Mixed Integer Programming Models for Hybrid Electric Vehicle Routing

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Christopher Bacher, Günther Raidl

bacher@ac.tuwien.ac.at

Institute of Computer Graphics and Algorithms
Vienna University of Technology

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Hybrid Electric Vehicles

- . . . combine Internal Combustion Engines w. Electric Machines
- . . . use different energy storage systems: fuel & batteries
- . . . are able to operate in different modes
- . . . different variants exists

Parallel Hybrid Electric Vehicles

Available operational modes:

- Combustion drive
- Pure electric drive
- Load Point Shifting
- Recuperation

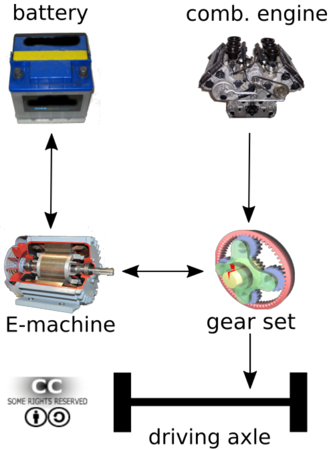

Figure: Structural HEV scheme

Tractive Power and Energy

Definition (Tractive Power)

To keep moving at a constant velocity v, a specific power level has to be reached by the engines at any point in time.

$$P_{\text{tract}} = \alpha m v + \beta v^3$$

Definition (Tractive Energy)

Moving a distance d at a constant velocity v requires a specific amount of energy.

$$\begin{aligned} E_{\text{tract}} &= \int (\alpha m v + \beta v^3) \delta \frac{d}{v} \\ &= \alpha m d + \beta v^2 d \end{aligned}$$

Model from (Barth, 2005), (Bektas, 2011)

Source Power and Energy

Difference between

- Tractive power and requested power (source power)
- Tractive energy and fuel energy equivalent (source energy)

due to

- Drive train efficiencies
- Combustion engine efficiencies
- Electric machine efficiencies
- Battery level
- Additional electr. consumers (heating, cooling, . . .)

Efficiency of Electric Machines

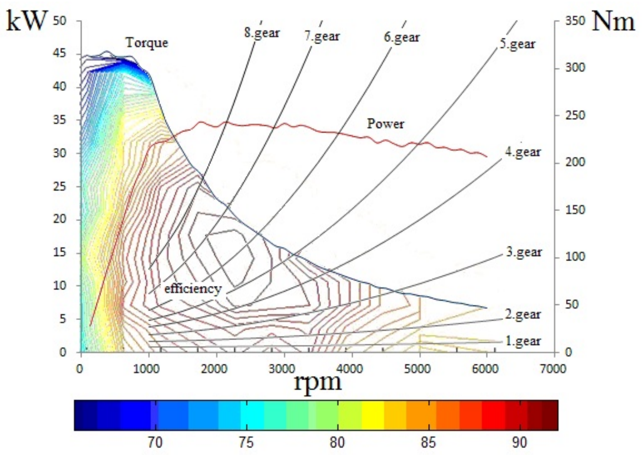

Figure: ICE efficiency map (Juraj, 2012, `posterus.sk/?p=12975`, accessed: June 2014)

Efficiency of Combustion Engines

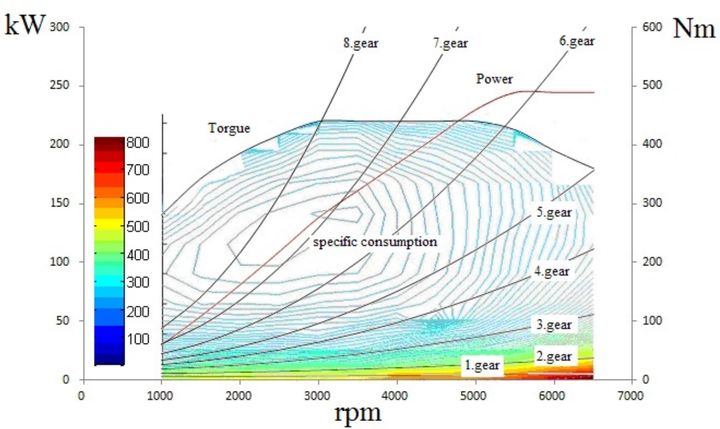

Figure: ICE efficiency map (Juraj, 2012, `posterus.sk/?p=12975`**, accessed: June 2014)**

Motivation

When minimizing transformed energy (fuel/charge), do routes for conventional vehicles and hybrid electric vehicles differ?

