\boldsymbol{R})$ and $y \in X \setminus F^{\text{ext}}(\boldsymbol{R})$ with $t \notin \{x,\ y\}$, we have

$$p_x(\boldsymbol{R}_{-i}) = p_x(\boldsymbol{R}) > p_y(\boldsymbol{R}) = p_y(\boldsymbol{R}_{-i}) \quad \text{and} \quad p_x(\boldsymbol{R}) \geq p_t(\boldsymbol{R}) > p_t(\boldsymbol{R}_{-i}).$$

   Thus, the plurality winners of $\boldsymbol{R}_{-i}$ are $F^{\text{ext}}(\boldsymbol{R}) \setminus \{t\}$. Hence, for all $\succ \in F^{\text{int}}(\boldsymbol{R}_{-i})$ we have $top(\succ) \in F^{\text{ext}}(\boldsymbol{R}) \setminus \{t\}$, which implies $top(B_i) \in F^{\text{ext}}(\boldsymbol{R})$.

2. $F^{\text{ext}}(\boldsymbol{R}) = \{t\}$. Here, for all $x \in X \setminus \{t\}$, we have

$$p_t(\boldsymbol{R}_{-i}) = p_t(\boldsymbol{R}) - 1 \geq p_x(\boldsymbol{R}) = p_x(\boldsymbol{R}_{-i}),$$

   where the inequality holds because t is the sole plurality winner in $\boldsymbol{R}$. Thus, $t \in F^{\text{ext}}(\boldsymbol{R}_{-i})$. By definition of $\mathcal{B}_i$, since $top(R_i) = t$, we get $top(B_i) = t \in F^{\text{ext}}(\boldsymbol{R})$.

This concludes the proof of Fact 1. Next, for a candidate $x \in X$ and profile $\boldsymbol{R}$, let

$$s_x(\boldsymbol{R}) = |\{i \in \text{Soc} \colon x = top(R_i)\}| \quad \text{and} \quad e_x(\boldsymbol{R}) = |\{i \in N \setminus \text{Soc} \colon x = top(R_i)\}|.$$

Clearly $p_x(\boldsymbol{R}) = s_x(\boldsymbol{R}) + e_x(\boldsymbol{R})$ and $e_x(\boldsymbol{R}) = e_x(\boldsymbol{B})$ for all x. By Fact 1,

$$\sum_{x \in F^{\text{ext}}(\boldsymbol{R})} s_x(\boldsymbol{B}) = |\text{Soc}| \geq \sum_{x \in F^{\text{ext}}(\boldsymbol{R})} s_x(\boldsymbol{R}).$$

Hence, there exists some $x^\star \in F^{\text{ext}}(\boldsymbol{R})$ such that $s_{x^\star}(\boldsymbol{B}) \geq s_{x^\star}(\boldsymbol{R})$. Crucially, this implies $p_{x^\star}(\boldsymbol{B}) \geq p_{x^\star}(\boldsymbol{R})$. For any $y \in X \setminus F^{\text{ext}}(\boldsymbol{R})$, we get

$$p_{x^\star}(\boldsymbol{B}) \geq p_{x^\star}(\boldsymbol{R}) > p_y(\boldsymbol{R}) \geq e_y(\boldsymbol{R}) = p_y(\boldsymbol{B}),$$

where the strict inequality follows because $x^\star$ is in $F^{\text{ext}}(\boldsymbol{R})$ while y is not, and the equality by Fact 1. Lastly, the above implies $y \notin F^{\text{ext}}(\boldsymbol{B})$ and, thus, $F^{\text{ext}}(\boldsymbol{B}) \subseteq F^{\text{ext}}(\boldsymbol{R})$. □

Proposition 5. *Let F^{ext} and F^{int} be neutral rules and let F^{int} also satisfy JR. Let $\mathcal{B}$ be* union. *Then, there exists a party-list profile $\boldsymbol{A}$ and a committee size k where all voters approve of at least k candidates such that $F^{ext}(\boldsymbol{B})$ does not satisfy JR with respect to the individual preferences in $\boldsymbol{A}$.*