Related Work

- (Gong et. al, 2008),
  Trip-based Optimal Power Management of PHEVs
  $\rightarrow$ fixed route: optimal operation strategy (DP)
- (Bektas & Laporte, 2011),
  Pollution Routing Problem
  $\rightarrow$ VRP: MILP minimizes fuel consumption + driver costs
- (Erdogan & Miller-Hooks, 2012),
  A Green Vehicle Routing Problem
  $\rightarrow$ VRP: minimizing fuel consumption with refueling
- (Arslan et. al, 2014),
  Minimum Cost Path for PHEVs $\rightarrow$ Quadratic minimum cost path model
- ...

Considered modelling aspects

Model based on PRP (Bektas, 2011) & (Franceschetti, 2013)

The intended HEVRP model should cover

- Different constant velocities as mean velocities
- Cargo mass and its influence on power/energy requirements
- Different efficiencies of the machines
- Modes: ICE-only, Electric, Load-Point-Shift, Recuperation
- Arbitrary percental energy splits on each arc

In following the model is split into different modules

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Problem definition

Given a single depot $0 \in \mathcal{V}$

- customers $i \in \mathcal{V}$ with demand q_i (given in mass),
- distances d_{ij} and mean velocities $v_r \in \mathcal{R}_{ij}$ between customers,
- time windows $[t_i^b, t_i^e]$ and service times t_i^s of customers,
- curb weight m_c, cargo capacity Q (mass), and
- machine efficiencies $\eta^{ICE}, \eta^{EM}, \eta^{LS}$,

what is

1. the **minimal energy** equivalent of the fuel consumed,
2. the **energy split** achieving it?

Compact VRP Module

Models basic connections between nodes and selection of travel speeeds

$$\sum_{j \in \mathcal{V}_0} \boxed{a_{0j}} = K \qquad \sum_{j \in \mathcal{V}} \boxed{a_{j0}} = K \tag{1}$$

$$\sum_{j \in \mathcal{V}_i} \boxed{a_{ij}} = 1 \qquad \forall i \in \mathcal{V} \setminus 0 \tag{2}$$

$$\sum_{j \in \mathcal{V}_i} \boxed{a_{ji}} = 1 \qquad \forall i \in \mathcal{V} \setminus 0 \tag{3}$$

$$\sum_{r \in \mathcal{R}_i j} \boxed{z_{ijr}} = \boxed{a_{ij}} \qquad \forall i, j \in \mathcal{V} \tag{4}$$

$$\text{Arc (i,j) is chosen} \quad \boxed{a_{ij}} \in \{0, 1\} \qquad \forall i, j \in \mathcal{V} \tag{5}$$

$$\text{Vel. } v_r \text{ on (i,j)} \quad \boxed{z_{ijr}} \in \{0, 1\} \qquad \forall i, j \in \mathcal{V}, r \in \mathcal{R}_{ij} \tag{6}$$

Time Window Module

Models time window restriction of customers

$$\sum_{j \in \mathcal{V}_i} \left(\boxed{t_{ji}^D} + \sum_{r \in \mathcal{R}_{ij}} \frac{d_{ij}}{v_r} \boxed{z_{jir}} \right) \leq \boxed{t_i^A} \qquad \forall i \in V \qquad (7)$$

$$\boxed{t_i^A} + t_i^s \leq \sum_{j \in \mathcal{V}_i} \boxed{t_{ij}^D} \qquad \forall i \in V \setminus 0 \qquad (8)$$

$$(t_i^b + t_i^s) \boxed{a_{ij}} \leq \boxed{t_{ij}^D} \leq (t_i^e + t_i^s) \boxed{a_{ij}} \qquad \forall i \in V \qquad (9)$$

$$\text{Arrival time at i} \quad \boxed{t_i^A} \in [0, t_i^e] \qquad \forall i \in V \qquad (10)$$

$$\text{Departure time for (i,j)} \quad \boxed{t_{ij}^D} \in \mathbb{R}^+ \qquad \forall i \in V \qquad (11)$$