Proof. Let $k = 3$ and suppose that two voters approve of $\{a,\ b,\ c\}$ and two of $\{d,\ e,\ f\}$. Since F^{int} satisfies JR, for any $i \in N$, all $C \in F^{\text{int}}(\boldsymbol{A}_{-i})$ must include one candidate from $\{a,\ b,\ c\}$ and one from $\{d,\ e,\ f\}$. By neutrality of F^{int} and by the *union* ballot function, this gives $B_i = \{a,\ \ldots,\ f\}$ for all $i \in N$. Thus, by neutrality of F^{ext}, there must be some $C \in F^{\text{ext}}(\boldsymbol{B})$ violating JR w.r.t. $\boldsymbol{A}$. □

Proposition 6. *Let F^{int} be a positional scoring rule. Then, there exists a profile $\boldsymbol{R}$ such that the Condorcet winner of $\boldsymbol{B}$ differs from the Condorcet winner of $\boldsymbol{R}$.*

Proof. Let $X = \{a_1,\ \ldots,\ a_m\}$ with $m \geq 3$. Let $\boldsymbol{\alpha} = (\alpha(1),\ \ldots,\ \alpha(m))$ (with $\alpha(1) > \alpha(m)$) be the score vector defining F^{int}. There are three cases.

1. $\alpha(1) = \alpha(2)$. Here, let ℓ indicate the largest number such that $\alpha(\ell) = \alpha(1)$. Consider profile $\boldsymbol{R}$:

$$\begin{aligned} R_1 &: a_1 \succ a_2 \succ \cdots \succ a_m, \\ R_2 &: a_1 \succ a_2 \succ \cdots \succ a_m, \\ R_3 &: a_2 \succ \underbrace{\cdots}_{\ell-1 \text{ canidates}} \succ a_1 \succ \cdots \succ a_m. \end{aligned}$$

   Where left unspecified, we assume any ordering of the candidates (it is not essential to the proof). Here, a_1 is the Condorcet winner. Consider voter 1. In profile $\boldsymbol{R}_{-1}$, candidates a_1 and a_2 receive $\alpha(1) + \alpha(\ell+1)$ and $\alpha(1) + \alpha(2)$ points, respectively. By assumption, $\alpha(1) = \alpha(2) > \alpha(\ell+1)$; hence, here a_2 receives a larger score than a_1. Moreover, $\alpha(1) + \alpha(2) = 2\alpha(1)$ is the highest score achievable in a two-voter profile. This, together with the fact that, for any candidate $x \in X \setminus \{a_1,\ a_2\}$ we have $a_2 \succ_1 x$, implies that a_2 is ranked first in B_1. A similar argument shows that a_2 is ranked first in B_2. Hence, a_2, and not a_1, is the strict Condorcet winner in $\boldsymbol{B}$.

2. $\alpha(1) > \alpha(2) = \alpha(3)$. Consider profile $\boldsymbol{R}$:

$$\begin{aligned} R_1 &: a_1 \succ a_2 \succ a_3 \succ \cdots \succ a_m, \\ R_2 &: a_1 \succ a_2 \succ a_3 \succ \cdots \succ a_m, \\ R_3 &: a_2 \succ a_3 \succ a_1 \succ \cdots \succ a_m, \\ R_4 &: a_3 \succ a_2 \succ a_1 \succ \cdots \succ a_m, \\ R_5 &: a_3 \succ a_2 \succ a_1 \succ \cdots \succ a_m. \end{aligned}$$

   Here, a_2 is the strict Condorcet winner. We show that in both B_1 and B_2 candidate a_3 is ranked first. We focus on voter 1 (for voter 2, we have an analogous argument). In $\boldsymbol{R}_{-1}$, candidate a_3

gets a score of $2\alpha(1) + \alpha(2) + \alpha(3)$. Alternative a_1, on the other hand, only gets a score of $\alpha(1)+3\alpha(3)$, and similarly a_2 receives a score of $\alpha(1)+3\alpha(2)$. By assumption, a_3 has the largest score of the three candidates; obviously also a greater score than any other candidate. Hence, a_3 is ranked first in B_1 and B_2. Furthermore, in subprofile $\boldsymbol{R}_{-3}$, we have that a_1 and a_3 have the same score, namely $2\alpha(1) + 2\alpha(3)$; this is strictly larger than the score of a_2, which is only $4\alpha(2)$. Again, any other candidate has a strictly smaller score. Hence, since $a_3 \succ_3 a_1$, we get that a_3 is ranked first in B_3. This is enough to show that a_3, and not a_2, is the strict Condorcet winner of $\boldsymbol{B}$.

3. $\alpha(1) > \alpha(2) > \alpha(3)$. Consider profile $\boldsymbol{R}$:

$$R_1\colon a_1 \succ a_2 \succ a_3 \succ \cdots \succ a_m,$$
$$R_2\colon a_1 \succ a_2 \succ a_3 \succ \cdots \succ a_m,$$
$$R_3\colon a_2 \succ a_3 \succ a_1 \succ \cdots \succ a_m.$$

In subprofile $\boldsymbol{R}_{-1}$, candidate a_2 receives a score of $\alpha(1) + \alpha(2)$. On the other hand, a_1 and a_3 receive a smaller score of $\alpha(1)+\alpha(3)$ and $\alpha(2)+\alpha(3)$, respectively. Any other candidate receive a score strictly lower than a_2. Hence, a_2 is ranked first in B_1. The same argument shows that a_2 is ranked first in B_2. Hence, a_2, and not a_1, is the strict Condorcet winner in $\boldsymbol{B}$.

This concludes the proof. □

Theorem 8. *Let F^{ext} be AV, F^{int} be MES without any completion method, and the ballot function be* lexicographic. *If $\boldsymbol{R}$ is a party-list profile and $n \geq k+1$, then $F^{ext}(\boldsymbol{B})$ satisfies JR.*

Proof. Let N' be a cohesive group in $\boldsymbol{R}$ of size $|N'| = n'$. By definition, this implies $k \cdot n'/n \geq 1$ and hence also

$$\frac{n'}{n-1}k \geq 1.$$

In particular, this implies for every voter $i \notin N'$ that in $\boldsymbol{R}_{-i}$ the voters in N' together receive a budget of $n' \cdot k/n{-}1 \geq 1$. As $\boldsymbol{R}$ is a party-list profile, the only way the voters can spend this budget is by buying a jointly approved candidate, which must exist by the cohesiveness of N'. Thus under $\boldsymbol{R}_{-i}$ we have that MES must return a committee that contains such a candidate. As we use the *lexicographic* ballot function, it must return the lexicographically minimal candidate $c_{\min}$ from $\bigcap_{j\in N'} A_j$. Thus $c_{\min}$ must receive at least $n-n'$ many approvals.

Assume now that we also have

$$\frac{n'-1}{n-1}k \geq 1.$$

Then, by the same argument as above, every voter in N' must also approve $c_{\min}$, which means that $c_{\min}$ is unanimously approved in $\boldsymbol{B}$. As under the *lexicographic* ballot function no voter can approve more than k candidates, there cannot be more than k unanimously approved candidates and $c_{\min}$ must be an AV winner under $\boldsymbol{B}$. Thus let us now assume that

$$\frac{n'-1}{n-1}k < 1. \tag{4}$$

We want to show that, in this case, there cannot be more than k candidates that receive at least as many approvals as $c_{\min}$, i.e., $n-n'$. For a candidate c to receive any approvals, there must be a large enough party $N^\star$ of size $n^\star$ supporting c. By the same argument as above, any candidate supported by party $N^\star$ that receives any votes is either unanimously supported or receives $n-n^\star$ many approvals. In particular, $\ell^\star$ candidates receive unanimous support if and only if

$$\frac{n^\star-1}{n-1}k \geq \ell^\star.$$

If $n^\star > n'$ then $n - n^\star < n - n'$ and these unanimously supported candidates are the only ones that receive as many votes as c_{min}. On the other hand, if $n^\star < n'$ then we know

$$\frac{n^\star}{n-1}k \leq \frac{n'-1}{n-1}k < 1.$$

Consequently, MES will never select a candidate supported by the voters in $N^\star$ for any $i \in N$ and $\boldsymbol{R}_{-i}$.