Mass Flow Module

Models a flow of mass on a route

$$(m_c + q_j)\boxed{a_{ij}} \leq \boxed{m_{ij}} \leq (Q - q_i)\boxed{a_{ij}} \qquad \forall i,j \in \mathcal{V} \qquad (12)$$

$$\sum_{j \in \mathcal{V}_i} \boxed{m_{ji}} - \sum_{j \in \mathcal{V}_i} \boxed{m_{ij}} = q_i \qquad \forall i,j \in \mathcal{V} \qquad (13)$$

$$\text{Mass transported on (i,j)} \quad \boxed{m_{ij}} \in \mathbb{R}^+ \qquad (14)$$

Battery Flow Module (I)

Models battery consumption and replenishment

$$\text{Battery level on (i,j)} \qquad \forall i,j \in \mathcal{V} \qquad (15)$$

$$0 \leq \boxed{b_{ij}} \leq B \boxed{a_{ij}}$$

$$\text{Source energy (Electric Mode) on (i,j)} \qquad \forall i,j \in \mathcal{V} \qquad (16)$$

$$0 \leq \boxed{e_{ij}^{EM}} \leq \boxed{b_{ij}} + \boxed{e_{ij}^{LSc}}$$

$$\text{Source energy (Load Shift charge) on (i,j)} \qquad \forall i,j \in \mathcal{V} \qquad (17)$$

$$0 \leq \boxed{e_{ij}^{LSc}} \leq E^{max} \boxed{a_{ij}}$$

$$\text{Source energy (Recuperation) on (i,j)} \qquad \forall i,j \in \mathcal{V} \qquad (18)$$

$$0 \leq \boxed{e_{ij}^{R}} \leq B \boxed{a_{ij}}$$

Battery Flow Module (II)

Models battery consumption and replenishment

$$\sum_{j \in \mathcal{V}_0} \boxed{b_{0j}} = BK \qquad (19)$$

$$\sum_{j \in \mathcal{V}_i} \left(\boxed{b_{ji}} - \boxed{e_{ji}^{\mathrm{EM}}} + \boxed{e_{ji}^{\mathrm{LS}c}} + \boxed{e_{ji}^{\mathrm{R}}} \right) = \sum_{j \in \mathcal{V}_i} \boxed{b_{ij}} \qquad \forall i, j \in \mathcal{V} \qquad (20)$$

$$\boxed{e_{ij}^{\mathrm{R}}} \leq \eta^R \frac{1}{2} \sum_{r \in \mathcal{R}_{ij}} v_r \boxed{m_{ijr}} \qquad \forall i, j \in \mathcal{V} \qquad (21)$$

$$0 \leq \boxed{m_{ijr}} \leq \boxed{m_{ij}} \qquad \forall i, j \in \mathcal{V}, r \in \mathcal{R}_{ij} \qquad (22)$$

$$0 \leq \boxed{m_{ijr}} \leq Q \boxed{z_{ijr}} \qquad \forall i, j \in \mathcal{V}, r \in \mathcal{R}_{ij} \qquad (23)$$

Energy Split Module (I)

Models objective and energy split

$$\min \sum_{i,j \in \mathcal{V}} \left(\boxed{e_{ij}^{ICE}} + \boxed{e_{ij}^{LS_D}} + \boxed{e_{ij}^{LS_C}} \right) \tag{24}$$

$$\boxed{E_{ij}} = \alpha \boxed{m_{ij}} d_{ij} + \beta \sum_{r \in \mathcal{R}_{ij}} d_{ij} v_r \boxed{z_{ijr}} \qquad \forall i,j \in \mathcal{V} \tag{25}$$

$$\boxed{E_{ij}} = \boxed{E_{ij}^{EM}} + \boxed{E_{ij}^{ICE}} + \boxed{E_{ij}^{LS_D}} \qquad \forall i,j \in \mathcal{V} \tag{26}$$

$$\text{Tract. energy (EM)} \quad 0 \leq \boxed{E_{ij}^{EM}} \leq E^{max} \tag{27}$$

$$\text{Tract. energy (ICE)} \quad 0 \leq \boxed{E_{ij}^{ICE}} \leq E^{max} \tag{28}$$

$$\text{Load Shift Drive part} \quad 0 \leq \boxed{E_{ij}^{LS_D}} \leq E^{max} \tag{29}$$

$$\text{Load Shift Charge part} \quad 0 \leq \boxed{E_{ij}^{LS_D}} \leq E^{max} \tag{30}$$

Energy Split Module (II)

Models relation to source energies

$$\boxed{E_{ij}^{\mathrm{EM}}} = \boxed{e_{ij}^{\mathrm{EM}}}\eta^{\mathrm{EM}} \qquad \forall i,j \in \mathcal{V} \tag{31}$$

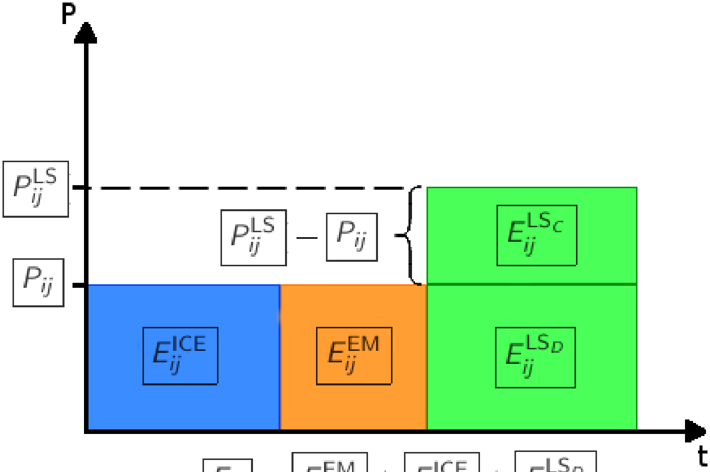
$$\boxed{E_{ij}^{\mathrm{ICE}}} = \boxed{e_{ij}^{\mathrm{ICE}}}\eta^{\mathrm{ICE}}(\boxed{P_{ij}}, \boxed{z_{ijr}}) \qquad \forall i,j \in \mathcal{V} \tag{32}$$

$$\boxed{E_{ij}^{\mathrm{LS}_D}} = \boxed{e_{ij}^{\mathrm{LS}_D}}\eta^{\mathrm{LS}}(\boxed{P_{ij}^{\mathrm{LS}}}) \qquad \forall i,j \in \mathcal{V} \tag{33}$$

$$\boxed{E_{ij}^{\mathrm{LS}_C}} = \boxed{e_{ij}^{\mathrm{LS}_C}}\eta^{\mathrm{LS}}(\boxed{P_{ij}^{\mathrm{LS}}}) \qquad \forall i,j \in \mathcal{V} \tag{34}$$

$$\frac{\boxed{E_{ij}^{\mathrm{LS}_C}}}{\boxed{P_{ij}^{\mathrm{LS}}} - \boxed{P_{ij}}} = \frac{\boxed{E_{ij}^{\mathrm{LS}_D}}}{\boxed{P_{ij}}} \tag{35}$$

Energy Split Module (III)



$$E_{ij} = E_{ij}^{EM} + E_{ij}^{ICE} + E_{ij}^{LS_D}$$

Figure: Graphical representation of the energy split for a single arc

Energy Split Module (III)

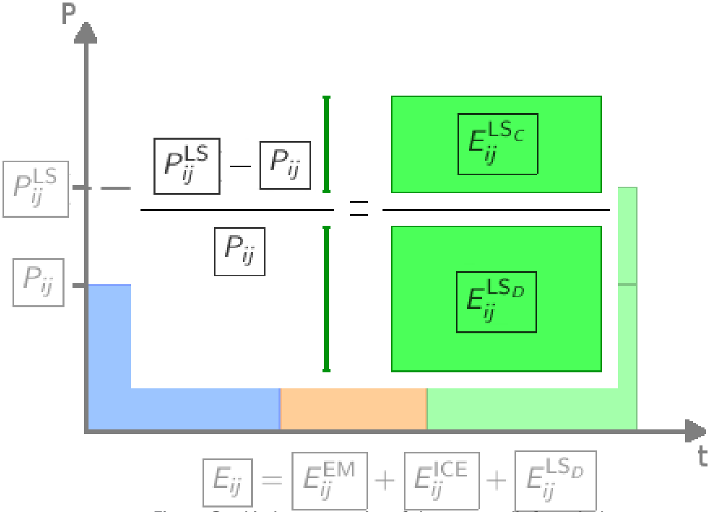


Figure: Graphical representation of the energy split for a single arc

Overview of Solution Approaches

- Constraint Programming (CP)
- Extension of CP-based Column Generation for VRPs (Gualandi, 2013)
- Branch-and-Price-and-Cut: theoretical model, likely infeasible in practice, able to treat all of the above constraints
- **Relaxing the model**
- **(Generalized) Benders Decomposition** (work in progress)

Mass Energy Relaxation (I)