In summary, the candidates that receive at least as many approvals as c_{min} are either unanimously approved or are approved by exactly n' voters. Now let x be the number of candidates that are approved by exactly n' voters, which includes c_{min}. Additionally, let $N_1, \dots, N_p$ be the cohesive groups with strictly more than n' members. For each such N_j we let y_j be the number of unanimously supported candidates from that list and n_j the size of N_j. Then, let us compute

$$n \geq xn' + \sum_{j=1}^{p} n_j \geq x\frac{n}{k} + \sum_{j=1}^{p}\left(\frac{y_j(n-1)}{k} + 1\right) = x\frac{n}{k} + \frac{\sum_{j=1}^{p} y_j(n-1)}{k} + p = \frac{xn + \sum_{j=1}^{p} y_j(n-1)}{k} + p$$

Now, we have to distinguish two cases. If p is 0 then there no unanimously approved candidates and the equation above tells us that $n \geq n \cdot x/k$, which means there are at most k candidates that receive $n - n'$ many approvals. Hence, c_{min} is an AV winner under $\boldsymbol{B}$. Assume now that $p \geq 1$. Then we get the following:

$$n \geq \frac{xn + \sum_{j=1}^{p} y_j(n-1)}{k} + p \geq \frac{xn + \sum_{j=1}^{p} y_j(n-1)}{k} + 1 \geq \frac{x(n-1) + \sum_{j=1}^{p} y_j(n-1)}{k} + 1 = \frac{x + \sum_{j=1}^{p} y_j}{k}(n-1) + 1.$$

As $n > 1$, this implies $x + \Sigma_{j=1}^{p} y_j \leq k$ and thus we again know that at most k candidates receive at least as much support as c_{min}. □

Proposition 11. *Let F^{int} be simple Condorcet. Then, for every Soc $\subseteq N$ and profile $\boldsymbol{R}$ with CW($\boldsymbol{R}$) $\neq \varnothing$, we have that $\varnothing \subsetneq$ CW($\boldsymbol{B}$) $\subseteq$ CW($\boldsymbol{R}$).*

Proof. Fix a profile $\boldsymbol{R}$ with CW$(\boldsymbol{R}) \neq \varnothing$. The proof has the following steps. (1) First, we deal with the simple case where $|N|$ is odd. Then, assuming an even $|N|$, (2) we characterize how sociotropic voters update their ballots. With this, we show (3) CW$(\boldsymbol{B}) \subseteq$ CW$(\boldsymbol{R})$ and (4) CW$(\boldsymbol{B}) \neq \varnothing$.

Step 1. Assume $|N|$ is odd. Here, there cannot be ties in majority contests. Thus, there must be some (strict) Condorcet winner x. Via arguments analogous to those in the proof of Proposition 9, one shows that x is ranked first in every B_i with $i \in$ Soc. Hence, for all $y \in X \setminus \{x\}$, we have $n_{xy}^{\boldsymbol{B}} \geq n_{xy}^{\boldsymbol{R}}$, and thus CW$(\boldsymbol{B}) =$ CW$(\boldsymbol{R}) = \{x\}$.

Step 2. Assume $|N|$ is even, fix some $i \in$ Soc, and define x_i as the unique

$$x_i \in \text{CW}(\boldsymbol{R}) \text{ such that } y \succ_i x_i \text{ for all } y \in \text{CW}(\boldsymbol{R}) \setminus \{x_i\}.$$

We show that CW$(\boldsymbol{R}_{-i}) = \{x_i\}$. First, assume some $y \in$ CW$(\boldsymbol{R}) \setminus \{x_i\}$ exists. By definition of x_i, we have that $y \succ_i x_i$, and hence

$$n_{x_i y}^{\boldsymbol{R}_{-i}} = n_{x_i y}^{\boldsymbol{R}} = n_{y x_i}^{\boldsymbol{R}} > n_{y x_i}^{\boldsymbol{R}_{-i}}.$$

Here, the second equality follows from the fact that $x_i,\ y \in$ CW$(\boldsymbol{R})$. Therefore, in $\boldsymbol{R}_{-i}$, candidate y loses to x_i in the pairwise majority contest. Hence no such y (if any exists) can be in CW$(\boldsymbol{R}_{-i})$. Next, consider some $y \in X \setminus$ CW$(\boldsymbol{R})$. Assume towards a contradiction that $y \in$ CW$(\boldsymbol{R}_{-i})$. By definition of y, there must be some $x \in X \setminus \{y\}$ such that $n^{\boldsymbol{R}}_{xy} > n^{\boldsymbol{R}}_{yx}$, and, by assumption, we must have that $n^{\boldsymbol{R}_{-i}}_{xy} \leq n^{\boldsymbol{R}_{-i}}_{yx}$. Therefore, we must have $n^{\boldsymbol{R}}_{xy} - 1 = n^{\boldsymbol{R}}_{yx}$. However, since $|N| = n^{\boldsymbol{R}}_{xy} + n^{\boldsymbol{R}}_{yx}$, we derive $|N| = 2n^{\boldsymbol{R}}_{xy} - 1$, an odd number. Since $|N|$ is even, we reached the desired contradiction.

We have shown that $\mathrm{CW}(\boldsymbol{R}_{-i})$ is either $\varnothing$ or $\{x_i\}$. By definition of simple Condorcet, in the former case we have $R_i = B_i$. In the latter, B_i is obtained by ranking x_i first while leaving the relative order between any $y,\ z \in X \setminus \{x_i\}$ as in R_i.

Step 3. Consider any $x \in \mathrm{CW}(\boldsymbol{R})$ and $y,\ z \in X \setminus \mathrm{CW}(\boldsymbol{R})$. By the conclusion of Step 2, we know $n^{\boldsymbol{B}}_{xy} \geq n^{\boldsymbol{R}}_{xy}$ and $n^{\boldsymbol{B}}_{yz} = n^{\boldsymbol{R}}_{yz}$. This implies $\mathrm{CW}(\boldsymbol{B}) \subseteq \mathrm{CW}(\boldsymbol{R})$.

Step 4. It remains to be shown that $\mathrm{CW}(\boldsymbol{B}) \neq \varnothing$. For all $x \in X$, define

$$s(x) = |\{i \in N \colon B_i \neq R_i \text{ and } x \text{ is ranked first in } B_i\}|$$

and let $x^\star \in \arg\max_{x \in \mathrm{CW}(\boldsymbol{R})} s(x)$. We now show that $x^\star$ never loses any pairwise majority contest in $\boldsymbol{B}$, proving the claim. By the arguments in Step 3, we know that $x^\star$ cannot lose to any $y \in X \setminus \mathrm{CW}(\boldsymbol{R})$. Then, consider an arbitrary candidate $y \in \mathrm{CW}(\boldsymbol{R}) \setminus \{x^\star\}$. By the conclusion of Step 2, for all $i \in N$ and $x \in \mathrm{CW}(\boldsymbol{R})$, $R_i \neq B_i$ and x being ranked first in B_i implies that x is ranked last in R_i among the members of $\mathrm{CW}(\boldsymbol{R})$. Thus

$$n^{\boldsymbol{B}}_{x^\star y} = n^{\boldsymbol{R}}_{x^\star y} + s(x^\star) - s(y) \quad \text{and} \quad n^{\boldsymbol{B}}_{yx^\star} = n^{\boldsymbol{R}}_{yx^\star} + s(y) - s(x^\star).$$

By definition of $x^\star$, we know that $s(x^\star) - s(y) \geq 0$. Hence, $n^{\boldsymbol{B}}_{x^\star y} \geq n^{\boldsymbol{R}}_{x^\star y} = n^{\boldsymbol{R}}_{yx^\star} \geq n^{\boldsymbol{B}}_{yx^\star}$, where the equality follows from the fact that $x^\star,\ y \in \mathrm{CW}(\boldsymbol{R})$. Since y was chosen arbitrarily, $x^\star$ never loses a pairwise majority contest in $\boldsymbol{B}$. This concludes Step 4 and, with it, the proof. $\square$

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