- Do not consider cargo mass for energy calculation
- Constant efficiencies
- Fixed P^{LS_C}

Replace energy module by ...

$$\boxed{E_{ij}} = \alpha m_c d_{ij} \boxed{z_{ijr}} + \beta \sum_{r \in \mathcal{R}_{ij}} d_{ij} v_r \boxed{z_{ijr}} \qquad \forall i,j \in \mathcal{V} \qquad (36)$$

$$\boxed{E_{ij}} = \boxed{E_{ij}^{\mathrm{EM}}} + \boxed{E_{ij}^{\mathrm{ICE}}} + \boxed{E_{ij}^{\mathrm{LS}_D}} \qquad \forall i,j \in \mathcal{V} \qquad (37)$$

$$(38)$$

Mass Energy Relaxation (II)

...and the following

$$0 \leq \boxed{y_{ijr}} \leq \boxed{z_{ijr}} \tag{39}$$

$$\boxed{E^{\text{ICE}}_{ij}} + \boxed{E^{\text{EM}}_{ij}} = \alpha m_c d_{ij} \sum_{r \in \mathcal{R}_{ij}} \boxed{y_{ijr}} + \beta \sum_{r \in \mathcal{R}_{ij}} d_{ij} v_r \boxed{y_{ijr}} \qquad \forall i,j \in \mathcal{V} \tag{40}$$

$$\boxed{E^{\text{LS}_D}_{ij}} = \alpha m_c d_{ij} \sum_{r \in \mathcal{R}_{ij}} (1 - \boxed{y_{ijr}}) + \beta \sum_{r \in \mathcal{R}_{ij}} d_{ij} v_r (1 - \boxed{y_{ijr}}) \qquad \forall i,j \in \mathcal{V} \tag{41}$$

$$\boxed{E^{\text{LS}_C}_{ij}} = E^{\max}(1 - \boxed{y_{ijr}}) - \boxed{E^{\text{LS}_D}_{ij}} \qquad \forall i,j \in \mathcal{V} \tag{42}$$

Towards a (Generalized) Benders Decomposition (I)

Benders Decompostion splits the problem into

- a master problem (MP): selects routes and velocities
- several linear subproblems (SPs): energy split calculation

by fixing complicating variables (arc and velocity selection).

Master Problem

$$\begin{aligned} \min z & \\ f(y) + \bar{u}_i(a - g(y)) \le z & \qquad \forall i \text{ SP feasible} \\ \bar{u}_j(a - g(y)) \le 0 & \qquad \forall j \text{ SP infeasible} \\ y \in \mathcal{Y} & \end{aligned}$$

Towards a (Generalized) Benders Decomposition (II)

Sub Problem

$$\min cx$$
$$Ax \geq a - g(\bar{y})$$
$$x \in \mathbb{R}$$

Problem: Load Point Shifting constraint is not $g(\bar{y})$ but $g(x, \bar{y})$
$\rightarrow$ would introduce a non-linear cut!

Towards a (Generalized) Benders Decomposition (II)

Sub Problem

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Generalized Benders Decomposition

(Geoffrion, 1972) extends Benders approach to non-linear functions

Master Problem

$$\begin{aligned} & \min z \\ \underset{x \in \mathbb{R}}{\text{infimum}} \{ f(x,y) + \bar{u}_i g(x,y) \} & \leq z \qquad \forall i \text{ SP feasible} \\ \underset{x \in \mathbb{R}}{\text{infimum}} \{ \bar{u}_j (a - g(x,y)) \} & \leq 0 \qquad \forall j \text{ SP infeasible} \\ y & \in \mathcal{Y} \end{aligned}$$

Generalized Benders Decomposition

Definition (Property P, (Geoffrion, 1972))

For any u the infimum has to be taken independently of y.

This is not possible for our problem.

We currently try to

- Separate the cut into two expressions:
- Expr. I: Property P holds
- Expr. II: Property P does not hold

$\rightarrow$ Try to relax Expr. II below the infimum
if the y values differ from those generating the cut.

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Conclusion & Future work

We have . . .

- Discussed a HEVRP model for parallel HEVs
- Seen the problems in modeling specfic characteristics of HEVs
- Shown some approaches trying to circumvent these problems

In future work we . . .

- Try to extend the Generalized Benders Decomposition
- Investigate further relaxations of the model
- Mean-velocity vs mean-energy models
- Integrate routing with vehicle design

Thank you for your attention!